

Nash Equilibrium

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I. Introduction

In game theory, the Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. If each player has chosen a strategy and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitutes a Nash equilibrium. The reality of the Nash equilibrium of a game can be tested using experimental economics method.

The cooperative solution of game theory, which focused on the coalition of players, had been the most popular method of solving the game. Since the inception of the game theory, a cooperative solution was considered to be the superior concept. However, in recent times, the Nash equilibrium- the non-cooperative solution par excellence- has been the dominant solution. Textbooks and publications now consider Nash equilibrium to be more important and give short shrift (or no shrift at all) to the cooperative side.

Definition

Let (S, f) be a game with n players, where S_i is the strategy set for player i , $S = S_1 \times S_2 \times \dots \times S_n$ is the set of strategy profiles and $f = (f_1(x), \dots, f_n(x))$ is the payoff function for $x \in S$. Let x_i be a strategy profile of player i and x_{-i} be a strategy profile of all players except for player i . When each player $i \in \{1, \dots, n\}$ chooses strategy x_i resulting in strategy profile $x = (x_1, \dots, x_n)$ then player i obtains payoff $f_i(x)$. Note that the payoff depends on the strategy profile chosen, i.e., on the strategy chosen by player i as well as the strategies chosen by all the other players. A strategy profile $x^* \in S$ is a Nash equilibrium (NE) if no unilateral deviation in strategy by any single player is profitable for that player, that is

$$\forall i, x_i \in S_i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*).$$

When the inequality above holds strictly (with $>$ instead of \geq) for all players and all feasible alternative strategies, then the equilibrium is classified as a *strict Nash equilibrium*. If instead, for some player, there is exact equality between x_i^* and some other strategy in the set S , then the equilibrium is classified as a *weak Nash equilibrium*.

II. Collusion

One of the most important concepts related to Nash Equilibrium is Collusion. Collusion is a non-competitive agreement between rivals that attempts to disrupt the market's equilibrium. By collaborating with each other, rival firms look to alter the price of a good to their advantage. The parties may collectively choose to restrict the supply of a good, and/or agree to increase its price in order to maximize profits. Groups may also collude by sharing private information, allowing them to benefit from insider knowledge.

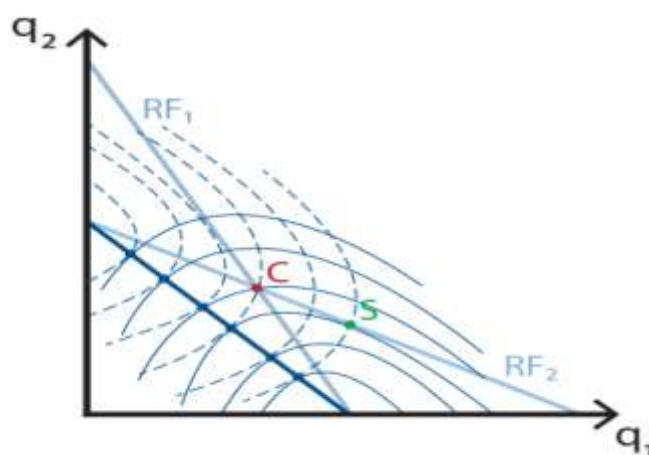
Collusion makes allusion to the cooperation between different firms. This cooperation leads to a restraint of market competition, in any of its forms, which translates into higher profits for the firms in detriment of consumer's welfare. A cartel is an example of firms belonging to the same industry structure which collude to some degree in setting prices and/or output levels. Agreements which have as their object or effect the prevention, restriction or distortion of perfect competition are prohibited. Such agreements include, but are not restricted to, activities such as:

- fixing purchase or selling prices or any other trading condition, directly or indirectly;
- controlling or limiting production levels, markets, technological advances or investments;
- sharing markets or resources supplies.

Legislation in different countries may consider different scenarios and penalties for such agreements, but the main idea is clear: firms behaviour shall not affect the correct functioning of market forces.

A clear example is to consider an industry where there are only two firms (duopoly). Both firms will set their levels of output and prices with the objective of maximizing their joint profits. Many strategies can be

used in order to maximize profits which would lead to a multiple Nash equilibria solution. As seen in the following figure, collusion maximises aggregate profits for both firms, since the isoprofit curves are tangent. It's a better equilibrium than the one in Cournot duopolies or Stackelberg duopolies.



