

Between Closed Sets and $g\omega$ -Closed Sets

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Abstract: Levine [7] introduced the notion of g -closed sets and further properties of g -closed sets are investigated. In 1982, the notions of ω -open and ω -closed sets were introduced and studied by Hdeib [5]. Khalid Y. Al-Zoubi [6] introduced the notion of $g\omega$ -closed sets and further properties of $g\omega$ -closed sets are investigated. In this paper, we introduce the notion of $mg\omega$ -closed sets and obtain the unified characterizations for certain families of subsets between closed sets and $g\omega$ -closed sets.

Key words and phrases: $g\omega$ -closed set, m -structure, m -space, $mg\omega$ -closed set.

I. Introduction

In 1970, Levine [7] introduced the notion of generalized closed (g -closed) sets in topological spaces. In 1982, Hdeib [5] introduced the notion of ω -closed sets in topological spaces. Recently, many variations of g -closed sets are introduced and investigated. One among them is $g\omega$ -closed sets which were introduced by Khalid Y. Al-Zoubi [6]. In 2006, Noiri and Popa [11] introduced the notion of mg^* -closed sets and studied the basic properties, characterizations and preservation properties. Also, they defined several subsets which lie between closed sets and g -closed sets. In this paper, we introduce the notion of $mg\omega$ -closed sets and obtain the basic properties, characterizations and preservation properties. In the last section, we define several new subsets which lie between closed sets and $g\omega$ -closed sets.

II. Preliminaries

Let (X, τ) be a topological space and A a subset of X . The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively. A subset A is said to be regular open if $int(cl(A))=A$. The finite union of regular open sets is said to be π -open.

Definition 2.1: A subset A of a topological space (X, τ) is said to be α -open if $A \subset int(cl(int(A)))$. The complement of an α -open set is said to be α -closed.

Note: The family of all α -open (resp. regular open, π -open) sets in X is denoted by τ^α (resp. $RO(X)$, $\pi O(X)$).

Definition 2.2: A subset A of a topological space (X, τ) is said to be g -closed [7] (resp. g^* -closed [20] or strongly g -closed [18], πg -closed [4], rg -closed [14]) if $cl(A) \subset U$ whenever $A \subset U$ and U is open (resp. g -open, π -open, regular open) in (X, τ) . The complements of the above closed sets are called their respective open sets.

The family of all g -open sets in (X, τ) is denoted by $gO(X)$. The g -closure (resp. α -closure) of a subset A of X , denoted by $gcl(A)$ (resp. $\alpha cl(A)$), is defined to be the intersection of all g -closed sets (resp. α -closed sets) containing A .

Definition 2.3: A subset A of a topological space (X, τ) is said to be αg -closed [8] (resp. $g^\# \alpha$ -closed [13], $\pi g \alpha$ -closed [2], $r \alpha g$ -closed [10]) if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is open (resp. g -open, π -open, regular open) in (X, τ) .

The complements of the above closed sets are called their respective open sets.

Definition 2.4 [21]: Let H be a subset of a space (X, τ) , a point p in X is called a condensation point of H if for each open set U containing p , $U \cap H$ is uncountable.

Definition 2.5 [5]: A subset H of a space (X, τ) is called ω -closed if it contains all its condensation points. The complement of an ω -closed set is called ω -open. It is well known that a subset W of a space (X, τ) is ω -open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. The family of all ω -open sets, denoted by ω , is a topology on X , which is finer than τ . The interior and closure operator in (X, ω) are denoted by int_ω and cl_ω respectively.

Lemma 2.1 [5]: Let H be a subset of a space (X, τ) . Then

- (1) H is ω -closed in X if and only if $H = cl_\omega(H)$.
- (2) $cl_\omega(X \setminus H) = X \setminus int_\omega(H)$.
- (3) $cl_\omega(H)$ is ω -closed in X .
- (4) $x \in cl_\omega(H)$ if and only if $H \cap G \neq \emptyset$ for each ω -open set G containing x .
- (5) $cl_\omega(H) \subset cl(H)$.
- (6) $int(H) \subset int_\omega(H)$.

Definition 2.6: Let A be a subset of a space (X, τ) . Then A is said to be

- (1) $g^*\omega$ -closed [17] if $cl_\omega(A) \subset U$ whenever $A \subset U$ and U is g -open in (X, τ) .
- (2) $g\omega$ -closed [6] if $cl_\omega(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ) .
- (3) $\pi g\omega$ -closed [3] if $cl_\omega(A) \subset U$ whenever $A \subset U$ and U is π -open in (X, τ) .
- (4) $r\omega$ -closed [1] if $cl_\omega(A) \subset U$ whenever $A \subset U$ and U is regular open in (X, τ) .

