

Contribution of Leelavathi to Prosody

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I. Introduction

Leelavathi, a treatise on Mathematics, is written by Bhaskara II who lived in 12th century A.D. Besides explaining the details of mathematical concepts that were in existence up to that period, the text introduces some new mathematical concepts. This paper is an attempt to analyze the metres employed in Leelavathi as well as the concepts of permutation and combination introduced by the author in the same text.

Key words:-Combination, Leelavathi, Meruprasthara, Metre, Permutation,

II. Metres In The Text Leelavathi

In poetry, metre has a significant role to contribute the emotive aspect. The spontaneous out pore of emotion always happens through a suitable metre that is revealed in the mind of the poet at the time of literary creation. Rhythm itself is the life of the metre as it transfuses the emotion. Varied compositions of diversified rhythms which are innumerable give birth to different metres in poetry. Early poeticians like Bhamaha, Dandin, Vamana, Rudrada and Rajasekhara have stated that erudition in prosody is essential for making poetical composition.

In Vedic period, the skill of Vedic Rishis in handling the language and metre for expressing their ideas is also equally attractive. The metres used are well suited to the types of poetry, the ideas expressed in them and the content exposed. At the time of Natyasastra, Bharata who has a good insight on the relation between the rhythmic scheme of metre and sentiment, says -

Chando heeno na shabdosthi na chandasshabdavarjithamⁱ

The three terms, Chandas, Vrittam and Jati are seen profusely used in Sanskrit Prosody. Among them, Chandas and Vrittam, though they are used as synonyms, have separate meanings in the context of Prosody. Halayudha, while commenting on the aphorism ' Vritham' says that the classical metre is called Vritham as it existed in the Vedic metres like Gayathri.

Here it is implied that though so many metres or vritha-s can be derived from each Chandas, those which are pleasing to hear by the laws of Guru-s and Laghu-s only can be actually considered as Vrithas.

Preethim bhakthajanasya yo janayathe vignam vinidnan smrithaha
Tham vrindarakavindavanditha padam natva mathamgananam
Pateem sadganithasya vachmi chathurapreethipradam prasphutam
samkshipthaksharakomalamalapadairlalithyaleelavatheemⁱⁱ

In the last line of the above mentioned benedictory verse, the author reveals that Leelavathi is a composition made concise and simple with charming words. In this verse, the 19 syllabled metre Sardulavikreedita is used. The beautiful rhythm of this metre is flexible and adaptable to depict wide varieties of emotions. Kshemendra in Suvrithathilaka says that Sardulavikreedita is highly suitable for solemn compositions like prayers and mystical meditations and it is also very much effective in depicting heroism.

Shouryasthavo nripatheenam shadurlavikreeditham mathamⁱⁱⁱ

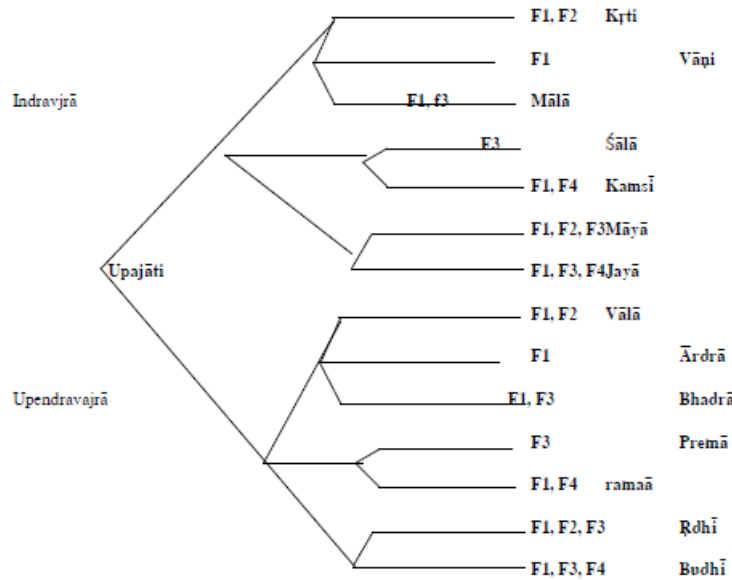
And harinapluta is effective to express terrific mood, praharshini has much ability to attract the minds of lovers .In the benedictory verse of the first sub-chapter, the metre employed is Anushtubh.

