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Pre-Service Teachers' Didactic Conceptual Structures in the Absolute and Quadratic Inequalities

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Abstract: *This paper examined the didactic conceptual structures of preservice teachers in the absolute and quadratic inequalities to deduce mistakes and errors. Quasi-experimental and mixed exploratory sequential designs were adopted on the participants who worked in 37 groups of 10 members in the Department of Basic Education, University of Education, Winneba in Ghana. The data collection instruments consisted of 15 open-ended items on the basic ideas of the absolute and quadratic inequalities to identify the didactic content knowledge in solving the problems. The thematic analysis of the conceptual structures as well as the marked scores revealed the errors and mistakes in the inner structures, inner relations, representations, and vertical-horizontal relations preservice teachers had confronted in solving problems in absolute and quadratic inequalities. The implications of these would impact negatively on the teaching and learning of mathematics curriculum, and inappropriately applied in a variety of daily lives.*

Keywords*: absolute and quadratic inequalities; didactic conceptual structures; didactic facets; pre-service teachers*

I. Introduction

There are several dimensions of knowledge that preservice teachers need to acquire from their education and training in order to ensure effective teaching and learning. Some of these are the content, profession, learners, curriculum, contextual and conceptual. The content is a body of knowledge that guarantees a teacher's expertise, determined by existing conditions and contexts, personal experiences and involvements, and boosted by teacher's beliefs and needs. This knowledge is considered prerequisite for every preservice and in-service teacher, and form the basic constituent of the teacher's professional knowledge. The various forms of content knowledge are subject, learners, methodology, curriculum, general pedagogical, context and self (Ball, Thames & Phelps, 2008; Gómez, 2009; Liakopoulou, 2011).

The subject knowledge requires familiarisation with scientific knowledge. The dimensions of this knowledge are axioms, theories, formulas and laws in mathematics. It relates to the facts and principles of the mathematical concepts being taught and learned; it involves the relations within one topic and between other topics; it follows the research methodology espoused by prominent scholars in the field, and it gives guides to the procedures and ways to generalizing true knowledge. The subject knowledge integrates the social norms, diagnoses errors, mistakes and misinterpretations, and fully comprehends the procedures required for the acquisition of the knowledge and skills connected to the mathematics being taught. This knowledge is basic to every subject in the curriculum of a particular level in order to adopt an interdisciplinary approach, and brings his/her influences and expertise to bare strongly on the opinions of other subjects in a more holistic outlook (Ball, Thames & Phelps, 2008; Gómez, 2009; Liakopoulou, 2011).

The knowledge of learners comprises the biological, social, psychological and cognitive development of pupils, on issues related to group dynamics and interaction between pupils as well as between teachers and pupils, pupils' behavioural problems, learning motivation, adjustment issues, and learning difficulties. This knowledge is being augmented by the professional methodology of the teacher. The teaching methodology entails the schematic presentation of the specific structural elements of instructions in lesson planning. This includes teacher's pre-lesson activities, organisation of content into thematic units, transformation of teaching materials into teachable knowledge, definition of teaching goals, methodological organisation of teaching, time planning, and selection of evaluation processes. The rests are teaching performance choices made during planning, didactic organisation, teaching paths, application of teaching forms, direct actions of the teacher, use of teaching methods and aids, and evaluation of teaching in the forms of assessing students' performances, curriculum goals, basic principles, and assessment techniques (Ball, Thames & Phelps, 2008; Gómez, 2009; Liakopoulou, 2011).

The curriculum is the knowledge of the school curriculum to determine the conceptual choices of the teacher. This entails the curriculum, textbooks, the rules and laws of the education system and, the role mathematics plays in general education (Gómez, 2009; Liakopoulou, 2011). Following closely with the curriculum is the pedagogical, which relates the organisation of the instructional activities and procedures in the

classroom to motivate and retain students' attention. This type of knowledge secures a framework of mental representations necessary for the comprehension and interpretation of the school classroom, and is absolutely essential for lesson planning to guide the teacher's didactic choices (Ernest, 1989; Liakopoulou, 2011).
The contextual knowledge is one in which the teacher applies to evaluate the school environment and the conditions of the teacher's work, vis-à-vis the school, district, the region and the students. This extends to the entire local community, education system, the school's organisation and management, the institutional framework and administrative structures (Ball, Thames & Phelps, 2008; Gómez, 2009; Liakopoulou, 2011). The knowledge of self relates to the personality of the teacher, the roles and responsibilities, training and qualifications, rights and professional development, and working conditions. The way teachers perceive their role defines not only their options, but also the way they comprehend, interpret and use this knowledge in the classroom (Clandinin & Connely, 1987; Liakopoulou, 2011).

