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# ADCSS-Labeling of 2-Tuple Graphs of Some Graphs

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***Abstract:*** *A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In this paper, we introduce the new concept, an absolute difference of cubic and square sum labeling of a graph. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize 2-tuple graphs of middle graph of paths and cycles, crown graph ,star graphs , the triangular snake graph, quadrilateral snake graph for adcss labeling.*

***Keywords:*** *Graph labeling, sum square graph, square sum graphs, cubic graphs, middle graphs, 2-tuple graph.*

## I. Introduction

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2] ,[3] , [4] and [5]. Some basic concepts are taken from Frank Harary [2]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph [6]. In [6],[7],[8],[9],[10],[11] , it is shown that planar grid, web graph, kayak paddle graph, snake graphs, friendship graph, armed crown, fan graph, cycle graphs ,wheel graph etc have an adcss labeling. In this paper we investigated ADCSS labeling of some 2-tuple graphs.

**Definition: 1.1 [6]**Let G = (V(G), E(G)) be a graph. A graph G is said to be absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection f : V(G) → {1,2,----------------,p} such that the induced function $f^*_{adcss}$ : E(G) → multiples of 2 is given by $f^*_{adcss}(uv) = \left|(f(u)^3 + f(v)^3) - (f(u)^2 + f(v)^2)\right|$ is injective.

**Definition: 1.2** A graph in which every edge associates distinct values with multiples of 2 is called the sum of the cubes of the vertices and the sum of the squares of the vertices. Such a labeling is called an absolute difference of cubic and square sum labeling or an absolute difference css-labeling.

## II. Main Results

**Definition 2.1** Let V(G) and X(G) denote the vertex set and the edge set of G, respectively. The middle graph M(G) of G whose vertex set is V(G) union X(G) where two vertices are adjacent if and only if

(i) They are adjacent edges of G or
(ii) One is a vertex and other is an edge incident with it.

**Definition 2.2** The crown graph $Cr_n$ is obtained by joining a pendant edge to each vertex of cycle $C_n$.

**Definition 2.3** The triangular snake graph $T_n$ is obtained from the path $P_n$ by replacing each edge $v_iv_{i+1}$ by cycle $C_3(v_iu_iv_{i+1})$ for $1 \le i \le n-1$.

**Definition 2.4** The quadrilateral snake graph $Q_n$ is obtained from the path $P_n$ by replacing each edge $v_iv_{i+1}$ by cycle $C_4(v_iu_iu_{i+1}v_{i+1})$ for $1 \le i \le n-1$

**Definition 2.5** Let G = (V,E) be a simple graph and G' = (V',E') be another copy of graph G. Join each vertex v of G to the corresponding vertex v' of G' by an edge. The new graph thus obtained is the 2- tuple graph of G. 2-tuple graph of G is denoted by $T^2(G)$. Further if G = (p; q) then $\left|V\{T^2(G)\}\right| = 2p$ and $\left|E\{T^2(G)\}\right| = 2q+p$

**Theorem: 2.1** 2-tuple graph of the middle graph of path $P_n$ ,$T^2\{M(P_n)\}$ admits ADCSS - labeling.

**Proof :** Let G = $T^2\{M(P_n)\}$ and let $v_1,v_2$,----------------,$v_{4n-2}$ are the vertices of G.

Here $|V(G)| = 4n-2$ and $|E(G)| = 8n-9$

Define a function f : V → {1,2,3,--------,4n-2} by

$f(v_i) = i$ , i = 1,2,------,4n-2.

For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\ v_{i+1}) = (i+1)^2 i + i^2(i-1)$, i = 1,2,3,-----------,4n-3

$f^*_{adcss}(v_{2i}\ v_{2i+2}) = (2i+2)^2(2i+1) + (2i)^2(2i-1)$ , i = 1,2,3,------------,n-2.

$f^*_{adcss}(v_{2i+1}\, v_{4n-2-2i})$ = $(2i+1)^2(2i) + (4n-2-2i)^2(4n-3-2i)$ , i = 0,1,2,3,----------,n-2.
$f^*_{adcss}(v_{2i+2}\, v_{4n-3-2i})$ = $(2i+2)^2(2i+1) + (4n-3-2i)^2(4n-4-2i)$ , i = 0,1,2,3,----------,n-2.
$f^*_{adcss}(v_{2n-1+2i}\, v_{2n+1+2i})$ = $(2n-1+2i)^2(2n-2+2i) + (2n+1+2i)^2(2n+2i)$ , i = 1,2,3,----------,n-2.

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $T^2\{M(P_n)\}$ admits adcss-labeling.

