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Coupled fixed point theorems for self maps on partially ordered multiplicative metric spaces

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Abstract: *In this paper we prove some theorems on the existence and uniqueness of coupled fixed pints, which are improvements of the results of Shanjit et al.[5].*
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I. Introduction and Preliminaries

The study for the existence of fixed points of contractive mappings is a famous topic in metric spaces. In 1922 Banach's contraction principle [2] guarantees the existence and uniqueness of fixed point of a contractive mapping on a complete metric spaces. This principle is applicable to a variety of subjects such as integral equations, differential equations, image processing and many others.

In the past years, many authors generalized Banach's contraction principle in various spaces, for example Quasi-metric spaces, Fuzzy metric spaces, Partial metric spaces and generalized metric spaces [8,9,11,13]. In 2008, Bashirov and Ozyapici [1] introduced the notion of multiplicative metric space and studied the concept of multiplicative calculus and illustrated the usefulness of multiplicative calculus with some interesting applications. In 2012, Ozavsar and Civikel [6] introduced the concept of multiplicative mapping in multiplicative metric space and proved some fixed point theorems for this type of mappings.

In 1987, Guo and Lakshmikantham [3] introduced the concept of coupled fixed point. Later, Bhaskar and Lakshmikantham [11] proved a new fixed oint theorem for a mixed monotone mapping in a metric space powered with partially order by using a weak contractive type assumption.

Recently X.He et al. [12] proved common fixed oint theorems for four self mappings in multiplicative metric space. In 2014, Ravi.P et al.[10] proved coupled coincidence point and common coupled fixed point theorem lacking the mixed monotone property.

In this paper we prove some theorems on the existence and uniqueness of coupled fixed pints, which are improvements of the results of Shanjit et al.[5].

Definition 1.1. (A.E.Bashirov, E.M.Kurplnara, A.Ozyapici [1]). Let X be a nonempty set. A multiplicative metric is a mapping $d : X \times X \to \mathbf{R}^+$ satisfying the following conditions:
(i) $d(x, y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$, if and only if $x = y$.
(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$.
(iii) $d(x, y) \leq d(x, z).d(z, y)$ for all $x, y, z \in X$.(Multiplicative triangle inequality)
Also (X, d) is called a multiplicative metric space.

Example 1.2. (M.Ozavser, A.C.Cevikel [6]). Let $d^* : (\mathbf{R}^+)^n \times (\mathbf{R}^+)^n \to \mathbf{R}^+$ be defined as follows

$$d^*(x, y) = \left|\frac{x_1}{y_1}\right|^* \left|\frac{x_2}{y_2}\right|^* ......\left|\frac{x_n}{y_n}\right|^*.$$

where $x = (x_1, x_2, ...x_n), y = (y_1, y_2, ...y_n) \in \mathbf{R}^+$ and

$|\,.\,|^{*}:\mathbf{R}^{+}\to\mathbf{R}^{+}$ is $|a|^{*}=\begin{cases} a & \text{if } a\geq 1 \\ \frac{1}{a} & \text{if } a\leq 1 \end{cases}$

Then $((\mathbf{R}^{+})^{n},d^{*})$ is a multiplicative metric space.

Example 1.3. (M.Ozavser, A.C.Cevikel [6]).Let a > 1 be fixed real number. then $d_a:\mathbf{R}^{+}\to\mathbf{R}^{+}$ defined by

$d_a(w,z)=a^{\sum_{i=1}^{n}|w_i-z_i|}$ where $w=(w_1,w_2,...w_n),z=(z_1,z_2,...z_n)\in\mathbf{R}^{+}$.

Obviously, $(\mathbf{R}^{+},d_a)$ is a multiplicative metric space. We can also extend multiplicative metric to $\mathbb{C}^{n}$ by the following definition:

$d_a(w,z)=a^{\sum_{i=1}^{n}|w_i-z_i|}$ where $w=(w_1,w_2,...w_n),z=(z_1,z_2,...z_n)\in\mathbb{C}^{n}$.