Conceptual knowledge is a connected network of knowledge, which makes relationships as prominent as the discrete bits of information. In effect, conceptual knowledge cannot be learned by rote but must be learned by thoughtful, reflective learning. This level of understanding allows learners to think generatively within that content area, enabling them to select appropriate procedures for each step when solving new problems, make predictions about the structure of solutions, and construct new understandings and problem-solving strategies. Common examples of such interconnections are linear equations, linear inequalities, quadratic equations, and quadratic inequalities. In conceptual structures, learners must understand the key important ideas, useful in contexts, systematic and logical, justifiable and flexible. (Richland, Stigler. & Holyoak, 2012; Wiggins, 2014)

Facets of Didactic Conceptual Structures

The word *didactics* originates from the Greek word *didaskein*, which means *to teach* or *to know how to teach* (Tchoshanov & Knyazeva, 2013). Didactic structures comprise of a three-fold relationship, called the didactic triangle or triad. This triad is made up of the teacher, the learner, and the content who actively engage in interwoven activities in the classroom. In the didactic triad, the teacher and learners are described as *the who*, content as *the what*, and the instructional methodology as *the how*. Theses relations make didactic structures necessary and sufficient tools in teaching, learning, transferring and transmitting the various kinds of knowledge required to promote lifelong understanding and applications. The triad also involves frameworks that integrate these three elements in the teaching and learning of mathematics (Klette, 2007; Tchoshanov & Knyazeva, 2013). Generally, didactic conceptual structures contain six components, namely epistemic, cognitive, affective, interactional, mediational, and ecological. One school of thought posits that the epistemic component involves the institutionally implemented ones (problems, languages, procedures, definitions, properties, and justifications), the cognitive involves the development of the individual learner (intelligence, readiness, maturation, and memory), the affective involves the emotional states of the learner (attitudes, emotions, and motivations), the interactional involves the sequence of interactions between the teachers and learners oriented at fixing and negotiating learning (methods, strategies, materials, and resources), the mediational involves the distribution of teaching aids, materials and resources used over time and the distribution of time for the actions and processes to mediate learning (periods, timetables, practical sessions, and extra curricula activities); and the ecological involves the system of relations with the social, political, and economic contexts that underlies and affects the teaching and learning process (Pino-Fan, Godino & Font, 2011; Pino-Fan, Godino, Font & Castro**,** 2012). Another school of thought, comprising Godino, Wilhelmi and Bencomo (2005), and Godino, Ortiz, Roa & Wilhelmi (2011) describe them as the six facets as follows:

1. Epistemic suitability to measure the extent to which the implemented curriculum (content implemented in a classroom or course) represents adequately the intended curriculum (curricular guidelines for this course or classroom). This evaluates the suitability of absolute and quadratic inequalities in the mathematics curricula.
2. Cognitive suitability to measure the degree to which the implemented is appropriate to the preservice teachers' cognitive development, the degree to which the implemented bridges the zone of proximal development, and whether learning is close to achieving the goals of the intended curriculum. This evaluates how suitable the absolute and quadratic inequalities improve cognitive learning in the mathematics classroom.
3. Emotional suitability to measure preservice teachers' involvement, interest, motivation, and attitudes in the teaching and learning process. This evaluates how well and appropriate absolute and quadratic inequalities contribute in promoting affective learning.
4. Media suitability to reflect the availability and adequacy of teaching-learning materials, teachers' resources and other internal support systems in the teaching and learning process. It evaluates the opportunity provided to learners to discuss issues in mathematics.

