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Numerical Solutions of Fingero-Imbibition in a Slightly Dipping Porous Media Involving Magnetic Fluid

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Abstract: *The present paper numerically discusses the phenomenon of fingero-imbibition in a double phase displacement process through slightly dipping porous medium with involvement of a layer of magnetic fluid in the injected phase. This phenomenon is assumed to occur in a double-phase displacement process involving two immiscible liquids of small viscosity difference in which the injection is initiated by imbibitions and the consequential displacement of the relatively more viscous native liquid produces fingers. The basic equations of the flow system coupled with analytical consideration for additional physical effects yields a nonlinear partial differential equation whose numerical solution has been obtained by reduced differential transform method.*
Keywords: *fingero-Imbibition phenomenon, Homogeneous porous medium, RDTM*

I. Introduction

The present paper deals with the phenomena of Fingero-Imbibition in double phase flow through porous homogeneous, cracked and slightly dipping porous media involving magnetic fluid. This phenomenon arises on account of simultaneous occurrence of two important phenomena imbibition and fingering. We have assumed that injection of preferentially wetting, less viscous fluid into porous medium saturated with resident fluid is initiated under imbibition and in consequence, the resident fluid is pushed by drive in secondary recovery process. Verma called these conjoint phenomena as Fingero-Imbibition. The phenomena of fingering and imbibition occurring simultaneously in displacement process, have gained much current importance due to their frequent occurrence in the problem of petroleum technology and many authors have discussed them from different point of view. In this paper, the underlying assumptions are that the two fluids are immiscible and the injected fluid is less viscous as well as preferentially wetting with respect to porous materials and with capillary pressure. The mathematical formulation of basic equations yields a Non-linear partial differential equation governing with fingero-imbibition in the investigated liquid-liquid displacement problem. A numerical solution is obtained by Successive over Relaxation Method.

Statement of the Problem:

We consider here a finite cylindrical mass of porous medium of length L(=1) saturated with native liquid (o) , is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibition face (x=0) and this end is exposed to an adjacent formation of ‘injected’ liquid (w) which involves a thin layer of suitable magnetic fluid. It is assumed that the later fluid is preferentially wetting and less viscous. This arrangement gives rise to a displacement process in which the injection of the fluid (w) is initiated by imbibition and the consequent displacement of native liquid (o) produces protuberances (fingers). This arrangement describes a one – dimensional phenomenon of Fingero-Imbibition. The cylindrical of porous matrix is inclined at small angle θ with horizontal only and remaining process of Fingero-Imbibition goes as it is.

Formulation of the Problem:

Assuming that the flow of two immiscible phases is governed by Darcy’s law, we may write the seepage velocity of injected and native fluid as,

$$V_w = -\left(\frac{K_w}{\delta_w}\right) K \left[\frac{\partial P_w}{\partial w} + \gamma H \frac{\partial H}{\partial x} + \rho_w g \sin\theta\right] \quad (1)$$

$$V_o = -\left(\frac{K_o}{\delta_o}\right) K \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin\theta\right] \quad (2)$$

Where

$$\gamma = \mu_o\lambda + \frac{16\pi\mu_o\lambda^2 r^3}{g(t+2)^3}$$

K = The permeability of the homogeneous medium
K_w = Relative permeability of injected fluid, which is function of S_w
K_o = Relative permeability of injected fluid, which is function of S_o
S_w = The saturation of injected fluid
S_o = The saturation of native fluid
P_w = Pressure of injected fluid
P_o = Pressure of native fluid
g = Acceleration due to gravity.

Where the phase densities ρ_w & ρ_o are regarded as constant, δ_w and δ_o are assumed to be invariant to magnetic field, and θ is inclination.
Neglecting the variation in phase densities, the equation of continuity for injected fluid can be written as:

$$p\left(\frac{\partial S_w}{\partial t}\right) + \left(\frac{\partial V_w}{\partial x}\right) = 0 \quad (3)$$