**Example 2.1** ADCSS labeling of the graph $T^2\{M(P_4)\}$ is shown in figure (i)

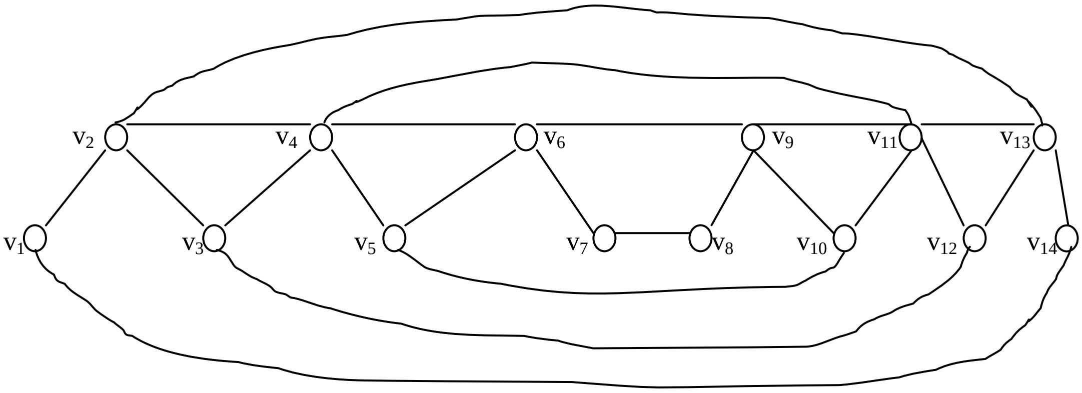


fig - i

**Theorem: 2.2** 2-tuple graph of the middle graph of cycle $C_n$,$T^2$ $M(C_n)$ admits ADCSS - labeling.
**Proof :** Let G = $T^2$ $M(C_n)$ and let $v_1,v_2$,----------------,$v_{4n}$ are the vertices of G.
Here $|V(G)| = 4n$ and $|E(G)| = 8n$
Define a function f : V → {1,2,3,--------,4n} by
$f(v_i) = i$ , i = 1,2,------,4n.
For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\, v_{i+1})$ = $(i+1)^2 i+i^2(i-1)$, i = 1,2,----,2n-1,2n+1,----,4n-1
$f^*_{adcss}(v_{2i}\, v_{2i+2})$ = $(2i+2)^2(2i+1) + (2i)^2(2i-1)$ , i = 1,2,3,---------------,n-1.
$f^*_{adcss}(v_2\, v_{2n})$ = $(2n)^2(2n-1) +4$
$f^*_{adcss}(v_1\, v_{2n})$ = $(2n)^2(2n-1)$
$f^*_{adcss}(v_{2n+2}\, v_{4n})$ = $(2n+2)^2(2n+1) +(4n)^2(4n-1)$
$f^*_{adcss}(v_{2n+1}\, v_{4n})$ = $(2n+1)^2(2n) +(4n)^2(4n-1)$
$f^*_{adcss}(v_{2n+2i}\, v_{2n+2i+2})$ = $(2n+2i)^2(2n+2i-1) +(2n+2i+2)^2(2n+2i+1)$, i = 1,2,-------------,n-1
$f^*_{adcss}(v_i\, v_{2n+i})$ = $(i)^2(i-1) +(2n+i)^2(2n)$, i = 1,2,--------------,2n

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $T^2$ $M(C_n)$ admits adcss-labeling.

**Example : 2.2** ADCSS labeling of the graph $T^2\{M(C_3)\}$ is shown in figure (ii)

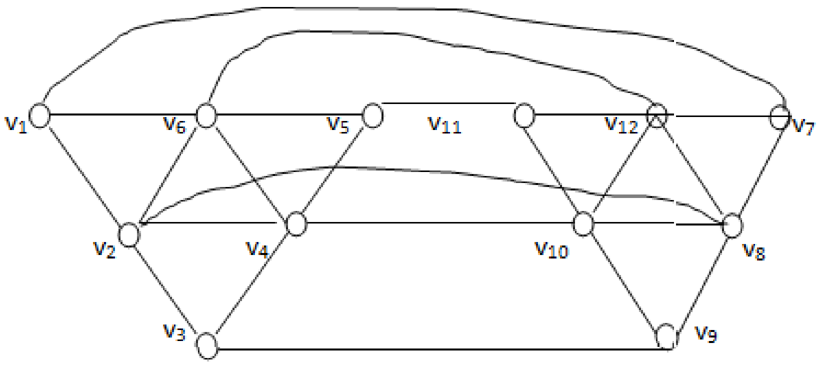


fig - ii

**Theorem: 2.3** $T^2(Cr_n)$ admits ADCSS - labeling.
**Proof :** Let G = $T^2(Cr_n)$ and let $v_1,v_2$,----------------,$v_{4n}$ are the vertices of G.
Here $|V(G)| = 4n$ and $|E(G)| = 6n$
Define a function f : V → {1,2,3,--------,4n} by

$f(v_i) = i$ , i = 1,2,------,4n.

