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Three Off-Steps Hybrid Method for Numerical Solution of Third Order Initial Value Problems.

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Abstract: *We developed three off-stephybrid methods for $k = 3$in block formation, which is of order nine by interpolation and collocation technique, using Legendre polynomials as our basis function of the approximation with constant step-size. The method was investigated and found to be convergent. The method was tested on third order numerical examples and compared with the existing methods;the superiority of our method overthe existing methods is established numerically.*
Keywords: *Interpolation, Collocation, Legendre Polynomials, Hybrid Method, Convergent.*

I. Introduction

Numerical methods are important tools for approximate solution of differential equations since the advent of computer. This paper consider an approximate method for the solution of general third order Initial Value Problems *(IVPs)* of the form

$$y''' = f(x, y, y', y''), \quad y^{(i)}(x_0) = y_0{}^i \quad i = 0,1,2 \ldots \quad (1)$$

Direct methods for the solution of higher order Ordinary Differential Equation *(ODEs)* has been established in literatures to be better than the method of reducing in terms of approximation, time of execution and cost of implementation ([4], [11]). The Linear Multistep Methods *(LMMs)* are usually applied to the *IVPs* as single formula but they are not self-starting and they advance the numerical integration of *ODEs* in one-step which lead to the overlapping the piecewise polynomials solution models. Thus there is need to develop a numerical method which is self-starting, eliminating the used of predictor with better accuracy and efficiency.

Scholars later developed block method which cater for some of the setbacks of the predictor corrector methods, ([4],[9],[10]) individually developed block methods using different approximation solution, which was found that the block methods is more efficient in terms of execution, effectiveness and accuracy. Themethods of collocation and interpolation of power series approximate, Legendre polynomials,Chebyshev polynomials,Orthogonal Polynomials, solution to generate a continuous LinearMultistep Methods*(LMMs)*has been discussed by many authors ranging from predictor-corrector method to hybrid block method,among them are: Awoyemi and Idowu [5], Majidetal. [13], Olabode and Yusuf [12], Adesanyaet al. [6], Yakusak et al. [3], Adeniyi et al. [1], Mohammed et al. [8]. The Block Method has advantage over predictor-corrector method ofbeing cost effective and gives better approximations. In the light of these, we derived three off-stepsHybrid Method for general third order *ODEs* using Legendre polynomials as basis function of the approximation.

In the next section we discuss the methodology, features and properties of the method and following with numerical evidences.

II. Methodology

In this section, we consider Legendre polynomial over an interval [0, 1], to develop the *LMMs* of the form

$$y(x) = \sum_{j=0}^{s+r-1} \alpha_j x^j \ (2)$$

Third derivative is obtained as

$$y'''(x) = \sum_{j=0}^{s+r-1} j(j-1)(j-2)\alpha_j x^{j-3} \ (3)$$

Substituting (3) into (1) gives

$$f\left(x, y(x), y'(x), y''(x)\right) = \sum_{j=0}^{s+r-1} j(j-1)(j-2)\alpha_j x^{j-3} \ (4)$$

Where s and r are the numbers of interpolations and collocations

We now consider the solution of (1) be soughed in partition of the form

$$\pi_N : a = x_0 < x_1 < \cdots . < x_n < x_{n+1} < \cdots x_N = b$$

With constant step size(h) gives

Interpolating (2) at $s = \frac{3}{2},\ 2,\ \frac{5}{2}$ and collocating (4) at $r = 0, \left(\frac{1}{2}\right), 3$ gives

$$\sum_{j=0}^{s+r-1} \alpha_j p^j(x) = y_{n+s} \ (5)$$

$$\sum_{j=0}^{s+r-1} j(j-1)(j-2)\alpha_j p^{j-3}(x) = f_{n+r} \ (6)$$

Solving (5) and (6) for α_j*'s* and substituting back into (2) gives a Continuous Linear Multistep Methods (CLMMs) of the form

$$Y_{n+j} = \sum_{j=0}^{s} \alpha_j(x) y_{n+j} + h^3 \sum_{j-0}^{r} \beta_j(x) f_{n+j} \ (7)$$