Remark 2.1 [17]: For a subset of a topological space, we obtain the following implications:

$$\begin{array}{ccccc}
 \text{closed} & \rightarrow & g^*\text{-closed} & \rightarrow & g\text{-closed} \\
 \downarrow & & \downarrow & & \downarrow \\
 \omega\text{-closed} & \rightarrow & g^*\omega\text{-closed} & \rightarrow & g\omega\text{-closed}
 \end{array}$$

None of the above implications is reversible.

Lemma 2.2 [6]: The open image of an ω -open set is ω -open.

Throughout the present paper, (X, τ) and (Y, σ) always denote topological spaces and $f : (X, \tau) \rightarrow (Y, \sigma)$ presents a function.

III. m-Structures

Definition 3.1: A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a Minimal Structure (briefly m -Structure) [15] on X if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X) , we denote a nonempty set X with a minimal structure m_X on X and call it an m -space. Each member of m_X is said to be m_X -open (or briefly m -open) and the complement of an m_X -open set is said to be m_X -closed (or briefly m -closed).

Remark 3.1: Let (X, τ) be a topological space. Then the families, $\tau_\omega, \tau^\alpha, \tau, RO(X), \pi O(X)$ and $gO(X)$ are all m -structures on X .

Definition 3.2: Let (X, m_X) be an m -space. For a subset A of X , the m_X -closure of A and the m_X -interior of A are defined in [9] as follows:

- (1) $m_X\text{-cl}(A) = \cap \{F : A \subset F, X - F \in m_X\}$,
- (2) $m_X\text{-int}(A) = \cup \{U : U \subset A, U \in m_X\}$.

Remark 3.2: Let (X, τ) be a topological space and A a subset of X . If $m_X = \tau$ (resp. $\tau_\omega, \tau^\alpha, gO(X)$), then we have $m_X\text{-cl}(A) = cl(A)$ (resp. $cl_\omega(A), \alpha cl(A), gcl(A)$).

Lemma 3.1 [15]: Let (X, m_X) be an m -space and A a subset of X . Then $x \in m_X\text{-cl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x .

Definition 3.3[9]: An m -structure m_X on a nonempty set X is said to have property (B) if the union of any family of subsets belonging to m_X belongs to m_X .

Remark 3.3: Let (X, τ) be a topological space. Then the families $\tau_\omega, \tau^\alpha, \tau$ and $\pi O(X)$ are all m -structures with property (B).

Lemma 3.2 [16]: Let X be a nonempty set and m_X an m -structure on X satisfying property (B). For a subset A of X , the following properties hold:

- (1) $A \in m_X$ if and only if $m_X\text{-int}(A) = A$,
- (2) A is m -closed if and only if $m_X\text{-cl}(A) = A$,
- (3) $m_X\text{-int}(A) \in m_X$ and $m_X\text{-cl}(A)$ is m -closed.

Definition 3.4 [11]: Let (X, τ) be a topological space and m_X an m -structure on X . A subset A of X is said to be

- (1) mg^* -closed if $cl(A) \subset U$ whenever $A \subset U$ and U is m_X -open,
- (2) mg^* -open if its complement is mg^* -closed.

Proposition 3.1 [11]: Let $\tau \subset m_X$. Then the following implications hold:

closed $\rightarrow mg^*$ -closed $\rightarrow g$ -closed

Proposition 3.2: Let $\tau \subset m_X$. Then the following implications hold:

closed $\rightarrow mg^*$ -closed $\rightarrow g$ -closed $\rightarrow g\omega$ -closed

Proof: It follows from Remark 2.1.

Theorem 3.1 [11]: Let $\tau \subset m_X$ and m_X have property (B). A subset A of X is mg^* -closed if and only if $cl(A) - A$ does not contain any nonempty m -closed set.

Theorem 3.2 [11]: Let m_X have property (B). A subset A of X is mg^* -closed if and only if $m_X-cl(\{x\} \cap A) \neq \emptyset$ for each $x \in cl(A)$.

IV. $mg\omega$ -Closed Sets

In this section, let (X, τ) be a topological space and m_X an m -structure on X . We obtain several basic properties of $mg\omega$ -closed sets.