However, cartels are not stable. There will always be incentives for each firm to trick the other, and change their output and/or price level in such a way, that they'll increase their own profit in detriment of the other's. To avoid this practice, any deviation by any party should be instantly punished; this is known as a trigger strategy. James W. Friedman demonstrated in his paper "A Non-cooperative Equilibrium for Supergames", 1971, that in this context of infinite interactions, it is possible that collusion occurs due to this punishment strategy. That is, the cartel may endure as long as punishment strategies are so devastating that the benefits derived from deviation would end up being smaller than the benefits of keep colluding. J. W. Friedman put this idea in what is known in game theory as Folk theorem.

Folk Theorem

The sustainability of the equation will depend mainly in two factors: the credibility of the threat of punishment, and the discount factor. The former is easily understood as a credible threat will ensure no deviations are made, and the latter is related with how much does each party value the profits obtained from the results of following a collusive strategy, compared to the possible profits of changing their strategy.

Factors that guarantee collusion stability:

There are number of factors that affect this collusive equilibrium, such as:

- Number of firms in the market: the higher the degree of concentration in a market the higher the incentives to collude. Firms in highly concentrated markets will tend to collude since all the profits will be distributed amongst fewer firms.
- Multimarket contact: if firms compete in more than one market, the collusive agreement will be more stable. Firms that compete with other firms over many markets can establish trigger strategies that can be applied in all these markets, which will create a more devastating punishment strategy.
- Market transparency: the more transparent a market is, the easier it is to ensure that every firm is following the same strategy and is not deviating from the deal. Collusion will be more difficult in industries where it is harder to detect changes in firm's prices or output.
- Asymmetry between firms: the bigger the asymmetry between firms, the harder it is for collusion to take place. If firms have different cost structures, the one with the lowest costs will be incentivised to lower its prices, and thus cause the other firm to have to exit the market.

Collusion Game

In game theory, collusion agreements can be described using the extensive form, as depicted in the adjacent game tree. In this case, two firms share the market, already colluding and maintaining high prices. Each firm can decide to stop colluding and start a price war, in order to increase their market share, even force the other to quit the market. Firm 1 can either keep colluding with firm 2, or start a price war. If firm 1 decides to keep colluding, firm 2 will need to make a decision. If they both agree to collude, they will get 5,5. However, if one of them decides to start a price war, the set of payoffs will be either 4,3 or 3,4, depending on which one starts the war (and therefore acquires a greater market share). It's easy to see that collude-collude is both the

Nash equilibrium and a Pareto optimum situation. This result may change when considering repeated games, as seen before.

Apple And Samsung



A threatening patent suit in the middle of Apple and Samsung began since the time that Apple blamed Samsung for replicating its outlines for cell phones and tablet PCs. Accordingly, Samsung avoided back with patent claims concerning the versatile innovation. As indicated by CNET News, this prosecution tumult increased into 50 claims against one another in 10 different nations. Apple got to be careful about Samsung's continually expanding piece of the pie of the cell phones and tablet PCs.

A cell phone or a tablet PC business sector can be said to be an oligopoly. There is just a modest bunch of firms offering the item: Apple, Samsung, HTC, Sony, etc. It is without a doubt not like the PC parts market where there are loads of firms giving the indistinguishable item.

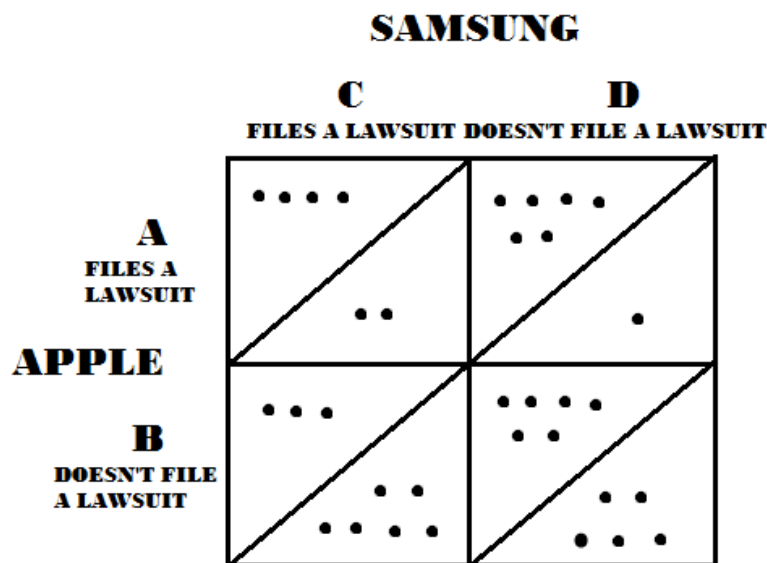
As Samsung's business sector force is expanding in both the cell phone and the tablet PC market, Apple has opened a Pandora's Box by documenting a claim against Samsung, as it was said above. This has set off the issue of Prisoner's Dilemma, of which the "players" in a "game" are compelled to pick the choice that exacerbates those two off. For this situation, the "players" are Apple and Samsung, and the "game" they are playing is the chicken session of patent cases.