Leelagalalulatola kalavyala vilasine

Ganeshaya namo neelakamalamala kanthaye^{iv}

In Sanskrit literary world, it is generally accepted that the metre Anushtubh is highly suitable for beginning Sargabandha as well as abridging long stories. Kshemendra is of the view that Anushtubh is also useful for advising on the matters relating to shama (s.t.III, 16).It might be because of this quality that Acharya made use of Anushtubh for Karanasuthra-s. Besides Anushtubh, the author has employed various other metres like Upendravajra and Upajati all of which are included in the eleven syllabled Chandas, Thrishtubh. Among them, Upajati is evolved from the combination of Indravajra (- - U / - - U / U - U / - -) and Upendravajra (U - U / - - U / U - U / - -). From the early period itself prosodists have made rigorous work to find out the number of various patterns and all the possible occurrences have been listed. By changing the positions of Indravajra and Upendravajra in a four lined verse, Pingala has accepted 14 possible patterns of Upajati metre and also given them names.

Feasible varieties of Upajati Metre



1. Illustrations

- (1) For example, Karanasutra I, which begins with the verse- Varaatakaanam dashakadvayam yath. The Upajati metre is Rama. Here, the 1st and 4th lines are in Upendravajra
- (2) In Karanasutra 14 , which is beginning with the verse dvidha bhaveth roopavibhag eva. the Upajati is Ardra .Here the first line only is in Upendravajra .
- (3) In Karanasutra 15 , where the first verse is bhajyadharaha shudhayathi yath gunaha syath. the metre is Prema. Here in the 3rd line only is in Upendravajra.
- (4) In the 4th example which starts with the verse- moolam chathurnam cha tatha navanam. the metre is In prema.

Thus, the author has employed various types of Upajati-s for Karanasutra-s and example verses.

The verses cited as examples reveal the poetic skill of the author. The metre Sragdhara which is included in Prakrti group and which is composed of seven trisyllabic Gana-s is profusely used by the author. In medieval age it was accepted as one of the foremost among the longer metres. Katyayana, one of the earliest authors on Prosody, has stated that this metre is much appropriate to depict Veera.

Veerasya bhujadandaanam varnane sragdhara matha

The example verses given in leelavathi reveal the multi faceted. Poetic genius of Bhaskaracharya. Saardulavikreeditha which is used in benedictory verse is seen employed profusely in the text.

Paartha karnnavadhaya margaganunam krudho rane sandadhe
 Tasyardvdhvena nivarya taccharaganunam moolaschathurbhihi hayaan
 Salyam shadbhiratheshubhihi thribhirapi chathram dhvajam kaarmukam
 Chichedasya shiraha sarena kathi the yaanarjunaha sandadhe^v

Here the author portraits the terrific scenes from battle field and the sentiment, Veera is enhanced in this description. The rhythm and melody are closely related. Example verses 21, 24, 36, 38, 44, 45.... are in sardulavikreeditha. Another metre employed by the poet is Sikharini.besides the metres saardulavikriditha, sikharini, anushtubh and Upajathi, the poet has employed the metres sikharini malini, drutavilambitha, vasantathilaka etc.

Thus the poetic imagination and scholarship in sastra goes hand in hand in this Sastrakavya.

Atha eva yamakanuprasyoriva vrithasyapi sabdalankaratvamabhyupagathamasmabhihi^{vi}

Mahimabhata has realised the rhythmic nature and potentiality of metre in communicating the emotive element in poetry. So he includes metre among the sabdalankara-s like Yamaka and Anuprasa.

The above concept advocated by Mahimabhata is consistent with the compositions in Leelavathi where the Sabdalankara-s like Yamaka and Anuprasa go in hand with all metres. These rhythmic compositions make the learners more interested in further studies.

2. Permutation

In Sanskrit, the word Prastara is used for permutation. Prastara means strewing or spreading out prasthanam vithanatham^{vii}. In music and musical instruments, the prastara-s are used in the elucidation of rhythm. In Prosody, this is a mathematical calculation that declares the possible number of metres in a Chandas. Pingala and Bharata have discussed two types of prastara-s namely Matragata and Varnagata. Most of the later writers described only the second type. The process of Prastara is as follows – write all the syllables in a quarter line according to metrical scale that is to be made the subject of Prastara, in Guru-s. This is the first Prastara of that metrical scale with 'Sarvaguru' (-----). Then write the first line, changing only the first Guru into Laghu (U -----). Now second prastara is formed. The process may be followed in the same manner filling up the left side of the changed Laghu with Guru-s. The right side should be the same as in the line just above. This continues until all the syllables in the line become Laghu-s. The process ends here and all the possible patterns in that metrical scale become explicit.

For example

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- - - - -
      U - - - - -
      - U - - - - -
      U U - - - - -
      - - U - - - - -
      U - U - - - - -
      - U U - - - - -
      U U U - - - - -
    
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In this way, the eight gana-s are the permutation of 3 syllables (- - -, U - -, U U -, U - U, U U U, - U U, - - U, - U -).