5. Interactive suitability to measure the extent to which the organisation of the teaching and the classroom environment serve to identify and solve possible errors, mistakes, and learning difficulties that appear during the instructional process. It evaluates the adequate of employing absolute and quadratic inequalities to identify errors and mitigate them.
6. Ecological suitability to measure the extent to which the teaching process is sacrosanct with the school and society educational goals, socio-cultural factors and states' needs. This evaluates how appropriate absolute and quadratic inequalities contribute in achieving the goals of the individual, the school, and the larger society.

While Pino-Fan, et. al (2012) conceptualized a four-level model, we have adopted only three levels of the triad for our models. These are the epistemic to measure the actions of the preservice teachers in ensuring that they equip themselves with the curricular guidelines in the classroom learning, the cognitive to ensure that the mathematical objects and themes are properly absorbed and emerged out of learning, and the ecological to ensure that preservice teachers become acquainted with the mathematics rules, theorems, and formulas that regulate and facilitate learning (Gómez, 2009; Pino-Fan, et. al, 2011; Pino-Fan, et. al, 2012).

The Three-Level Relational Didactic Triad

The three-level relational didactic triad consists of three interrelated elements surrounding the activities of the teacher, student and content in solving problems in the absolute and quadratic inequalities (Pino-Fan, et. al, 2012; Østergaard, 2013).

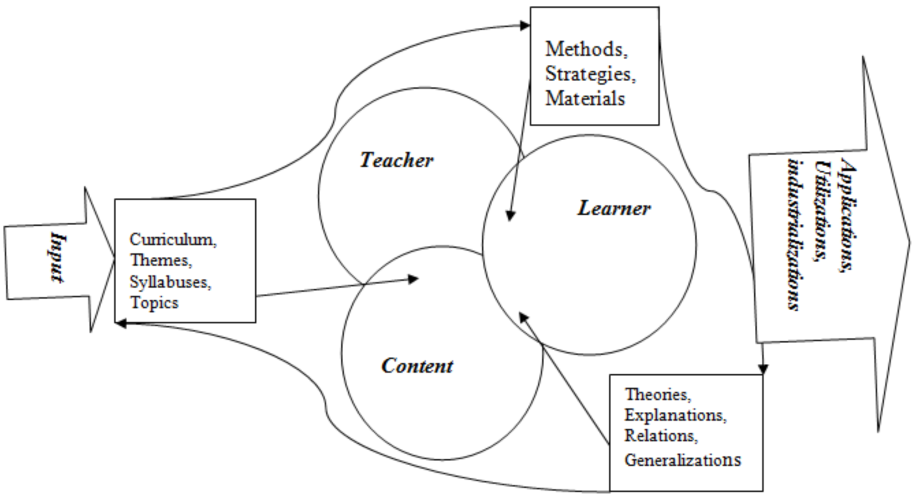

Figure 1: Relational Didactic Triad **(**Source: Østergaard, 2013**)**

The Figure 1 illustrates a simple relational triad in dealing with the didactic conceptual structures in the absolute value and quadratic inequalities. Teacher education is characterized by the fact that preservice teachers are supposed to learn mathematics and to learn to teach mathematics. Therefore, their knowledge in this model is very relevant (Gómez, 2009; Pino-Fan, et. al, 2012; Østergaard, 2013). The triad commences with the teacher and the content preplanned in the forms of the mathematics curriculum, themes, and topics from the intended mathematics curriculum. The teacher and the learner interact and implement the curriculum through methods, strategies, and materials. The leaner applies the various interventions and implementations through the conceptualizations of the mathematics concepts, theories, explanations, and relations. The interplay and interaction between the teacher and learner, coupled with the processes, procedures and institutionalizations is called the epistemic facet because it focuses on the set of problems, procedures, concepts, properties, language, and arguments intended and executed over a period of time (Godino, Ortiz, Roa & Wilhelmi, 2011).