The war between Apple and Samsung is well understood and here for some time now. It is fascinating to watch that both the organizations face prisoner's dilemma when they wish to make a move against the other in their patent war. As you read further, you would see the Nash Equilibrium and Nash Solution for the Patent war.

The circumstance Apple and Samsung are currently confronting can be delineated as a Duopolistic market, when "just two or three firms give a considerable measure of the yield" In this kind of business sectors, firms are very worried about how its opponent is responding. This interest is one of the drivers behind not only the 'Apple and Samsung Patent war', but also many other industrial fights that have taken place.

Choice making in these circumstances requires key thinking and suggests that organizations consider all other choices. On the same lines as the Prisoner's Dilemma, each of the two organizations would need to choose whether to document a claim against the other. Expecting that an organization that documents a claim additionally wins it against its rival, the rival would need to either permit the innovation or put intensely in R&D for exchange advancements. This would push up the cost of its items bringing about a drop in benefits.

The following would be results of every organization's individual decision on regardless of whether to document a claim against the other:



The dots represent the profits made by a firm. The higher the number of dots – the greater the profits made.

Case I

On the off chance that Apple records a suit against Samsung – given Apple's activity, Samsung's benefits augment if it documents a suit accordingly.

This is bolstered by incidents that have already taken place -Samsung needed to pay \$ 1 Billion in harms to Apple. The expense brought about on its items would increment in future due to authorizing/R&D. Likewise there would be a movement in clients from Samsung to Apple because of loss of reputation. (This discloses more noteworthy benefits to Apple when Samsung does not react by documenting a suit)

Case II

On the off chance that Apple does not record a suit against Samsung – given Apple's activity Samsung's benefits expand on the off chance that it documents a suit accordingly.

As we see : Whether or not Apple records a suit, Samsung documents a suit in response. Along these lines, recording a suit is Samsung's Dominant procedure.

Case III

In the event that Samsung records a suit against Apple – given Samsung's activity Apple's benefits expand in the event that it documents a suit accordingly.

Case IV

In the event that Samsung does not record a suit against Apple – given Samsung's activity Apple's benefits expand in the event that it documents a suit accordingly.

As we see : Whether or not Samsung documents a suit, Apple records a suit in response. Along these lines, documenting a suit is Samsung's Dominant system.

The Nash Equilibrium is for both organizations to record a suit. Given the adjustments, it is dependably in light of a legitimate concern for every firm to build yield and make their activities non-agreeable

The Nash solution is for both organizations to connive and withdraw from recording a suit along these lines and expanding their joint benefits.

Apple's technique is not to contend on expense but rather on separation. The organization needs to ensure its item's uniqueness, securing its capacities of positional point of preference through licenses that cover Design and Operating System highlights. Licenses are confinements that shield Apple's advancements from being replicated. They give the firm an extraordinary business sector power furthermore the first mover point of interest, the consequence of spearheading certain business sector fragments.

From a plan of action point of view Apple's key decision is Premium Price Positioning which is upheld by a high Willingness to Pay. Apple needs to forestall impersonation of its resources that drive the most elevated Reservation Price for clients, along these lines supporting it's R&D expenses and a higher upper hand than Samsung.

They achieve a Nash harmony, in which them two record a claim against one another, aggravating them off. The patent claim can be seen as a deadweight misfortune that may be "squandered" in a belligerent procedure. A few individuals contend that the main individuals picking up from this circumstance are the legal counsellors. Shoppers are a definitive casualties of this patent war on the grounds that the ligation weights are gone through higher costs for the items Apple and Samsung produce.

In any case, it ought to be noticed that this "game" of patent claims is rehashed variously, 50 claims as it was specified. Meritz Investment Bank's investigator Lee Secheol expected in April that Apple and Samsung would stop and accommodate with one another as the "diversion" is rehashed. He expected that both firms would understand that this circumstance is aggravating them off and that they would take a seat at the arranging table.

As indicated by CNET News, CEOs of Apple and Samsung did have a meeting. Then again, they have never thought of an understanding. The way that they had a meeting to accommodate demonstrated that the two acknowledged they were in a circumstance of prisoner's dilemma. On the other hand, their difference over pulling back from a patent war additionally demonstrated that this issue has turned out to be to some degree emotional, which makes it past the issue of prisoner's dilemma.