In Leelavathi, mathematical concepts related to metres also have been discussed. In the 10th sub chapter entitled 'Chandashcityadi', the author has introduced mathematical formula for calculating the permutations of metres. In the 57th and 58th the Karanasutra –

padaksharamithagache gunavargaphalam chaye dvgune
samavritthanam samkhya tadvarge vargavargascha\sva sva padononou syathamardha samanam cha
vishamanaam.

The author states how to arrive at the possible number of permutation in sama, ardhhasama and vishama metres. The number of syllables in a line is named as Gaccha padaksharamitha gache. For example: 8 is the Gaccha in the eight syllabled Anustubh. Then the number of permutations in this metre is 2⁸. Here, '2' denotes the basic constitute i.e., Guru and Laghu and the 8 denotes the number of syllables or Gaccha. If the Gaccha of a sama metre is 'n', the number of permutation is 2ⁿ, i.e., 256. Thus, in an 8 syllabled Ardhhasama metre, the permutation is 2²ⁿ – 2ⁿ i.e., 2¹⁶ – 2⁸ = 65280. The permutation of Visama metre is 2⁴ⁿ – 2²ⁿ i.e., 2³² – 2¹⁶ = 4294901760. Thus different arrangement or ordered set of a metre is called the permutation of that metre. Thus the total number of permutations of 'n' objects taken 'r' at a time is denoted by ⁿp_r or p(n, r)

3. Combination

In the Karanasutra-s 59, 60, and 61 are related to 'combination'. The author narrates how to find out the number of metres on the basis of the number of Guru-s from permutation.

Ekadyekouthara anka vyavastha bhajyaha kramsthithai
Paraha poorvane samgunyaha tatparasthena tena cha
Ekdvithriyadi bhedaha syuhu idam sadharanam smritham
Chndaschithyutrhare chandasyupayoge asya tadvidadam
Musha vahanabhedadou khandmerou cha silpakae
Vaidyake rasabhedeye tannoktham visthritharbhayath^{viii}

From the above verse, Acharya says that for choosing 'r' out of 'n' number of items, subtract 1, 2, 3 and divide with 1, 2, 3 Respectively.

$$\text{Then } {}^n C_r = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$$

$$= \frac{n!}{(n-r)!r!}$$

i.e., in eight syllabled Anustubh –

$$\frac{n}{1}, \left(\frac{n-1}{2}\right), \left(\frac{n-2}{3}\right) \dots \left(\frac{n-7}{8}\right)$$

By multiplying the items we get

$$\frac{n(n-1)(n-2)\dots(n-7)}{1 \times 2 \times 3 \times \dots \times 8}$$

Then, if there is guru, the number of metres = $\frac{8}{1} = 8$

$$\text{For 2 Guru } ({}^8C_2) = \frac{8 \times 7}{2} = 28$$

$$\text{For 3 Guru } ({}^8C_3) = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

$$\text{For 4 Guru } ({}^8C_4) = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 70$$

$$\text{For 5 Guru } ({}^8C_5) = ({}^8C_3) = 45$$

$$\text{For 6 Guru } ({}^8C_6) = ({}^8C_2) = 28$$

$$\text{For 7 Guru } ({}^8C_7) = ({}^8C_1) = 8$$

$$\text{For 8 Guru } ({}^8C_8) = 1$$

With the sum of combinations, add that is for one pattern with all short syllables. Thus, we get total number $2^8 = 256$ permutations of Anustubh metre.

To illustrate combination theory, Acharya takes the specimen Gayatri metre as example.

Prasthara mitra gayathryaha

Suhu pade vyakthayaha kathi

Ekadi gurvath chashu

Kathyatham that prithak prithak.

O friend, there are six letters in one line of Gayatri metre. If we choose only one Guru, 2 Gurus, or 3 Gurus, how many metres are possible? The answer is 6, 15 and 20 respectively. Here, the 'n' is 6. Then the total permutation is 2^n i.e., $2^6 = 64$. If we take the combinations of all the lines the total permutation is 16777216 i.e., $64 \times 64 \times 64 \times 64$.

Bhaskaracharya gives two another questions to illustrate the combination theory.

Ekadvithryadi musha vahanamithi maho bruhi me bhumirbharthu-

Rhamyaramye ashtamushe chathuravirachithe slshnasaalavishaale

Ekadvithryadi yuktha madhukathukashaymlaka ksharthikthai

Ekasmine shdrasahi syurganaka, kathi vada vyanjane vyakthi bhedaha^{ix}

A king had a beautiful palace with eight doors skilled engineers had constructed four open squares which were highly polished and huge in order to get fresh air, 1 door, 2 doors, 3 doors are opened. How many different types of breeze arrangements are possible?