Where p is porosity of the medium.
From the definition of phase saturation it is obvious that $S_w + S_o = 1$. The analytical condition (Scheidegger, 1960) governing imbibition phenomenon is

$$V_w + V_o = 0 \quad (4)$$

From the definition of capillary pressure P_c as the pressure discontinuity between two phases yields

$$P_c = P_o - P_w \quad (5)$$

On Simplifying Equation (1)&(2) by using Equation (4) & (5) we obtain

$$\frac{\partial P_o}{\partial t} = \left[\frac{-K_w\delta_o\left[\gamma H\frac{\partial H}{\partial x} + \rho_w g\sin\theta\right] - K_o\delta_w\left[\frac{\partial P_o}{\partial x} + \rho_o g\sin\theta\right]}{\left(\frac{K_o}{\delta_o} + \frac{K_w}{\delta_w}\right)}\right]\left(\frac{\partial P_c}{\partial x}\right) \quad (6)$$

Substituting above equation into equation (1), we obtain

$$V_o = \frac{KK_oK_w}{(K_o/\delta_o + K_w/\delta_w)}\left[\frac{\partial P_c}{\partial x} - \gamma H\frac{\partial H}{\partial x} + (\rho_o - \rho_w)g\sin\theta\right] \quad (7)$$

Since $\frac{K_oK_w}{(K_o/\delta_o + K_w/\delta_w)} \approx \frac{K_o}{\delta_o}$, Above equation reduces to the form

$$V_o = \frac{KK_o}{\delta_o}\left[\frac{\partial P_c}{\partial x} - \gamma H\frac{\partial H}{\partial x} + (\rho_o - \rho_w)g\sin\theta\right] \quad (8)$$

Substituting equation (8) into (3) we get,

$$p\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial}{\partial x}\left(\frac{KK_o}{\delta_o}\left[\frac{\partial P_c}{\partial x} - \gamma H\frac{\partial H}{\partial x} + (\rho_o - \rho_w)g\sin\theta\right]\right) = 0 \quad (9)$$

At this state, for definiteness of the mathematical analysis, we assume standard relationship due to Scheidegger and Johnson [1], Muskat [2], between phase saturation and relative permeability as

$$K_w = S_w, S_o = 1 - S_w \text{ and } P_c = -\beta_o S_w$$

Where β_o is capillary pressure coefficient. Considering the magnetic fluid H in the x-direction only, we may write [3], $H = \frac{\wedge}{x^n}$ where $\wedge$ is a constant parameter and n is an integer. Using the value of H for $n = -1$. Substituting all values in equation (9), we get,

$$p\left(\frac{\partial S_w}{\partial t}\right) + \frac{K}{\delta_o}\frac{\partial}{\partial x}\left((1 - S_w)\left[-\beta_o\frac{\partial S_w}{\partial x} - \gamma\wedge^2 x + (\rho_o - \rho_w)g\sin\theta\right]\right) = 0 \quad (10)$$

A set of suitable initial and boundary conditions associated to equation (10) are

$$S_w(x,0) = S_1 \ for\ all\ x > 0 \quad (11)$$

$$S_w(0,t) = S_{w_0}; \quad S_w(L,t) = S_{w_1} \quad for\ all\ t \geq 0 \quad (12)$$

Equation (10) is reduced to dimensionless form by setting

$$X = \frac{x}{L}, \quad T = \frac{t}{L^2(C_1/C_2)}, S_w(x,t) = 1 - S_w(x,t)$$

So that

$$\frac{\partial S_w}{\partial T} - \frac{\partial}{\partial x}\left(S_w\frac{\partial S_w}{\partial x}\right) + C_0\frac{\partial}{\partial x}(S_w x) - C_1\left(\frac{\partial s_w}{\partial x}\right) = 0 \quad (13)$$

Where $C_0 = \frac{\alpha\wedge^2 L^2}{\beta_0}$, $C_1 = \frac{L}{\beta_0}(\rho_o - \rho_w)g\ \sin\theta$. Asterisks are dropped for simplicity.
With auxiliary

$$S_w(x,0) = 1 - S_1 \quad for\ all\ x > 0 \quad (13(a))$$

$$S_w(0,t) = 1 - S_{w_0} \qquad for\ all\ t \geq 0 \qquad (13(b))$$
$$S_w(L,t) = 1 - S_{w_1} \qquad for\ all\ t \geq 0 \qquad (13(c))$$

Equation (13) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium with effect of magnetic fluid.

The problem is solved by using Differential Transform method. The numerical values are shown by table. Curves indicate the behavior of saturation of water corresponding to various time periods.