Advantages Of Nash Equilibrium

Nash Equilibrium is one of the most important concepts in economics. It has a lot of benefits, not just in economics but also in other fields where rational decision making has to take place.

This theory is the most effective when the game at hand is the outcome of mechanism design. The mechanism design, also called the reverse game theory, is the engineering part of the theory, which involves starting with a particular goal and then finding a pathway to achieve it. In this way the game is not given; it is chosen.

Nash Equilibrium also helps us to make predictions that are scientifically rejected. It is one of the few topics in economics that goes beyond pure explanations and takes into consideration many factors that might be indirectly influencing the equilibrium.

Nash Equilibrium possess the attributes of being simple enough to compute solutions for games while at the same time being complex enough to make it difficult to program into a computer.

Nash equilibrium also provides the optimum value or solution for a given problem that will be the most sustainable in the long run. It aims at achieving efficiency and the least amount of damage to either of the players. Thus it always aims at reaching the Pareto optimum level, but it may not always be successful.

Shortcomings Of Nash Equilibrium

First, many games have multiple equilibria, and players may not be clear about which one to focus on. If the players can communicate with each other before the game is played, they may be able to select equilibrium through negotiation (that is why Nash equilibrium is sometimes referred to as a “self-enforcing agreement”). But negotiation does not always suffice to resolve multiplicity. Consider, for example, the game of Table 1 from the earlier example, in which the options with Apple and Samsung are to file a lawsuit or to not file a lawsuit. Players may attempt to negotiate the outcome (A, C), which Pareto dominates as the equilibrium. Thus, Apple will announce that it plans to file a lawsuit and Samsung that it also plans to file a lawsuit. Notice, however, that these professions may not be credible. In particular, regardless of what it does on its own, Apple is better off if Samsung files a lawsuit. But Samsung will do so only if he thinks there is a sufficiently high

		SAMSUNG	
		C FILES A LAWSUIT	D DOESNT FILE A LAWSUIT
APPLE	A FILES A LAWSUIT	9,9	0,8
	B DOESNT FILE A LAWSUIT	8,0	7,7

Table 1

Probability that Apple will also file a lawsuit. Thus, Apple has the incentive to say that it intends to file a lawsuit regardless of whether that is actually true. Moreover, because A is a risky strategy for Apple, it might well play B if it is not very confident that Samsung will play C. In other words, Apple’s announcement that it will play A is not really believable, and neither is the announcement C by Samsung. Negotiation between the two players may, therefore, not accomplish much. Of course, even without communication, multiple equilibria do not always cause a problem. Consider, for example, the game of Table 2. There are two equilibria:

		SAMSUNG	
		C FILES A LAWSUIT	D DOESNT FILE A LAWSUIT
APPLE	A FILES A LAWSUIT	9,9	0,0
	B DOESNT FILE A LAWSUIT	0,0	1,1

Table 2

(A, C) and (B, D) (plus a mixed strategy). However, (U, L) stands out as obviously superior. Unfortunately, not all games have one particular equilibrium toward which players will naturally gravitate. For instance, in the game of Table 3—the classic Battle of the Sexes—the equilibria

		SAMSUNG		
		C	D	
APPLE		A FILES A LAWSUIT	2,1	0,0
		B DOESNT FILE A LAWSUIT	0,0	1,2

Table 3

(A, C) and (B, D) are exactly symmetric, and so there is no obvious criterion that would direct players to one equilibrium rather than the other.

III. Conclusion

Nash Equilibrium has been one of the most important and controversial topics in economics. However, it is a very theoretical concept and has a very few applications in the real world. It is easy to formulate examples that involve the application of this theory but it is a difficult task to be able to apply this theory to a practical situation.

In this paper, I have taken up the task of finding out the Nash equilibrium and Nash solution in the Prisoner’s Dilemma between Apple and Samsung on filing or not filing a lawsuit against each other. By applying the game theory, I concluded that the Nash Equilibrium will be reached when both the companies file a law suit whereas the Nash Solution is for both the companies to not file a lawsuit.

Nash Equilibrium is a concept that has been successful in many fields and has helped economists, politicians, bureaucrats, and business strategists understand the world around us in a better way. It has been able to show how the activities of one company can affect the entire industry in the most unbelievable way possible. However, just like a coin has two faces, even the Nash equilibrium has a good and a bad side. Thus there are a few disadvantages of having or reaching an equilibrium condition through this process. But all in all the discovery of Nash Equilibrium has changed the face of economics and of price fixing in many industries in today’s time.

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