Definition 4.1: Let (X, τ) be a topological space and m_X an m -structure on X . A subset A of X is said to be

- (1) $mg\omega$ -closed if $cl_\omega(A) \subset U$ whenever $A \subset U$ and U is m_X -open,
- (2) $mg\omega$ -open if its complement is $mg\omega$ -closed.

Remark 4.1: Let (X, τ) be a topological space and A a subset of X . If $m_X = gO(X)$ (resp. $\tau, \pi O(X), RO(X)$) and A is $mg\omega$ -closed, then A is $g^*\omega$ -closed (resp. $g\omega$ -closed, $\pi g\omega$ -closed, $rg\omega$ -closed).

Proposition 4.1: Let $\tau \subset m_X$. Then the following implications hold: closed $\rightarrow \omega$ -closed $\rightarrow mg\omega$ -closed $\rightarrow g\omega$ -closed

Proof: It is obvious that every closed set is ω -closed [1,5] and every ω -closed set is $mg\omega$ -closed by Lemma 2.1(1). Suppose that A is an $mg\omega$ -closed set. Let $A \subset U$ and

$U \in \tau$. Since $\tau \subset m_X$, $cl_\omega(A) \subset U$ and hence A is $g\omega$ -closed.

Proposition 4.2:

Let m_X be an m -structure

on X in the topological space (X, τ) . If A and B are $mg\omega$ -closed, then $A \cup B$ is $mg\omega$ -closed.

Proof: Let $A \cup B \subset U$ and $U \in m_X$. Then $A \subset U$ and $B \subset U$. Since A and B are $mg\omega$ -closed, we have $cl_\omega(A \cup B) = cl_\omega(A) \cup cl_\omega(B) \subset U$. Therefore, $A \cup B$ is $mg\omega$ -closed.

Proposition 4.3: Let m_X be an m -structure on X in the topological space (X, τ) . If A is $mg\omega$ -closed and m -open, then A is ω -closed.

Proof: This is obvious.

Proposition 4.4: Let (X, m_X) be an m -space and $A \subseteq X$. If A is $mg\omega$ -closed and $A \subset B \subset cl_\omega(A)$, then B is $mg\omega$ -closed.

Proof: Let $B \subset U$ and $U \in m_X$. Then $A \subset U$ and A is $mg\omega$ -closed. Hence $cl_\omega(B) = cl_\omega(A) \subset U$ and B is $mg\omega$ -closed.

Definition 4.2: [12] Let (X, m_X) be an m -space and A a subset of X . The m_X -frontier of A , $m_X-Fr(A)$, is defined as follows:

$$m_X-Fr(A) = m_X-cl(A) \cap m_X-cl(X-A).$$

Proposition 4.5: If A is an $mg\omega$ -closed subset of X and $A \subset U \in m_X$, then $m_X-Fr(U) \subset int_\omega(X-A)$.

Proof:

Let A be $mg\omega$ -closed and $A \subset U \in m_X$. Then $cl_\omega(A) \subset U$.

Suppose that $x \in m_X-Fr(U)$.

Since $U \in m_X$, $m_X-Fr(U) = m_X-cl(U) \cap m_X-cl(X-U)$

$$= m_X-cl(U) \cap (X-U)$$

$$= m_X-cl(U) - U.$$

Therefore, $x \notin U$ and $x \notin cl_\omega(A)$.

This shows that $x \in int_\omega(X-A)$ and hence $m_X-Fr(U) \subset int_\omega(X-A)$.

Proposition 4.6: In the m -space (X, m_X) , a subset A of X is $mg\omega$ -open if and only if $F \subset \text{int}_\omega(A)$ whenever $F \subset A$ and F is m -closed.

Proof:

Suppose that A is $mg\omega$ -open. Let $F \subset A$ and F be m -closed.

Then $X - A \subset X - F \in m_X$ and $X - A$ is $mg\omega$ -closed.

Therefore, we have $X - \text{int}_\omega(A) = \text{cl}_\omega(X - A) \subset X - F$ and hence $F \subset \text{int}_\omega(A)$.

Conversely, let $X - A \subset G$ and $G \in m_X$.

Then $X - G \subset A$ and $X - G$ is m -closed.

By the hypothesis, we have $X - G \subset \text{int}_\omega(A)$ and hence $\text{cl}_\omega(X - A) = X - \text{int}_\omega(A) \subset G$. Therefore, $X - A$ is $mg\omega$ -closed and A is $mg\omega$ -open.