Where y_{n+j} and f_{n+k} are given as

$$\alpha_{\frac{3}{2}} = 10 - 9t + 2t^2$$
$$\alpha_2 = -15 + 16t - 4t^2$$
$$\alpha_{\frac{5}{2}} = 6 - 7t + 2t^2$$
$$\beta_{\frac{1}{2}} = \frac{327713}{1209600}t - \frac{29}{50}t^5 + \frac{29}{50}t^6 - \frac{7381}{20160}t^2 - \frac{2599}{40320} - \frac{31}{315}t^7 + \frac{1}{63}t^8 + \frac{1}{2}t^4 + \frac{1}{945}t^9$$
$$\beta_1 = \frac{221359}{4838040}t + \frac{39}{40}t^5 - \frac{3827}{16128} - \frac{461}{720}t^6 - \frac{17683}{161280}t^2 + \frac{137}{630}t^7 - \frac{19}{504}t^8 - \frac{5}{8}t^4 + \frac{1}{378}t^9$$
$$\beta_{\frac{3}{2}} = \frac{290177}{362880}t - \frac{127}{135}t^5 + \frac{31}{45}t^6 - \frac{22621}{60480}t^2 - \frac{7051}{12096} - \frac{242}{945}t^7 + \frac{1}{21}t^8 + \frac{5}{9}t^4 - \frac{2}{567}t^9$$
$$\beta_2 = \frac{196771}{483840}t + \frac{11}{20}t^5 - \frac{307}{720}t^6 - \frac{8929}{161280}t^2 - \frac{5843}{16128} + \frac{107}{630}t^7 - \frac{17}{504}t^8 - \frac{5}{16}t^4 + \frac{1}{375}t^9$$
$$\beta_{\frac{5}{2}} = \frac{9473}{1209600}t - \frac{9}{50}t^5 + \frac{13}{90}t^6 - \frac{109}{5040}t^2 - \frac{79}{40320} - \frac{19}{315}t^7 + \frac{4}{315}t^8 + \frac{1}{10}t^4 - \frac{1}{945}t^9$$
$$\beta_3 = -\frac{953}{7257600}t + \frac{137}{5400}t^5 - \frac{1}{18}t^6 + \frac{187}{69120}t^2 - \frac{131}{241920} + \frac{17}{1890}t^7 - \frac{1}{504}t^8 - \frac{1}{72}t^4 + \frac{1}{5679}t^9$$

Solving independently the solution of (7) forf_{n+1} gives the continuous block formula in the form

$$Y_{n+j} = \sum_{j=0}^{\mu-1} \frac{(jh)^i}{i!} y_i^{(l)} + h^{\mu} \sum_{j=o}^{r} \sigma_j(x) f_{n+k} \ (8)$$

Where$i = 0, 1, 2\ \ and\ y_n\ , f_{n+1}$are

$$\sigma_{\frac{1}{2}} = \frac{203}{1350}t^5 - \frac{49}{720}t^6 + \frac{1}{6}t^3 + \frac{1}{54}t^7 - \frac{1}{360}t^8 - \frac{49}{240}t^4 + \frac{1}{5670}t^9$$
$$\sigma_1 = \frac{39}{40}t^5 - \frac{461}{720}t^6 + \frac{137}{630}t^7 - \frac{19}{504}t^8 - \frac{5}{8}t^4 + \frac{1}{378}t^9$$
$$\sigma_{\frac{3}{2}} = -\frac{127}{135}t^5 + \frac{31}{45}t^6 - \frac{242}{945}t^7 + \frac{1}{21}t^8 + \frac{5}{9}t^4 - \frac{2}{567}t^9$$
$$\sigma_2 = \frac{11}{20}t^5 - \frac{307}{720}t^6 + \frac{107}{630}t^7 - \frac{17}{504}t^8 - \frac{5}{16}t^4 + \frac{1}{375}t^9$$
$$\sigma_{\frac{5}{2}} = -\frac{9}{50}t^5 + \frac{13}{90}t^6 - \frac{19}{315}t^7 + \frac{4}{315}t^8 + \frac{1}{10}t^4 - \frac{1}{945}t^9$$
$$\sigma_3 = \frac{137}{5400}t^5 - \frac{1}{18}t^6 + \frac{17}{1890}t^7 - \frac{1}{504}t^8 - \frac{1}{72}t^4 + \frac{1}{5679}t^9$$

Evaluating the first and second derivative of (8) at $t = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$gives a discrete block in the form

$$A^0 Y_m{}^{(i)} = \sum_{i=0}^{2-i} h^i e_i y_n{}^{(i)} + h^{3-i}(df(y_n) + bF(Y_m)) \qquad (9)$$