Following closely is the cognitive facet of the learner. The learner utilizes the cognitive facet to conceptualize the concepts, relations, theories and generalizations from both the teacher and the content. This cognitive enables the preservice teacher to identify learning and learners levels of development and understanding of the concepts, topics and themes, and to confront the difficulties, mistakes and errors embedded in the intended curriculum. Therefore, the learner is not only to understand and transfer effective and appropriate teacher's methodology but also to absorb and recall facts from the content (Godino, Ortiz, Roa & Wilhelmi, 2011).The model ends with the ecological facet, where the input tools are being processed, transformed and churned out to the society to enable everyone apply, utilize and industrialize the mathematics. It is also called ecological, because the outcomes of the intended and executed curricula influence the social, political and economical settings, which support and condition the teaching and learning to derive maximum benefits (Godino, Batanero, Roa & Wilhelmi, 2008; Gómez, 2009; -Godino, Ortiz, Roa & Wilhelmi, 2011; Østergaard, 2013).

Didactic Conceptual Structures of Teaching and Learning Mathematics

Gómez (2009) identifies three conceptual structures enshrined in the concept of every school mathematics curriculum. The first conceptual structure is the mathematics structures involved. Every mathematics concept is related to at least two mathematical structures--the mathematical structure that the concept configures and the mathematical structures of which it forms part. For example, the concept of the quadratic inequality configures a mathematical structure in which structural relations are established between equations, inequalities, and absolute values. In addition, the concept of quadratic inequality forms part of graphs of functions. The second structure is the conceptual relations. Various relations are established between the i). concept itself and the concepts of the mathematical structure that this concept configures (e.g., the relation between the quadratic inequalities and the quadratic constant 'a'). ii). the objects that are specific cases of this concept (that is, the objects that saturate the predicate (e.g., $ax^2+bx+c\le 0$ and $x^2+10x+25\le 0$ as a specific case of quadratic inequality functions of the form $ax^2+bx+c\le 0$.

iii). the concepts that belong to the mathematical structure of which the concept forms part (e.g., the relation between the quadratic inequality and absolute inequality). The third structure is the relations of representations. Exploring a concept requires systems of representation to identify the ways in which the concept appears. There are three main types of relational structures identified in inequalities. The first is the relation between two signs that designate the same concept and within the same system of representation such as within each of $\pm a|x\pm k|\neq c$, and $\pm ax^2+bx+c\neq 0$ (called invariant syntactical transformations) in the absolute and quadratic inequalities respectively. The second is the relation between two signs that designate the same concept but belongs to different systems of representation such as $\pm a|x\pm k|\neq c$ and $\pm ax^2+bx+c\neq 0$ (called translation between systems of representation) in both the absolute and quadratic inequalities. The third is the relation between two signs that designate two different concepts within the same system of representations such within $a|x\pm k|\neq c$ itself, and $\pm ax^2+bx+c\neq 0$ itself (called variant syntactical transformations).

Gómez (2009) opines that the second and third structures can be grouped into two categories, called vertical relations and horizontal relations. Vertical relations refer to relations between the three kinds of elements such as Object→Concept→Mathematical structure, and the horizontal relations refer to the relations between signs in their different systems of representation (relations between representations such as absolute→ quadratic. It is therefore, prudent that we understand these conceptual didactic structures to help identify and solve problems in the absolute and quadratic inequalities (Appleby, Letal, & Ranieri, 2006; Gómez, 2007; Gómez, 2009; Schmitz, 2012).

Didactic Conceptual Structures in Absolute and Quadratic Inequalities

The absolute value of a real number x, denoted by $\pm a|x\pm k|\neq c$, is defined geometrically, as the distance between ±k (the origin or origin when k=0) and the graph of the real number (x) is c. Thus, $|x|\le 5$ means 5 units between 0 and x are -5 and 5, and $|x-3|\le 2$ means 2 units between x and 3 are 1 and 5 (Appleby, Letal, & Ranieri, 2006; Schmitz, 2012).

Alternatively, the absolute value of a real number can be defined algebraically in a piecewise manner, where $|x|=\begin{cases}-x \text{ if } x<0\\ x \text{ if } x\ge 0\end{cases}$. This piecewise definition is directly consistent with the idea of keeping a positive number the same, but changing a negative number to positive. Thus, if a real number x is nonnegative, then the absolute value will be that number x. If x is negative, then the absolute value will be the opposite of that number (Schultz, Ellis, Jr., Hollowell, & Kennedy, 2004; Appleby, Letal, & Ranieri, 2006; Cook & Tx, 2009; Schmitz, 2012).