Corollary 4.1: Let $\tau \subset m_X$. Then the following properties hold:

- (1) Every open set is $mg\omega$ -open and every $mg\omega$ -open set is $g\omega$ -open,
- (2) If A and B are $mg\omega$ -open, then $A \cap B$ is $mg\omega$ -open,
- (3) If A is $mg\omega$ -open and m -closed, then A is ω -open,
- (4) If A is $mg\omega$ -open and $\text{int}_\omega(A) \subset B \subset A$, then B is $mg\omega$ -open.

Proof: This follows from Propositions 4.1, 4.2, 4.3 and 4.4.

Proposition 4.7: Every mg^* -closed set is $mg\omega$ -closed.

Proof: It follows from Lemma 2.1(5).

Proposition 4.8: Let $\tau \subset m_X$. Then every mg^* -closed set is $g\omega$ -closed.

Proof: It follows from Propositions 4.1 and 4.7.

Proposition 4.9: Let $\tau \subset m_X$. Then the following implications hold:

$\text{closed} \rightarrow mg^*\text{-closed} \rightarrow mg\omega\text{-closed} \rightarrow g\omega\text{-closed}$

Proof: It follows from Propositions 3.2, 4.1 and 4.7.

V. Characterizations Of $mg\omega$ -Closed Sets

In this section, let (X, τ) be a topological space and m_X an m -structure on X . We obtain some characterizations of $mg\omega$ -closed sets.

Theorem 5.1: A subset A of X is $mg\omega$ -closed if and only if $\text{cl}_\omega(A) \cap F = \emptyset$ whenever $A \cap F = \emptyset$ and F is m -closed.

Proof: Suppose that A is $mg\omega$ -closed. Let $A \cap F = \emptyset$ and F be m -closed. Then

$A \subset X - F \in m_X$ and $\text{cl}_\omega(A) \subset X - F$.

Therefore, we have $\text{cl}_\omega(A) \cap F = \emptyset$.

Conversely, let $A \subset U$ and $U \in m_X$. Then

$A \cap (X - U) = \emptyset$ and $X - U$ is m -closed.

By the hypothesis, $\text{cl}_\omega(A) \cap (X - U) = \emptyset$ and hence $\text{cl}_\omega(A) \subset U$.

Therefore, A is $mg\omega$ -closed.

Theorem 5.2: Let $\tau_\omega \subset m_X$ and m_X have property (B). A subset A of X is $mg\omega$ -closed if and only if $\text{cl}_\omega(A) - A$ does not contain any nonempty m -closed set.

Proof: Suppose that A is $mg\omega$ -closed. Let $F \subset \text{cl}_\omega(A) - A$ and F be m -closed. Then

$F \subset \text{cl}_\omega(A)$ and $A \subset X - F \in m_X$.

Hence $\text{cl}_\omega(A) \subset X - F$.

Therefore, we have $F \subset X - \text{cl}_\omega(A)$.

Hence $F \subset \text{cl}_\omega(A) \cap (X - \text{cl}_\omega(A)) = \emptyset$.

Conversely, suppose that A is not $mg\omega$ -closed. Then

$\emptyset \neq \text{cl}_\omega(A) - U$ for some $U \in m_X$ containing A .

Since $\tau_\omega \subset m_X$ and m_X has property (B), $\text{cl}_\omega(A) - U$ is m -closed.

Moreover, $\text{cl}_\omega(A) - U \subset \text{cl}_\omega(A) - A$.

Thus $\text{cl}_\omega(A) - A$ contains a nonempty m -closed set which is a contradiction.

Hence A is $mg\omega$ -closed.

Theorem 5.3: Let $\tau_\omega \subset m_X$ and m_X have property (B). A subset A of X is $mg\omega$ -closed if and only if $\text{cl}_\omega(A) - A$ is $mg\omega$ -open.

Proof: Suppose that A is $mg\omega$ -closed. Let $F \subset cl_\omega(A) - A$ and F be m-closed.

By Theorem 5.2, we have $F = \emptyset$ and $F \subset int_\omega(cl_\omega(A) - A)$.

It follows from Proposition 4.6, $cl_\omega(A) - A$ is $mg\omega$ -open.

Conversely, let $A \subset U$ and $U \in m_X$.

Then $cl_\omega(A) \cap (X - U) \subset cl_\omega(A) - A$ and $cl_\omega(A) - A$ is $mg\omega$ -open.

Since $\tau_\omega \subset m_X$ and m_X has property (B), $cl_\omega(A) \cap (X - U)$ is m-closed and by Proposition 4.6, $cl_\omega(A) \cap$

$(X - U) \subset int_\omega(cl_\omega(A) - A)$.