II. Solution Using Rdtm Method

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial x}\left(S_w \frac{\partial S_w}{\partial x}\right) - C_0 \frac{\partial}{\partial x}(S_w x) + C_1\left(\frac{\partial S_w}{\partial x}\right) \qquad (13)$$
$$\therefore (S_w)_T = (S_w(S_w)_x)_x - C_0(S_w x)_x + C_1(S_w)_x$$

Taking the initial condition $S_w(x,0) = S_{w0} = f(x)$

$$f(x) = \frac{e^x - 1}{e - 1} \qquad (14)$$

The problem is solved by reduced differential transform method because our equation is non-linear partial differential equation.

Reduced differential Transform Method

The Basic definition of RDTM is given below

If the function u(x,t) is analytic and differential continuously with respect to time t and space x in the domain of interest then let

$$U_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]$$

Where the t-dimensional spectrum function $U_k(x)$ is the transformed function . u(x,t) represent transformed function. The differential inverse transform of $U_k(x)$ is defined as follow

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k$$
$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]t^k$$

Apply RDTM on (13)

$$(k+1)S_{w(k+1)}(x) = \left[\sum_{r=0}^{k}(S_{wr})\left(S_{w(k-r)}\right)_{xx}\right] + \ [(S_{wk})_x)^2] \ - C_0\left([(S_{wk})_x(kx)] + S_{wk}(x)\right) + C_1(S_{wk})_x \qquad (15)$$

Now let $k = 0$ & $C_0 = 1, C_1 = 1$ then put initial condition (14) into eq. (15), So we get $S_{w1}(x)$ as following

$$(1)S_{w1}(x) = [(S_{w0})(S_{w0})_{xx}] + [((S_{w0})_x)^2] - \left[[(S_{w0})_x(0)x] + S_{w0}(x)\right] + ((S_{w0})_x$$
$$S_{w1}(x) = \left(\left(\frac{e^x-1}{e-1}\right)\left(\frac{e^x}{e-1}\right)\right) + \left(\left(\frac{e^x}{e-1}\right)^2\right) - \left(0 + \left(\frac{e^x-1}{e-1}\right) + \left(\frac{e^x}{e-1}\right)\right)$$
$$S_{w1}(x) = \frac{2e^{2x} - e^x + e - 1}{(e-1)^2}$$

For second iteration let $k = 1$

$$(k+1)S_{w(k+1)}(x) = \left[\sum_{r=0}^{1}(S_{wr})\left(S_{w(k-r)}\right)_{xx}\right] + [(S_{wk})_x)^2] \ - \left([(S_{wk})_x(kx)] + S_{wk}(x)\right) + \ (S_{wk})_x$$
$$(2)S_{w2}(x) = \left[(S_{w0})\left(S_{w(1-0)}\right)_{xx} + (S_{w1})\left(S_{w(1-1)}\right)_{xx}\right] + [(S_{w1})_x)^2] \ - \left([(S_{w1})_x(1.x)] + S_{w1}(x)\right) + (S_{w1})_x$$
$$(2)S_{w2}(x) = \left(\left(\frac{e^x-1}{e-1}\right)\left(\frac{8e^{2x}-e^x}{(e-1)^2}\right) + \left(\frac{2e^{2x}-e^x+e-1}{(e-1)^2}\right)\left(\frac{e^x}{e-1}\right)\right) + \left(\frac{4e^{2x}-e^x}{(e-1)^2}\right)^2$$
$$- \left[\left(\frac{4e^{2x}-e^x}{(e-1)^2}\right)x + \left(\frac{2e^{2x}-e^x+e-1}{(e-1)^2}\right)\right] + \left(\frac{4e^{2x}-e^x}{(e-1)^2}\right)$$

$$= \frac{10e^{3x+1}-(14-8x)e^{2x+1}-(2-4x)e^{2(x+1)}-18e^{3x}+(13-4x)e^{2x}-(1+2x)e^{x+1}+xe^{x}+16e^{4x}+(x+1)e^{x+2}-e^{3}+3e^{2}-3e+1}{(e-1)^{4}}$$

In this way we can generated other polynomials by putting different values in equation (18)
Now by inverse transform