How many kinds of relishes can be made by using 1,2,3,4,5 or 6 types from the sweet, bitter, astringent, soar, salty, hot substances.

As per the permutations of Anushtubh, we get the total number, $2^8 = 256$ from $({}^8C_0), ({}^8C_1), ({}^8C_2) \dots ({}^8C_8)$. These are 1, 8, 28, 56, 70, 56, 28, 8, and 1 respectively. The number of different relishes = $1+8+28+56+70+56+28+8+1 = 64 = 2^6$

4. Meruprastara

In Leelavathi, there are indications about 'Khandameruprastara' in Karanasutra 61. Since the graphic representation of the permutation of one syllables Ukta to 26 syllabled Utkrthi metres has a slight resemblance to Meru (mountain), this pattern is known as Meruprastara. The last sutra of the 8th chapter of Chandassastra pare purnamithi^x. Pingala introduces Meruprastara. Commenting on this aphorism Halayudha gives more details on the structural elements. tasya prathame koshte yadvrithasamkhyajatham tat poovakoshtayoho poornam niveshayeth. Tathrobhayoho koshkayorekaikamankam dadyat, madhye koshte tu parakoshtdvayankamekikrithya poornam niveshayedithi poornasabdarthaha. Chaturthyam panktavapi paryanthakoshtakayorekaikameva sthapayeth/madhyamakoshtyosthu parakoshtdvayankamekikrithya poornam thrisamkhyaroopam sthapayeth. Utharathrapyayameva nyasaha.

As per the view of Pingala the permutation of the metres Ukta to Utkrthi is as follows

Ukta - 2

Atyukta - 4

Madhya - 8

Pratistka - 16

Supratista - 32

Gayatri - 64

Usnik - 128

Anushtubh - 256

This construction is known as Meruprastara. From the sum of each line we get the actual permutation of metres. If a syllable in a line is increased the number of prastara will be doubled. Thus the nine syllabled Brhati will have 512 Prastara-s. This idea has stated by Pingala in the aphorism pare purnam.^{xi} Commenting on this Halayudha says- tatsamkhyajatam dvigunitam parasya chandaso vrithanam samkhyā bhavati. Tatyatha-chathushshthirgayathri samavrithanam samkhyā dvigunikritha sathi parasyoshnishta samavrithasamkhyā sashstavimshasatham bhavathi.

The Meruprastara which is used in Prosody is now known as Pascal's triangle in modern mathematics.

$(x+y)^0$	0								
$(x+y)^1$	1	1							
$(x+y)^2$	1	2	1						
$(x+y)^3$	1	3	3	1					
$(x+y)^4$	1	4	6	4	1				
$(x+y)^5$	1	5	10	10	5	1			
$(x+y)^6$	1	6	15	20	15	6	1		
$(x+y)^7$	1	7	21	35	35	21	7	1	
$(x+y)^8$	1	8	28	56	70	56	28	8	1

For example :

$$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

The binomial expansion of Meruprastara is as follows:

$$\begin{aligned} (x+y)^0 &= 1 \\ (x+y)^1 &= x + y \\ (x+y)^2 &= x^2 + 2xy + y^2 \\ (x+y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 \\ (x+y)^4 &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \\ (x+y)^5 &= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 \\ (x+y)^6 &= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6 \\ (x+y)^7 &= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7 \\ (x+y)^8 &= x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8 \end{aligned}$$

III. Conclusion

By referring to the mathematical concepts, put forth by Pingala, Bhaskaracharya pays tribute to the preceptors. The syntagm of words seen in Karanasutra-s and example verses amply reveal the poetic genius and erudition of the author. The rhythmic harmony of the syllables employed in the metres helps to communicate the emotive elements marvelously. The presence of Sabdalankara-s like Anuprasa and Yamaka enhances the melodious texture of the Leelavathi. Bhaskaracarya's deep knowledge on metrical and mathematical concepts like Prastara, Samayojita and Meruprastara makes the text Leelavathi unique in the field of Mathematics. There is no doubt that the text Leelavathi is a good read to scholarly people as well as ordinary learners.

ⁱ Natyasatra , XIV, 47

ⁱⁱ Mangala sloka of leelavathi

ⁱⁱⁱ Natyasatra XIV, 68, 84

^{iv} Leelavathi prathamakhanda,1

^v Leelavathi example verse 250

^{vi} Vyakthiviveka, p.191

^{vii} N.S., Abhinavabharati, XIV, commentary on V.112.

^{viii} Karanasutra, 59,60

^{ix} Karanasutra, 59,60,61

^x Chandassatra , VIII , 35

^{xi} Ibid, VIII, 34

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