Example 1.4. (M.Ozavser, A.C.Cevikel [6]). Let $X=\{(x,1)\in\mathbf{R}^{2}:1\leq x\leq 2\}\bigcup\{(1,x)\in\mathbf{R}^{2}:1\leq x\leq 2\}$.

Consider a mapping $d:X\times X\to\mathbb{R}$ defined by $d((a,b),(c,d))=(|\frac{a}{c}|^{*}|\frac{b}{d}|^{*})^{\frac{1}{3}}$. Then (X,d) is a multiplicative metric space.

Definition 1.5. (M.Ozavser, A.C.Cevikel [6]).(Multiplicative convergence). Let (X,d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x\in X$. If for every multiplicative open ball $B_{\varepsilon}(x)=\{y/d(x,y)<\varepsilon\},\varepsilon>1$ there exists a natural number N such that $n\geq N$, $x_n\in B_{\varepsilon}(x)$, the sequence $\{x_n\}$ is said to be multiplicative converging to x, denoted by $x_n\to x$ $(n\to\infty)$.

Definition 1.6. (M.Ozavsar, A.C.Civikel [6]). Let (X,d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x\in X$. The sequence $\{x_n\}$ is called a multiplicative Cauchy sequence if, for each $\varepsilon>1$, there exists $N\in\mathbf{N}$ such that $d(x_n,x_m)<\varepsilon$, for all $m,n\geq N$

Definition 1.7. (M.Ozavsar, A.C.Civikel [6]). Let (X,d) be a multiplicative metric space. A mapping $f:X\to X$ is called a multiplicative contraction if there exists a real constant $\lambda\in[0,1)$ such that $d(fx,fy)\leq d(x,y)^{\lambda}$ for all $x,y\in X$.

Definition 1.8. (M.Ozavsar, A.C.Civikel [6])(Multiplicative continuity). Let (X,d_X) and (Y,d_Y) be two multiplicative metric spaces and $f:X\to Y$ be a function. If for every $\varepsilon>1$, there exists $\delta>1$ such that $f(B_{\delta}(x))\subset B_{\varepsilon}(f(x))$, then we call f multiplicative continuous at $x\in X$.

Definition 1.9. (M.Ozavsar, A.C.Civikel [6]). Let (X,d) be a multiplicative metric space. We call (X,d) is complete if every multiplicative Cauchy sequence in X is multiplicative convergent to $x\in X$.

Definition 1.10. (T.Gnana Bhaskar, V.Lakshmikantham[11]). Let $(X,\preceq)$ be a partially ordered set and $S:X\times X\to X$. The mapping S is said to have the mixed monotone property if S is monotone non-decreasing in its first argument and monotone non-increasing in its second argument. that is, for any $x,y\in X$,

$x_1,x_2\in X$, $x_1\preceq x_2\Rightarrow S(x_1,y)\preceq S(x_2,y)$,

$y_1,y_2\in X$, $y_1\preceq y_2\Rightarrow S(x,y_1)\succeq S(x,y_2)$.

Definition 1.11. (T.Gnana Bhaskar, V.Lakshmikantham[11]). An element $(x,y)\in X\times X$ is called coupled fixed point of the mapping $S:X\times X\to X$ if $S(x,y)=x$ and $S(y,x)=y$.

Recently L.Shanjit, Y.Rohan et al [5] proved the following coupled fixed point theorem.