Where

$$Y_m = [y_{n+\frac{1}{2}}, y_{n+1}, y_{n+\frac{3}{2}}, y_{n+2}, y_{n+\frac{5}{2}}, y_{n+3}]^T$$
$$F(Y_m) = [f_{n+\frac{1}{2}}, f_{n+1}, f_{n+\frac{3}{2}}, f_{n+2}, f_{n+\frac{5}{2}}, f_{n+3}]^T$$
$$f(y_n) = [f_{n-\frac{1}{2}}, f_{n-1}, f_{n-\frac{3}{2}}, f_{n-2}, f_{n-\frac{5}{2}}, f_{n-3}]^T$$
$$y_m = [y_{n-\frac{1}{2}}, y_{n-1}, y_{n-\frac{3}{2}}, y_{n-2}, y_{n-\frac{5}{2}}, y_{n-3}]^T$$
$$A^0 = 6 \times 6$$

If $i = 0$,we obtain

$$e_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} e_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & \frac{5}{2} \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{8} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{9}{8} \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & \frac{25}{8} \\ 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix} d_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{343801}{29030400} \\ 0 & 0 & 0 & 0 & 0 & \frac{6887}{113400} \\ 0 & 0 & 0 & 0 & 0 & \frac{53893}{358400} \\ 0 & 0 & 0 & 0 & 0 & \frac{3863}{14175} \\ 0 & 0 & 0 & 0 & 0 & \frac{505625}{1161216} \\ 0 & 0 & 0 & 0 & 0 & \frac{891}{1400} \end{bmatrix}$$

$$b_0 = \begin{bmatrix} \frac{6031}{345600} & -\frac{329081}{1935360} & \frac{5177}{362880} & -\frac{15107}{1935360} & \frac{5947}{2419200} & -\frac{9809}{2903400} \\ \frac{1499}{9450} & -\frac{233}{2160} & \frac{52}{567} & -\frac{379}{7560} & \frac{149}{9450} & -\frac{491}{226800} \\ \frac{43173}{89600} & -\frac{14499}{71680} & \frac{9}{40} & -\frac{8829}{7560} & \frac{3483}{89600} & -\frac{1917}{358400} \\ \frac{4664}{4725} & -\frac{226}{946} & \frac{272}{567} & -\frac{31}{135} & \frac{344}{4725} & -\frac{142}{14175} \\ \frac{162125}{96768} & -\frac{85625}{387072} & \frac{66875}{72576} & -\frac{119375}{387072} & \frac{1625}{13824} & -\frac{18625}{1161216} \\ \frac{891}{350} & -\frac{81}{560} & \frac{54}{35} & -\frac{81}{280} & \frac{81}{350} & -\frac{9}{400} \end{bmatrix}$$

If $i = 1$, weobtain

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} e_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & \frac{5}{2} \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

$$d_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{28549}{483840} \\ 0 & 0 & 0 & 0 & 0 & \frac{1027}{7450} \\ 0 & 0 & 0 & 0 & 0 & \frac{759}{3584} \\ 0 & 0 & 0 & 0 & 0 & \frac{272}{945} \\ 0 & 0 & 0 & 0 & 0 & \frac{35225}{96768} \\ 0 & 0 & 0 & 0 & 0 & \frac{123}{280} \end{bmatrix}$$

$$b_1 = \begin{bmatrix} \frac{275}{2804} & -\frac{5717}{53760} & \frac{10621}{120960} & -\frac{7703}{161280} & \frac{403}{26880} & -\frac{199}{96768} \\ \frac{97}{210} & -\frac{2}{9} & \frac{197}{945} & -\frac{97}{840} & \frac{23}{630} & -\frac{19}{3780} \\ \frac{1485}{1792} & -\frac{2403}{17920} & \frac{45}{128} & -\frac{3267}{17820} & \frac{513}{8960} & -\frac{141}{17920} \\ \frac{376}{315} & -\frac{2}{105} & \frac{656}{945} & -\frac{2}{9} & \frac{8}{105} & -\frac{2}{189} \\ \frac{8375}{5375} & \frac{3125}{32256} & \frac{25625}{24192} & -\frac{625}{10752} & \frac{275}{2304} & -\frac{1375}{96768} \\ \frac{27}{14} & \frac{27}{140} & \frac{51}{35} & \frac{27}{280} & \frac{27}{70} & 0 \end{bmatrix}$$