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k$$

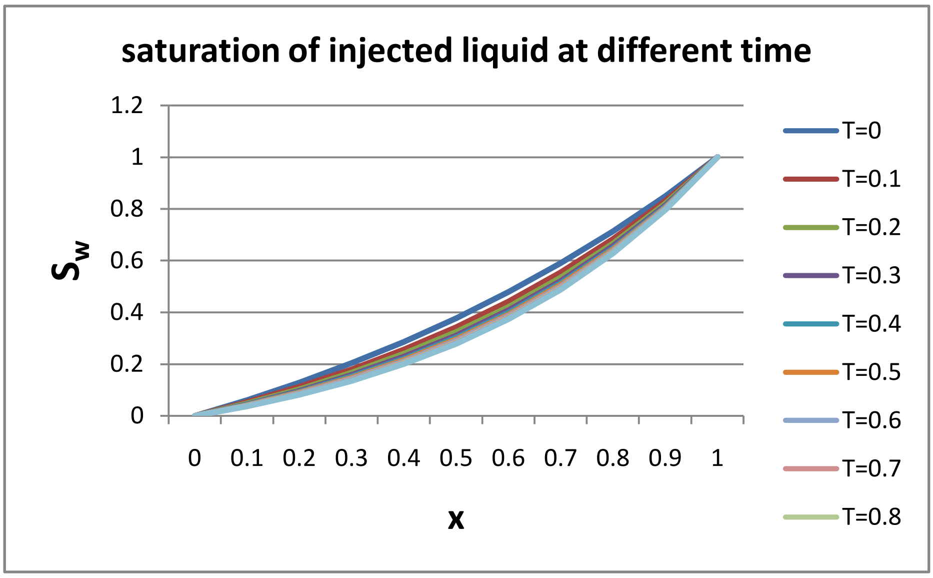
$$S_w(x,T) = S_{w0}(x)T^0 + S_{w1}(x)T^1 + S_{w2}(x)T^2 + \cdots$$

$$S_w(x,t) = \frac{e^x-1}{e-1}T^0 + \frac{2e^{2x}-e^x+e-1}{(e-1)^2}T^1 + \frac{10e^{3x+1}-(14-8x)e^{2x+1}-(2-4x)e^{2(x+1)}-(1+2x)e^{x+1}+(x+1)e^{x+2}}{(e-1)^4}\frac{T^2}{2} + \frac{16e^{4x}-18e^{3x}+(13-4x)e^{2x}+xe^{x}-e^{3}+3e^{2}-3e+1}{(e-1)^4}\frac{T^2}{2}\cdots$$

III. Table And Figure

The following table shows the approximate solution for saturation of injected liquid for different values of x at different time using RDTM

X	T=0	T=0.1	T=0.2	T=0.3	T=0.4	T=0.5	T=0.6	T=0.7	T=0.8	T=0.9	T=1
0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.0612	0.0528	0.048	0.0449	0.0428	0.0412	0.04	0.039	0.0382	0.0375	0.037
0.2	0.1289	0.1125	0.1032	0.0971	0.0929	0.0898	0.0874	0.0855	0.0839	0.0826	0.0815
0.3	0.2036	0.18	0.1665	0.1578	0.1517	0.1472	0.1437	0.141	0.1387	0.1368	0.1353
0.4	0.2862	0.2565	0.2395	0.2285	0.2208	0.2151	0.2107	0.2073	0.2044	0.2021	0.2001
0.5	0.3775	0.3433	0.3238	0.3111	0.3023	0.2958	0.2907	0.2867	0.2834	0.2807	0.2785
0.6	0.4785	0.4421	0.4214	0.4079	0.3986	0.3916	0.3862	0.382	0.3785	0.3757	0.3733
0.7	0.59	0.5548	0.5346	0.5216	0.5125	0.5058	0.5006	0.4965	0.4931	0.4903	0.488
0.8	0.7132	0.6835	0.6664	0.6554	0.6477	0.6421	0.6377	0.6342	0.6313	0.629	0.627
0.9	0.8495	0.8308	0.8202	0.8133	0.8085	0.805	0.8022	0.8001	0.7982	0.7968	0.7956
1	1	1	1	1	1	1	1	1	1	1	1



IV. Conclusion

In the graph X-axis represents the values of x and Y-axis represents the saturation of injected liquid involving magnetic fluid sw in porous media of length one. It is clear from graph that, at particular time, saturation of injected liquid involving magnetic fluid decrease with increase in value of x (or as we move ahead) and at x=1, saturation is decreased to zero and as time increases, rate of increase of the saturation of injected liquid decreases at each layer.

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