Theorem 1.12. (L.Shanjt, Y.Rohan et al [5]). Let $(X,\preceq)$ be a partially ordered set and suppose that there is a multiplicative metric d on X such that (X,d) is a multiplicative metric space. Let $S:X\times X\to X$ be a

continuous mapping having the mixed monotone property on X. Assume that there exists a $\lambda \in [0,1)$ with $d(S(x,y),S(u,v)) \le [d(x,u).d(y,v)]^{\frac{\lambda}{2}}$ for each $x \succeq u$ and $y \preceq v$.
If there exists $x_0, y_0 \in X$ such that $x_0 \preceq S(x_0, y_0)$ and $y_0 \succeq S(y_0, x_0)$, then there exists $x, y \in X$ such that $x = S(x,y)$ and $y = S(y,x)$. If for every $(x,y),(x^*,y^*) \in X \times X$, there exists $(z_1,z_2) \in X \times X$ such that $(S(z_1,z_2),S(z_2,z_1))$ is comparable with $(S(x,y),S(y,x))$ and $(S(x^*,y^*),S(y^*,x^*))$, then S has a unique coupled fixed point.

II. Main Result

In this section we improve and extend theorem 1.12.

Definition 2.1. Suppose $(X,\preceq)$ is a partially ordered set. Suppose $(x,y) \in X \times X$ and $(u,v) \in X \times X$. Then we write $(x,y) \preceq (u,v)$ if $x \preceq u$ and $y \succeq v$. Also we write $(x,y) \succeq (u,v)$ if $x \succeq u$ and $y \preceq v$. If either $(x,y) \preceq (u,v)$ or $(x,y) \succeq (u,v)$ then we say that (x,y) is comparable with (u,v). It may be observed that $(X \times X,\preceq)$ is a partially ordered set with the above ordering on $X \times X$.

Note: Let (X,d) be a multiplicative metric space. Define $D: X^2 \times X^2 \to \mathbf{R}^+$ by $D((x,y),(u,v)) = d(x,u).d(y,v)$ for all $x,y,u,v \in X$. Then D is a multiplicative metric on $X \times X$ and $(X \times X, D)$ is a multiplicative metric space.

Lemma 2.2. Suppose $(X \times X, D)$ is a complete multiplicative metric space then $x_n \to x$ and $y_n \to y$ in X iff $(x_n, y_n) \to (x,y)$ in $X \times X$.

Proof: Suppose $x_n \to x$ and $y_n \to y$ in X

$$D((x_n,y_n),(x,y)) = d((x_n,x).d(y_n,y)$$

$$\to 1 \qquad (since\ d(x_n,x) \to 1 \text{ and } d(y_n,y) \to 1)$$

$\therefore (x_n,y_n) \to (x,y)$.

Conversely suppose that $(x_n,y_n) \to (x,y)$

Now $d((x_n,x).d(y_n,y) = D((x_n,y_n),(x,y)) \to 1$

$\therefore d((x_n,x).d(y_n,y) \to 1$

that is $d(x_n,x) \to 1$ and $d(y_n,y) \to 1$.

$\therefore x_n \to x$ and $y_n \to y$.

Lemma 2.3. Suppose $(X \times X, D)$ is a multiplicative metric space and $S: X \times X \to X$ is a continuous map. If $(x_n,y_n) \to (x,y)$ in $X \times X$. Then $S(x_n,y_n) \to S(x,y)$ w.r.to D in $X \times X$.

Lemma 2.4. Let $(X,\preceq)$ be a partially ordered set and suppose that (X,d) is a multiplicative metric space. Let $S: X \times X \to X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists $\lambda \in [0,1)$ with $d(S(x,y),S(u,v)) \le max\{d(x,u),d(y,v)\}^{\lambda}$ whenever (x,y) and (u,v) are comparable. Suppose (x,y) is a coupled fixed point of S which is comparable with (u,v). Define the sequence (u_n,v_n) such that $u_1 = S(u,v)$ and $v_1 = S(v,u)$ and inductively $u_{n+1} = S(u_n,v_n)$ and $v_{n+1} = S(v_n,u_n)$. Then $u_n \to x$ and $v_n \to y$.

Proof: Define $S(u,v) = u_1$ and $S(v,u) = v_1$, and suppose (x,y) is comparable to (u,v).

Without loss of generality we may assume that $x \preceq u$ and $y \succeq v$.