Now, $int_\omega(cl_\omega(A) - A) = int_\omega(cl_\omega(A)) \cap int_\omega(X - A)$

$\subset cl_\omega(A) \cap int_\omega(X - A)$

$= cl_\omega(A) \cap (X - cl_\omega(A)) = \emptyset$.

Thus $cl_\omega(A) \cap (X - U) = \emptyset$ and hence $cl_\omega(A) \subset U$.

This shows that A is $mg\omega$ -closed.

Theorem 5.4: Let m_X have property (B). A subset A of X is $mg\omega$ -closed if and only if

$m_X-cl(\{x\}) \cap A \neq \emptyset$ for each $x \in cl_\omega(A)$.

Proof: Suppose $m_X-cl(\{x\}) \cap A = \emptyset$ for some $x \in cl_\omega(A)$.

By Lemma 3.2, $m_X-cl(\{x\})$ is m-closed and $A \subset X - (m_X-cl(\{x\})) \in m_X$.

If $cl_\omega(A) \subset X - (m_X-cl(\{x\}))$ then

$x \in cl_\omega(A) \subset X - (m_X-cl(\{x\})) \subset X - \{x\}$ is a contradiction.

Thus $cl_\omega(A) \not\subset X - (m_X-cl(\{x\}))$ and hence A is not $mg\omega$ -closed.

Conversely, suppose that A is not $mg\omega$ -closed.

Then there exists $U \in m_X$ such that $A \subset U$, but $cl_\omega(A) \not\subset U$.

So there exists $x \in cl_\omega(A)$ but $x \notin U$. Then $x \in U^c$ which is m_X -closed.

Thus $m_X-cl(\{x\}) \subset m_X-cl(U^c) = U^c$.

This implies $m_X-cl(\{x\}) \cap U = \emptyset$.

Hence $m_X-cl(\{x\}) \cap A \subset m_X-cl(\{x\}) \cap U = \emptyset$.

Thus there exists $x \in cl_\omega(A)$ such that $m_X-cl(\{x\}) \cap A = \emptyset$. This proves the converse.

Corollary 5.1: Let $\tau_\omega \subset m_X$ and m_X have property (B). For a subset A of X, the following properties are equivalent:

- (1) A is $mg\omega$ -open,
- (2) $A - int_\omega(A)$ does not contain any nonempty m-closed set,
- (3) $A - int_\omega(A)$ is $mg\omega$ -open,
- (4) $m_X-cl(\{x\}) \cap (X - A) = \emptyset$ for each $x \in X - int_\omega(A)$.

Proof: This follows from Proposition 4.6 and Theorems 5.2, 5.3 and 5.4.

VI. Preservation Theorems

Definition 6.1 [11]: A function $f : (X, m_X) \rightarrow (Y, m_Y)$ is said to be

- (1) M-continuous if for each $x \in X$ and each $V \in m_Y$ containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$.
- (2) M-closed if for each m-closed set F of (X, m_X) , $f(F)$ is m-closed in (Y, m_Y) .

Theorem 6.1 [15]: Let m_X be an m-structure on X with property (B) and m_Y be a minimal structure on Y. Let $f : (X, m_X) \rightarrow (Y, m_Y)$ be a function. Then the following are equivalent:

- (1) f is M-continuous,
- (2) $f^{-1}(V) \in m_X$ for every $V \in m_Y$.

Lemma 6.1 [11]: A function $f : (X, m_X) \rightarrow (Y, m_Y)$ is M-closed if and only if for each subset B of Y and each $U \in m_X$ containing $f^{-1}(B)$, there exists $V \in m_Y$ such that $B \subset V$ and $f^{-1}(V) \subset U$.

Theorem 6.2: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is closed and $f : (X, m_X) \rightarrow (Y, m_Y)$ is M-continuous, where m_X has property (B), then $f(A)$ is $mg\omega$ -closed in (Y, m_Y) for each $mg\omega$ -closed set A of (X, m_X) .

Proof: Let A be any $mg\omega$ -closed set of (X, m_X) and $f(A) \subset V \in m_Y$.

Since m_X has property (B), $A \subset f^{-1}(V) \in m_X$ by Theorem 6.1.

Since A is $mg\omega$ -closed, $cl_\omega(A) \subset f^{-1}(V)$ and $f(cl_\omega(A)) \subset V$.

Since f is closed, by Lemma 2.2, $cl_\omega(f(A)) \subset f(cl_\omega(A)) \subset V$.