Let c be the unit distance between 0 and x, and p and q be the endpoints, then:

1. $a|x|\le -c=\{\ \}$ because |x| is always non-negative.
2. $a|x|\le c=\{x \mid p\le x\le q\}$ because the continuous operator is used.
3. $a|x|\ge c=\{x \mid x\le p\cup x\ge q\}$ because the discontinuous operator is used.
4. $-a|x|\le c=\{x \mid x\le p\cup x\ge q\}$ because the reverse operator is used.
5. $-a|x|\ge c=\{x \mid p\le x\le q\}$ because the reverse operator is used. (Appleby, Letal, & Ranieri, 2006; Cook & Tx, 2009).

A quadratic inequality is a mathematical statement that relates a quadratic expression as either less than or greater than another. Solutions to that quadratic inequality are two real number, p and q that produce true statements when substituted for the variable, x. The real values, p and q in the domain of the function are called critical numbers or roots (p and q), and can be obtained by factorizing or sketching the graph of the inequality (Schultz, Ellis, Jr., Hollowell, & Kennedy, 2004; Appleby, Letal, & Ranieri, 2006; Cook & Tx, 2009; Schmitz, 2012).

Let p and q be he roots of the quadratic inequality, $\pm ax^2 + bx + c \neq 0$, where p<q, then:

6. $ax^2 + bx + c \leq 0 \Rightarrow \{p \leq x \leq q\}$ because the continuous operator is used.
7. $ax^2 + bx + c \geq 0 \Rightarrow \{x \leq p \cup x \geq q\}$ because the discontinuous operator is used.
8. $-ax^2 + bx + c \geq 0 \Rightarrow \{x \leq p \cup x \geq q\}$ because the discontinuous operator is used.
9. $-ax^2 + bx + c \leq 0 \Rightarrow \{p \leq x \leq q\}$ because the continuous operator is used.
10. $|ax^2 + bx + c \leq 0| < -k \Rightarrow \{\ \}$ because |x| is always non-negative **(**Schultz, Ellis, Jr., Hollowell, & Kennedy, 2004; Appleby, Letal, & Ranieri, 2006).

Having examined the conceptual structures, didactic errors and mistakes could occur if preservice teachers compare the solutions of the following pairs of inequalities:

11. a$|x| \leq c$ and $ax^2 + bx + c \leq 0$.
12. a$|x| \geq c$ and $ax^2 + bx + c \geq 0$.
13. $-a|x| \leq c$ and $-ax^2 + bx + c \leq 0$.
14. $-a|x| \geq c$ and $-ax^2 + bx + c \geq 0$ (Appleby, Letal, & Ranieri, 2006; Schmitz, 2012).

Statement of the Problem

First, the relation between the teacher and (who), content issues (what), and instructional procedures and activities (how) has changed over time. While the traditional perspective of the *what,* backed up with arguments of *why* questions, have been more focussed on the issues of the teacher's teaching styles and teaching methods (the how), and characteristics of the new didactic conceptual structures have always been refocused. There is a renewed interest in the interplay with the content and the activities of the learner in relation to teaching and learning (Klette, 2007).

Second, some studies hold strong views in the teaching and learning of only the content in the classrooms and as a consequence, the other two segments of the triad are underdeveloped. Even though content and subject matter may be important, the presentation of their relational dynamics between the concepts and themes continue to be vague and obscure (Klette, 2007). There is therefore, the need for preservice teachers to teach, learn and understand conceptual structures to provide intra-connect and interconnect of mathematics concepts.

Third, the role of conceptual structures has been underestimated in studies of teaching and learning of absolute and quadratic inequalities. Such neglect does not fully address the critical issues of knowing what to teach, who to teach, and how to teach during the teaching and learning process. Didactic conceptual structures allow for smooth flow of methods, strategies, teacher-learner interactions and teaching-learning manipulation in the classroom. We therefore require this framework to integrate the teaching and learning of absolute and quadratic inequalities (Klette, 2007; Bäckman & Attorps, 2012).

II. Materials and Methods

Research questions

1. What didactic conceptual structures do preservice teachers experience in solving problems in absolute and quadratic inequalities?
2. How do the conceptual structures influence their performance in absolute inequalities to quadratic inequalities?