For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\ v_{i+1}) = i^2(i-1)+(i+1)^2 i$, i = 1,2,------,n-1,n+1,---------,2n-1.

$f^*_{adcss}(v_i\ v_{i+n}) = (i+n)^2(i+n-1) + (i)^2(i-1)$ , i = 1,2,3,----------------,n.

$f^*_{adcss}(v_n\ v_1) = (n)^2(n-1)$

$f^*_{adcss}(v_{n+1}\ v_{2n}) = (n+1)^2(n) + (2n)^2(2n-1)$

$f^*_{adcss}(v_{2n+i}\ v_i) = (i)^2(i-1) + (2n+i)^2(2n + i -1)$, i = 1,2,-------------,n

$f^*_{adcss}(v_{2n+i}\ v_{3n+i}) = (3n+i)^2(3n+i-1) + (2n+i)^2(2n + i -1)$, i = 1,2,-------------,n

$f^*_{adcss}(v_{n+i}\ v_{3n+i}) = (n+i)^2(n+i-1) + (3n+i)^2(3n + i -1)$, i = 1,2,-------------,n

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $T^2(Cr_n)$ admits adcss-labeling.

**Example 2.3** ADCSS labeling of the graph $T^2(Cr_4)$ is shown in figure (iii)

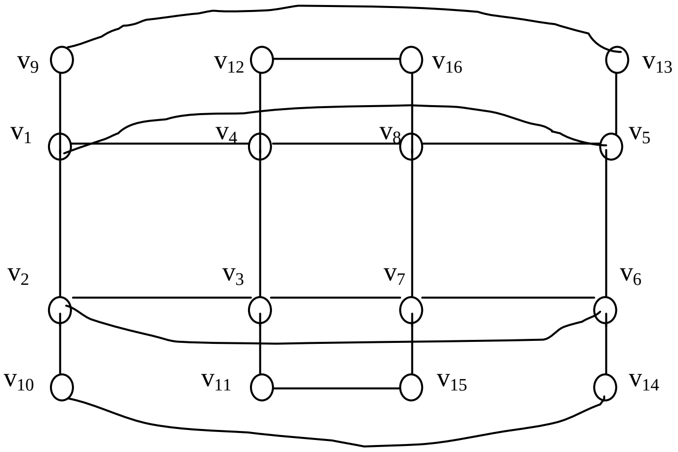


fig - iii

**Theorem: 2.4** $T^2(K_{1,n})$ admits ADCSS – labeling, where $K_{1,n}$ is the star graph.

**Proof :** Let G = $T^2(K_{1,n})$ and let $v_1, v_2$,----------------,$v_{2n+2}$ are the vertices of G.

Here $|V(G)| = 2n+2$ and $|E(G)| = 3n+1$

Define a function f : V → {1,2,3,--------,2n+2} by

$f(v_i) = i$ , i = 1,2,------,2n+2.

For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_1\ v_{i+1}) = (i+1)^2 i$, i = 1,2,-----------,n.

$f^*_{adcss}(v_{n+2}\ v_{n+2+i}) = (n+2)^2(n+1) + (n+2+i)^2(n+1+i)$ , i = 1,2,3,---------,n.

$f^*_{adcss}(v_i\ v_{n+i+1}) = (i)^2(i-1) + (n+i+1)^2(n+i)$, i = 1,2,3,---------,n+1.

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $T^2(K_{1,n})$ admits adcss-labeling.

**Example 2.4** ADCSS labeling of the graph $T^2(K_{1,4})$ is shown in figure (iv)

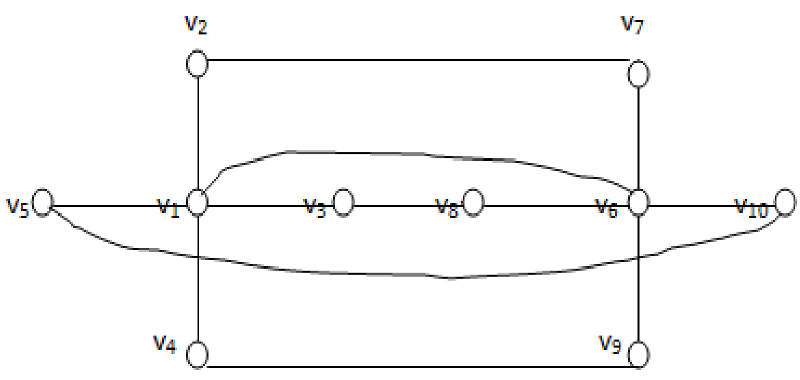


fig - iv

**Theorem: 2.5** $T^2(T_n)$ admits ADCSS – labeling, where $T_n$ is the triangular snake graph.

**Proof :** Let G = $T^2(T_n)$ and let $v_1, v_2$,----------------,$v_{4n-2}$ are the vertices of G.

Here $|V(G)| = 4n-2$ and $|E(G)| = 8n-7$

Define a function $f : V \rightarrow \{1,2,3,\text{--------},4n-2\}$ by

$f(v_i) = i$ , $i = 1,2,\text{------},4n-2$.