Now $x = S(x,y) \preceq S(u,y) \preceq S(u,v) = u_1$

and $y = S(y,x) \succeq S(v,x) \succeq S(v,u) = v_1$.

$\therefore (x,y) \preceq (S(u,v),S(v,u)) = (u_1,v_1)$

also $S(x,y) \preceq S(S(u,v),S(v,u)) = S(u_1,v_1) = u_2$

and $S(y,x) \succeq S(S(v,u),S(u,v)) = S(v_1,u_1) = v_2$

Similarly $(x,y) = (S(x,y),S(y,x)) \preceq (u_2,v_2)$. i.e., $x \preceq u_2$ and $y \pm v_2$.

$\therefore (x,y) \preceq (u_2,v_2)$.

Now $x = S(x,y) \preceq S(u_2,v_2) = u_3$

and $y = S(y,x) \succeq S(v_2,u_2) = v_3$

$\therefore (x,y) \preceq (u_3,v_3)$.

By induction $x \preceq u_n$ and $y \succeq v_n \; \forall n$. i.e., $(x,y) \preceq (u_n,v_n)$.

Now $d(x,u_2) = d(S(x,y),S(u_1,v_1)) \le max\{d(x,u_1),d(y,v_1)\}^\lambda$

$$\le max\{d(x,S(u,v)),d(y,S(v,u))\}^\lambda . \qquad (2.4.1)$$

$$d(x,S(u,v)) = d(S(x,y),S(u,v))$$
$$\le max\{d(x,u),d(y,v)\}^\lambda$$
$$= \alpha^\lambda \qquad \text{(where } \alpha = max\{d(x,u),d(y,v)\})$$

Similarly $d(y,S(v,u)) = \alpha^\lambda$

Hence, from (2.4.1), we have

$d(x,u_2) \le \max\left\{\alpha^\lambda,\alpha^\lambda\right\}^\lambda = \alpha^{\lambda^2}$.

and $d(y,v_2) = d(S(y,x),S(v_1,u_1)) \le max\{d(y,v_1),d(x,u_1)\}^\lambda$

$$= max\{d(y,S(v,u)),d(x,S(u,v))\}^\lambda$$
$$\le max\{\alpha^\lambda,\alpha^\lambda\}^\lambda$$
$$= \alpha^{\lambda^2} .$$

Similarly $d(x,u_3) \le \alpha^{\lambda^3}$ and $d(y,v_3) \le \alpha^{\lambda^3}$

By induction $d(x,u_n) \le \alpha^{\lambda^n}$ and $d(y,v_n) \le \alpha^{\lambda^n}$

$\therefore d(x,u_n) \le \alpha^{\lambda^n} \to 1$ (as $n \to \infty, 0 \le \lambda < 1$)

$\therefore u_n \to x$.

and $d(y,v_n) \le \alpha^{\lambda^n} \to 1$ (as $n \to \infty, 0 \le \lambda < 1$)

$\therefore v_n \to y$.

Theorem 2.5. Let $(X,\preceq)$ be a partially ordered set and suppose that (X,d) is a multiplicative metric space. Let $S : X \times X \to X$ be a continuous mapping having the mixed monotone property on X . Assume that there exists $\lambda \in [0,1)$ with $d(S(x,y),S(u,v)) \le max\{d(x,u),d(y,v)\}^\lambda$ whenever (x,y) and (u,v) are comparable. If there exist $x_0,y_0 \in X$ such that $(x_0,y_0) \preceq (S(x_0,y_0),S(y_0,x_0))$, then there exist $x,y \in X$ such that $x = S(x,y)$ and $y = S(y,x)$. That is, (x,y) is a coupled fixed point of S . If for every $(x,y),(x^*,y^*) \in X \times X$, there exists $(u,v) \in X \times X$ such that (u,v) is comparable with (x,y) and (x^*,y^*) , then S has a unique coupled fixed point.