Research Design

This was quasi-experimental and exploratory mixed design to explore and analyze preservice teachers' didactic conceptual structures in the absolute and the quadratic inequalities.

Participants

The study involved 37 groups of preservice teachers in the Department of Basic Education, University of Education, Winneba in Ghana. These 37 groups consisted of ten members in each group. These 37 groups were already constituted by the class for the purposes of all group assignments, and not just for mathematics. Moreover, there were always males and females in each group. The department runs several undergraduate and post graduate courses, among which is mathematics. Absolute values and quadratic inequalities are among the nine broad areas covered in a semester.

Data collection method

The data was gathered through semi-structured questionnaire that contained only open-ended items but precoded to suit both qualitative and quantitative analysis. The qualitative items were thematically categorized into absolute, quadratic, and both inequalities while the quantitative items graded the appropriate answers provided by the respondents. The length of time to complete each question was approximately between 45 and 60 minutes (Pino-Fan, et. al, 2012).

Data analysis method

The analyses were equally carried out in two phases. The first phase was transcriptions of preservice teachers' responses of the qualitative items to assess and evaluate their content conceptual structures, and the second was a single-subject t-test analysis of the scores to support whether there were significant differences in their didactic conceptual structures. In the end, the two phases concurrently explored the impacts of the didactic conceptual structures to the teaching and learning of absolute and quadratic inequalities.

III. Results

Analysis of Didactic Conceptual Structures in Solving Problems in Absolute Inequalities

The negative influence of some conceptual structures in solving problems in absolute inequalities depicted a lot of errors and mistakes in the conceptual structure of the absolute inequality. Many groups did not understand the concept of 'c' unit distance from x, and provided the following inappropriate and incorrect responses.

Group A: *The operation signs are the same*.
Group B: *They are all bounded values with the less than sign (<)*.
Group C: *All end with the given value that 6, 11, 5 and 22*.
Group D: *They have bounded numbers*.
Group E: *Their responses in common are the units from the value.*
They are all talking about inequalities which are less than sign (<).
The ranges end with the number given.
Group F: *They are all measured in units*.

Clearly, the preservice teachers lacked the knowledge of conceptual structure of the absolute inequality involved. The concept of the absolute inequality is related to the concept of linear inequality. However, the preservice teachers failed to relate absolute inequalities to linear inequalities and linear equations. In addition, the concept of number line was very integral in defining the absolute inequality and solving these categories of problems.

Analysis of Didactic Conceptual Structures in Solving Quadratic Inequalities

In using the solution set, {x: x = -3, 4} as a hypothetical case for the four ways of obtaining the quadratic inequalities, the following responses were provided:

Group A: *The problems are inequalities too*.
Group B: *They have factors of -3 and 4 which have are less than and greater than signs*.
Group C: *All end with the given values of 0 and -c*.
Group D: *Some have no bars and one have bars*.
Group E: *The answers are found in -3 and 4.*
Group F: *They are all inequalities and when you work them you will get numbers*.

There were clearly lack of conceptual relations between roots themselves, and between the roots and quadratic inequalities. This arose because of the lack of knowledge of appropriate conceptual structures in the quadratic inequalities themselves, and the mathematical structures that characterise the solutions of quadratic inequalities involving the constant 'a'. Preservice teachers failed to conceptualize their experiences with the positive and negative values of 'a' in quadratic equations to the quadratic inequalities.

Didactic Conceptual Structures in Connecting Absolute and Quadratic Inequalities

In linking the solutions of $\pm a|x \pm k| \neq c$ to those of quadratic inequality $\pm | ax^2 + bx + c | \neq d$, the respondents provided the following answers:

Group A: *The first one is absolute inequalities and the second one is quadratic inequalities*.
Group B: *Anytime there is greater than in absolute question, there is also greater than in quadratic questions*.
Group C: *The absolute questions have only 'c' but the quadratic ones have a, b, c*.
Group D: *If you can solve inequalities questions, you can solve the quadratic inequalities too*.