If $i = 2$,we obtain

$$e_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad d_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{19087}{120960} \\ 0 & 0 & 0 & 0 & 0 & \frac{1139}{7560} \\ 0 & 0 & 0 & 0 & 0 & \frac{137}{896} \\ 0 & 0 & 0 & 0 & 0 & \frac{143}{945} \\ 0 & 0 & 0 & 0 & 0 & \frac{3715}{24192} \\ 0 & 0 & 0 & 0 & 0 & \frac{41}{280} \end{bmatrix}$$

$$b_2 = \begin{bmatrix} \frac{2713}{5040} & -\frac{15487}{40320} & \frac{293}{945} & -\frac{6737}{40320} & \frac{263}{5040} & -\frac{863}{120960} \\ \frac{47}{63} & \frac{11}{2520} & \frac{166}{945} & -\frac{269}{2520} & \frac{11}{315} & -\frac{37}{7560} \\ \frac{81}{112} & \frac{1161}{4480} & \frac{17}{35} & -\frac{729}{4480} & \frac{27}{560} & -\frac{29}{8880} \\ \frac{322}{315} & \frac{64}{315} & \frac{752}{945} & \frac{29}{315} & \frac{8}{315} & -\frac{4}{945} \\ \frac{725}{1008} & \frac{2125}{8064} & \frac{125}{189} & \frac{3875}{8064} & \frac{235}{1008} & -\frac{275}{24192} \\ \frac{27}{35} & \frac{27}{280} & \frac{34}{35} & \frac{27}{2800} & \frac{27}{35} & \frac{41}{280} \end{bmatrix}$$

III. Analysis Of The Method

3.1 Order of the Method.

Let the Linear operator $L\{y(x): h\}$associated with the block formula (9) be defined as

$L\{y(x): h\}= A^0 y_m{}^{(i)} - \sum_{i=0}^{2-i} h^i\, e_i y_n{}^{(i)} - h^{3-i}\big(df(y_n) + bf(y_m)\big)$(10)

Expanding (9) in Taylor's series and comparing the coefficient of h gives

$L\{y(x): h\} = c_0 y(x) + c_1 h y'(x) + \cdots + c_p h^p y^p(x) + c_{p+1} h^{p+1} y^{p+1} + +..$

Definition: The linear operator Land associated *LMMs* are said to be of order p if

$$c_0 = c_1 = c_2 = c_p = c_{p+1} = 0 \; and \; C_{P+2} \neq 0$$

C_{P+2} Called the error constant comparing the coefficient of (9) at$i = 0$

$$c_0 = c_1 = c_2 = \ldots c_8 = c_9 = 0$$

$$C_{10} = \left[\frac{400}{1857945600}, \frac{47}{2903040}, \frac{783}{22937600}, \frac{29}{453600}, \frac{7625}{74317824}, \frac{27}{179200}\right]$$

3.2 Zero Stability

The block method is said to be zero stable if $h \to 0,$ the root $r_{ij} = 1(1)k$ of the first characteristic polynomial $\rho(R) = 0$ that is

$$\rho(R) = \det\left[\sum A^0 R^{k-1}\right] = 0$$

Satisfying /R/≤ 1 and for those root with /R/≤ 1must satisfies multiplicity equal to unity.

The method (9) has

$$\rho(R) = R^5(R - 1) = 0$$

R=0, 0, 0, 0, 0, 1

Thus our method is zero stable

3.3 Convergence

A block method is said to be convergent if and only if it is consistent and zero stable.

From above, it shows clearly that our method is convergent.

IV. Numerical Examples

In this section we implement our proposed method in solving third order ordinary differential equations.