Proof: Define the sequence (x_n,y_n) inductively, by $x_1 = S(x_0,y_0)$, $y_1 = S(y_0,x_0)$ and $x_{n+1} = S(x_n,y_n)$, $y_{n+1} = S(y_n,x_n)$ for $n = 1,2,.....$

By hypothesis $x_0 \preceq S(x_0,y_0) = x_1$ and $y_0 \succeq S(y_0,x_0) = y_1$.

Let $x_2 = S(x_1, y_1)$ and $y_2 = S(y_1, x_1)$
By mixed monotone property of S, we have
$x_1 = S(x_0, y_0) \preceq S(x_1, y_0) \preceq S(x_1, y_1) = x_2$
and $y_1 = S(y_0, x_0) \succeq S(y_1, x_0) \succeq S(y_1, x_1) = y_2$
By induction we can show that
$x_{n+1} = S(x_n, y_n) \succeq S(x_{n-1}, y_n) \succeq S(x_{n-1}, y_{n-1}) = x_n$
$y_{n+1} = S(y_n, x_n) \preceq S(y_{n-1}, x_n) \preceq S(y_{n-1}, x_{n-1}) = y_n$
Now for all $n \in N$, we have
$d(x_{n+1}, x_n) = d(S(x_n, y_n), S(x_{n-1}, y_{n-1})) \le max\{d(x_n, x_{n-1}), d(y_n, y_{n-1})\}^\lambda$
and $d(y_{n+1}, y_n) = d(S(y_n, x_n), S(y_{n-1}, x_{n-1})) \le max\{d(y_n, y_{n-1}), d(x_n, x_{n-1})\}^\lambda$
Write $\alpha_n = max\{d(x_n, x_{n-1}), d(y_n, y_{n-1})\}$

therefore

$$(\alpha_{n+1})^\lambda = max\{d(x_{n+1}, x_n), d(y_{n+1}, y_n)\}^\lambda$$
$$= max\{\{d(x_{n+1}, x_n)\}^\lambda, \{d(y_{n+1}, y_n)\}^\lambda\}^\lambda$$
$$\le max\{(\alpha_n{}^\lambda)^\lambda, (\alpha_n{}^\lambda)^\lambda\}$$
$$\le max\{(\alpha_n)^{\lambda^2}, (\alpha_n)^{\lambda^2}\} = (\alpha_n)^{\lambda^2}$$

$$\therefore (\alpha_{n+1})^\lambda \le (\alpha_n)^{\lambda^2} \le (\alpha_{n-1})^{\lambda^3} \le (\alpha_{n-2})^{\lambda^4} \le ....(\alpha_1)^{\lambda^{n+1}} \to 1 \text{ as } n \to \infty$$

$(since \quad \alpha \ge 1$ and $\lambda \in [0,1))$.
$\therefore (\alpha_{n+1})^\lambda \to 1$, $1 \le \alpha_{n+1}^\lambda \to 1$
Hence $1 \le \alpha_{n+1} \to 1$.
$\therefore 1 \le max\{d(x_{n+1}, x_n), d(y_{n+1}, y_n)\} \to 1$
$\therefore d(x_{n+1}, x_n) \to 1$ and $d(y_{n+1}, y_n) \to 1$.
Now we show that the sequences $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X.
Write $z_n = (x_n, y_n)$ and $D(z_n, z_m) = d(x_n, x_m).d(y_n, y_m)$
Now