Group E: *All the questions cannot be solved because there are no factors like -3 and 4.*
Group F: *There is no links because some are absolute inequalities and some are quadratic inequalities.*

The mistakes and errors occurred here mainly because the preservice teachers had little or no knowledge of the desired and appropriate conceptual structures in the representations of relations. They had errors in the relations between two signs that designate the same concept, and within the same system of representation as witnessed in $|x|<c$ and $-|x|>c$, and $ax^2+bx+c<0$ and $-ax^2+bx+c>0$. There were also errors in relating between two signs that designate the same concept belonging to different systems of representation as witnessed in $|x|<c$ and $ax^2+bx+c<0$, and $-|x|>c$ and $-ax^2+bx+c>0$, and the relation between two signs that designate two different concepts within the same system of representation as in $|x|<c$ and $-|x|<c$, and $ax^2+bx+c<0$ and $- ax^2+bx+c<0$ .This analysis confirms that conceptual structures in absolute and quadratic inequalities can be grouped as either vertical relations or horizontal relations. The vertical relations showcase the structures involved in detecting and diagnosing the concepts themselves in the absolute and quadratic inequalities. On the other hand, the knowledge of conceptual structures in the horizontal relations would have helped to discover the dynamics of the solutions and the signs of 'a' in both inequalities. Therefore, in advancing and promoting didactic conceptual structures for these two inequalities, the vertical and horizontal structures must be always embedded to facilitate teaching and learning.

One-Sample T-Test Analysis of Responses of Absolute and Quadratic Inequalities

The single within-subject t-test comparison of the mean difference in the preservice teachers responses were scored, coded and analyzed. The main statistics in the table were the t-statistic (t), degree of freedom (df), significance level (two-tailed), mean difference, and confidence interval around the mean difference.

Table 1: Summary of One-Sample T-Test Statistics of Scores

T-Statistics			***95% Confidence Interval of Mean Difference***		
t	df	Sign. level (2-tailed)	Mean Difference	Lower	Upper
47.83	36	0.000	15.57	14.91	16.23

We observed that the scores of the preservice teachers were significantly different from zero (t[47.83] = 36, p =0.000), and means that each group had entirely different conceptual structures that influenced the ways they responded to the problems. These phenomena, if not checked and ameliorated, would negatively impact on teaching and learning, and performance and achievements of the students they teach.

Discussion of Findings

All problems of the absolute inequality deal with a distance from a number, in which two solutions must always be found to each problem. It was observed that the absolute inequalities take four different forms compared with the value, *c*. To solve such a problem, preservice teachers adopted negative conceptual structures that lead them to provide the inaccurate responses and obtained incorrect answers. If the preservice teachers had discovered appropriate conceptual models, they would have discovered that, the less than inequalities are generally considered conjunctions and the greater than are disjunctions. The discovery of this general conceptual structure that notwithstanding, depends on the value of the constant 'a', in which case the solutions and may be linked to those of linear equations. The same patterns of errors were committed in the solving problems in quadratic inequalities. The real numbers, p and q, as two solution sets $\pm ax^2 + bx + c \neq 0$ could be conceptualized as solving for the distance between p in one part of the parabola to q in another part. It was revealed that the solution of quadratic inequalities take four different forms, and preservice teachers are expected to conceptualize the different structures within each of them, and relate the structures to absolute inequalities to as conceptual structures of conjunctions, disjunctions, and the constant 'a' have a lot of impact to bear in enhancing effective teaching and learning of the inequalities.

IV. Conclusion

The results showed the different errors and mistakes the preservice teachers committed in solving the problems in absolute inequality, the quadratic inequality, and between the absolute and quadratic inequalities. We discovered that the preservice teachers lacked appropriate and accurate conceptual structures to deal with these types of problems. This explained why some groups that formed good conceptual structures scored higher than those that demonstrated little knowledge and understanding. Therefore, training and educating preservice teachers on models that are best rooted on didactic conceptual structures could enhance effective teaching and learning. Secondly, the didactic conceptual framework bridged the teaching and learning within the themes themselves, and between other themes. This discovery can be linked to recent research works in higher thematic patterns that incorporate activities of the teacher, the learner, and the content in the triad.

We therefore suggested the need to improve upon our three-level triad model to enhance teaching and learning. And improving this model for teacher education and training would definitely require significant modifications in the initial inputs channelled into the triad to produce the desired results that are worthy of society's applications, utilizations and industrializations of mathematics.

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