Hence $f(A)$ is $mg\omega$ -closed in (Y, m_Y) .

Definition 6.2 [6]: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called ω -irresolute if $f^{-1}(B)$ is ω -open in (X, τ) for every ω -open set B of (Y, σ) .

Theorem 6.3: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is ω -irresolute and $f : (X, m_X) \rightarrow (Y, m_Y)$ is M -closed, then $f^{-1}(B)$ is $mg\omega$ -closed in (X, m_X) for each $mg\omega$ -closed set B of (Y, m_Y) .

Proof: Let B be any $mg\omega$ -closed set of (Y, m_Y) and $f^{-1}(B) \subset U \in m_X$.

Since f is M -closed, by Lemma 6.1, there exists $V \in m_Y$ such that

$B \subset V$ and $f^{-1}(V) \subset U$.

Since B is $mg\omega$ -closed in Y , $cl_\omega(B) \subset V$ and

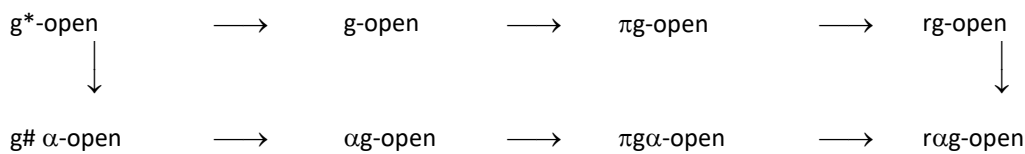
since f is ω -irresolute, $cl_\omega(f^{-1}(B)) \subset f^{-1}(cl_\omega(B)) \subset f^{-1}(V) \subset U$.

Hence $f^{-1}(B)$ is $mg\omega$ -closed in (X, m_X) .

VII. New Forms Of $mg\omega$ -Closed Sets

In a topological space (X, τ) , from the definitions, we obtain the following diagram.

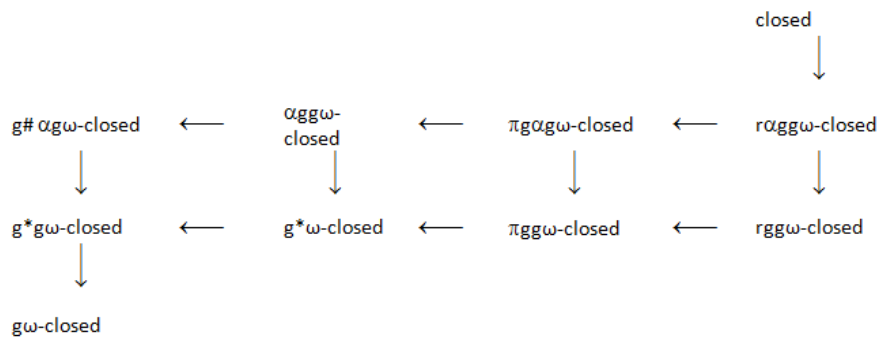
Diagram – I



In (X, τ) we denote the collection of all g -open (resp. g^* -open, πg -open, rg -open, $g^\# \alpha$ -open, αg -open, $\pi g \alpha$ -open, $rg \alpha$ -open) sets by $gO(X)$ (resp. $g^*O(X)$, $\pi gO(X)$, $rgO(X)$, $g^\# \alpha O(X)$, $\alpha gO(X)$, $\pi g \alpha O(X)$, $rg \alpha O(X)$). These collections of are all m -structure on X . Using these m -structures $gO(X)$ $g^*O(X)$, $\pi gO(X)$, $rgO(X)$, $g^\# \alpha O(X)$, $\alpha gO(X)$, $\pi g \alpha O(X)$, $rg \alpha O(X)$ for a subset A , we define new types of $g\omega$ -closed sets as follows.

Dfinition 7.1: A subset A of a topological space (X, τ) is said to be $g^*g\omega$ -closed (resp. $g^*\omega$ -closed [18], $\pi gg\omega$ -closed, $rgg\omega$ -closed, $g^\# \alpha g\omega$ -closed, $\alpha gg\omega$ -closed, $\pi g \alpha g\omega$ -closed, $r g g\omega$ -closed) if $cl_\omega(A) \subset U$ whenever $A \subset U$ and U is g^* -open (resp. g -open, πg -open, rg -open, $g^\# \alpha$ -open, αg -open, $\pi g \alpha$ -open, $r g \alpha$ -open) in (X, τ) . By Diagram I and Definition 7.1, we have the following diagram:

Diagram - II



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