For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) = (i+1)^2 i + i^2(i-1)$, $\quad i = 1,2,\text{-------------------},2n-2$

$f^*_{adcss}(v_{2n-1+i}\, v_{2n+i}) = (2n-1+i)^2(2n-2+i) + (2n+i)^2(2n+i-1)$ , $\quad i = 1,2,3,\text{----------------},2n-2$.

$f^*_{adcss}(v_{2n-2+2i}\, v_{2n+2i}) = (2n-2+2i)^2(2n-3+2i) + (2n+2i)^2(2n+2i-1)$ , $\quad i = 1,2,3,\text{----------------},n-1$.

$f^*_{adcss}(v_{2i-1}\, v_{2i+1}) = (2i+1)^2 2i + (2i-1)^2(2i-2)$ $\quad i = 1,2,\text{-------------------},n-1$

$f^*_{adcss}(v_{2i}\, v_{2i+2n-1}) = (2i)^2(2i-1) + (2i+2n-1)^2(2i+2n-2)$, $\quad i = 1,2,\text{-------------------},n-1$

$f^*_{adcss}(v_{2i-1}\, v_{2i+2n-2}) = (2i-1)^2(2i-2) + (2i+2n-2)^2(2i+2n-3)$, $\quad i = 1,2,\text{-------------------},n$

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $T^2(T_n)$ admits adcss-labeling.

**Example 2.5** ADCSS labeling of the graph $T^2(T_4)$ is shown in figure (v)

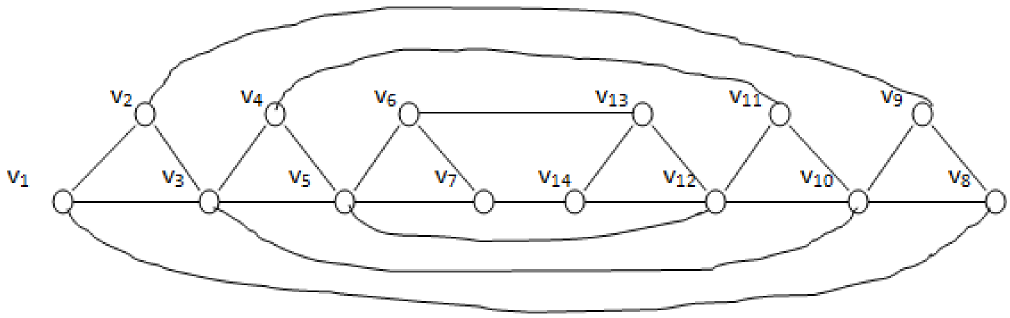


fig -v

**Theorem: 2.6** $T^2(Q_n)$ admits ADCSS – labeling, where $Q_n$ is the quadrilateral snake graph.

**Proof :** Let G = $T^2(Q_n)$ and let $v_1, v_2, \text{----------------}, v_{6n-4}$ are the vertices of G.

Here $|V(G)| = 6n-4$ and $|E(G)| = 11n-10$

Define a function $f : V \rightarrow \{1,2,3,\text{--------},6n-4\}$ by

$f(v_i) = i$ , $i = 1,2,\text{------},6n-4$.

For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) = (i+1)^2 i + i^2(i-1)$ $\quad i = 1,2,\text{-----------},3n-3$

$f^*_{adcss}(v_{3n-2+i}\, v_{3n-1+i}) = (3n-2+i)^2(3n-3+i) + (3n-1+i)^2(3n-2+i)$ , $\quad i = 1,2,3,\text{---------},3n-3$.

$f^*_{adcss}(v_{3i-2}\, v_{3i+1}) = (3i-2)^2(3i-3) + (3i+1)^2(3i)$ , $\quad i = 1,2,3,\text{---------},n-1$.

$f^*_{adcss}(v_{3n-4+3i}\, v_{3n-1+3i}) = (3n-4+3i)^2(3n-5+3i) + (3n-1+3i)^2(3n-2+3i)$, $\quad i = 1,2,\text{------------},n-1$

$f^*_{adcss}(v_i\, v_{3n-2+i}) = (3n-2+i)^2(3n-3+i) + i^2(i-1)$ $\quad i = 1,2,\text{-----------},3n-2$

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $T^2(Q_n)$ admits adcss-labeling.

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