Examples 1:

$$y'''(x) - y''(x) + y'(x) - y(x) = 0$$

$$y(0) = 1, y'(0) = 0, y''(0) = -1, h = 0.01$$

Theoretical Solution $y(x) = \cos x$

Table 1a: Showing the exact solution, the numerical solution and comparison of error for example 1

x	$EXACT$	$NUMERICALSOLUTION$	$ERROR$	[8]
O.01	0.9999500004	0.9999500004	0.0	6.7200 E –07
0.02	0.9998000067	0.9998000067	0.0	1.34410 E -06
0.03	0.9995500337	0.9995500337	0.0	2.01700 E -06
0.04	0.9992001067	0.9992001066	1.0 E -10	2.68840 E -06
0.05	0.9987502604	0.9987502603	1.0 E -10	3.35940 E-06

Examples 2:

$$y'''(x) = e^x, y'(0) = 1, y''(0) = 5, h = 0.1$$

Theoretical Solution $y(x) = 2 + 2x^2 + e^x$

Table 1b: Showing the exact solution, the numerical solution and comparison of error for example 2

x	$EXACT$	$NUMERICAL\ SOLUTION$	$ERROR$	[8]
0.0	3	3	0.0	0.0
0.1	3.125170918	3.125170918	0.0	0.0
0.2	3.301402758	3.301402758	0.0	0.0
0.3	3.529858808	3.529858808	0.0	1.000000083 E -09
0.4	3.811824698	3.811824698	0.0	1.000000083 E -09
0.5	4.148721271	4.148721271	0.0	1.000000083 E -09
0.6	4.542118800	4.542118801	1.0 E -09	1.000000083 E -09
0.7	4.993752707	4.993752708	1.0 E -09	9.999991946 E -10
0.8	5.505540928	5.505540929	1.0 E -09	1.000000083 E -10
0.9	6.079603111	6.079603112	1.0 E -09	2.000000165 E -09
1.0	6.718281830	6.718281829	1.0 E -09	1.000000083 E -09

Examples 3:

$$y'''(x) = x - 4y', \ \ y'(0) = 0, y''(0) = 1, h = 0.1$$

Theoretical Solution: $y(x) = -\frac{3}{16}\cos 2x + \frac{5}{16}$

Table 1c: Showing the exact solution, the numerical solution and comparison of error for example 3

x	$EXACT$	$NUMERICAL\ SOLUTION$	$ERROR$	[8]
0.0	0.0	0.0	0.0	0.0
0.1	0.0049875167	0.0049875167	0.0	9.61000 E -10
0.2	0.0198010636	0.0198010636	0.0	6.50000 E -09
0.3	0.0439995722	0.0439995722	0.0	1.59700 E -08
0.4	0.0768674920	0.0768674919	1.0 E -11	1.66400 E -08
0.5	0.1174433176	0.1174433176	0.0	2.03000 E -08
0.6	0.1645579210	0.1645579210	0.0	2.66400 E -08
0.7	0.2168811607	0.2168811606	1.0 E -10	2.67000 E -08
0.8	0.2729749104	0.2729749104	0.0	2.71000 E -08
0.9	0.3313503928	0.3313503927	1.0 E -10	2.7700 E -08
1.0	0.3905275319	0.3905275318	1.0 E -10	2.7200 E -08

Examples 4:

$$y'''(x) + y'(x) = 0, y'(0) = 1, y''(0) = 2, h = 0.1$$

Theoretical Solution: $y(x) = 2(1 - \cos x) + \sin x$

Table 1d: Showing the exact solution, the numerical solution and comparison of error for example 4

x	$EXACT$	$NUMERICAL\ SOLUTION$	$ERROR$	[4]
0.0	0.0	0.0	0.0	0.0
0.1	0.109825086	0.109825086	0.0	1.6613 E -12
0.2	0.238536175	0.238536175	0.0	7.5411 E -12
0.3	0.384847228	0.384847228	0.0	1.3843 E -09
0.4	0.547296354	0.547296354	0.0	4.5000 E -08
0.5	0.724260414	0.724260415	1.0 E -10	1.0520 E -08
0.6	0.913971243	0.913971244	1.0 E -10	1.9815 E -08
0.7	1.114533313	1.114533313	0.0	5.2968 E -08
0.8	1.323942672	1.323942672	0.0	5.0419 E -08
0.9	1.540106973	1.540106973	0.0	7.2608 E -08
1.0	1.760866373	1.760866373	0.0	9.9511 E -08

V. Discussion And Conclusion

We have developed threeoff-step order nine hybrid method for the solution of third order ordinary differential equations in this paper. Our new method is convergent;the method is cost effective in termof cost of developing the scheme and the time of execution. It must be noted that the method adopted is Legendre polynomialto generate a self-starting block method. The results showed that themethod is more accurate than [8] and [4] when compared to the theoretical solution and hence our method is therefore favorable.

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