$$D((x_n, y_n), (x_{n+k}, y_{n+k})) \le D((x_n, y_n), (x_{n+1}, y_{n+1})).D((x_{n+1}, y_{n+1}), (x_{n+k}, y_{n+k}))$$
$$= D(z_n, z_{n+1}).D(z_{n+1}, z_{n+k})$$
$$\le D(z_n, z_{n+1}).D(z_{n+1}, z_{n+2}).D(z_{n+2}, z_{n+k})$$
$$\le D(z_n, z_{n+1}).D(z_{n+1}, z_{n+2}).........D(z_{n+k-1}, z_{n+k})$$
$$= D((x_n, y_n), (x_{n+1}, y_{n+1})).D((x_{n+1}, y_{n+1}), (x_{n+2}, y_{n+2}))...........D((x_{n+k-1}, y_{n+k-1}), (x_{n+k}, y_{n+k}))$$
$$= d(x_n, x_{n+1}).d(y_n, y_{n+1}).d(x_{n+1}, x_{n+2}).d(y_{n+1}, y_{n+2})..........d(x_{n+k-1}, x_{n+k}).d(y_{n+k-1}, y_{n+k})$$
$$\le max\{d(x_n, x_{n+1}), d(y_n, y_{n+1})\}^2.max\{d(x_{n+1}, x_{n+2}).d(y_{n+1}, y_{n+2})\}^2...........max\{d(x_{n+k-1}, x_{n+k}).d(y_{n+k-1}, y_{n+k})\}^2$$
$$= (\alpha_{n+1})^2.(\alpha_{n+2})^2...............(\alpha_{n+k})^2.$$
$$\le [(\alpha_1)^{\lambda^{(n)}}]^2.[(\alpha_1)^{\lambda^{(n+1)}}]^2.[(\alpha_1)^{\lambda^{(n+2)}}]^2..........[(\alpha_1)^{\lambda^{(n+k-1)}}]^2.$$
$$= \alpha_1^{2[\lambda^n + \lambda^{(n+1)} + \lambda^{(n+2)} + ... + \lambda^{(n+k-1)}]}.$$
$$\le \alpha_1^{2[\frac{\lambda^n}{1-\lambda}]} \to 1 \text{ as } n \to \infty.$$

$\therefore$ Sequence (x_n, y_n) is a Cauchy sequence in $X \times X$ w.r.to D.
Suppose $(x_n, y_n) \to (x, y)$. Then $x_n \to x$ and $y_n \to y$.

Now we show that $d(x_{n+1}, S(x,y)) \to 1$

$i.e., d(S(x_n, y_n), S(x,y)) \to 1$.

Since S is continuous, and $x_n \to x$, $y_n \to y$ then $S(x_n, y_n) \to S(x,y)$

Now $x_{n+1} = S(x_n, y_n) \to S(x,y)$.

$\therefore x = S(x,y)$.

And similarly $y_{n+1} = S(y_n, x_n) \to S(y,x)$.

$\therefore y = S(y,x)$.

$\therefore (x,y)$ is a coupled foxed point of S.

Suppose (x,y) and (x^*, y^*) are coupled fixed points of S. Then there exists $(u,v) \in X \times X$ such that (u,v) is comparable with (x,y) and (x^*, y^*)

$\underline{Case I:}$ Suppose (x,y) and (x^*, y^*) are coupled fixed points of S and (x,y) is comparable to (x^*, y^*) w.r.to the ordering in $X \times X$.

Claim : $x = x^*, y = y^*$.

$d(x,x^*) = d(S(x,y), S(x^*,y^*)) \leq max\{d(x,x^*), d(y,y^*)\}^{\lambda}$

$(\because (x,y) and\ (x^*,y^*)$ are comparable)

Similarly $d(y,y^*) = d(S(y,x), S(y^*,x^*)) \leq max\{d(y,y^*), d(x,x^*)\}^{\lambda}$

$\therefore max\{d(x,x^*), d(y,y^*)\} \leq max\{d(x,x^*), d(y,y^*)\}^{\lambda}$

$\therefore max\{d(x,x^*), d(y,y^*)\} = 1 \qquad (0 \leq \lambda < 1)$

$\therefore d(x,x^*) = 1, d(y,y^*) = 1$

$\therefore x = x^*, y = y^*$. Hence $(x,y) = (x^*,y^*)$

$\underline{Case II:}$ If (x,y) is not comparable to (x^*,y^*), then there exists an upper bound (or) lower bound $z = (u,v) \in X \times X$ then for all $n = 0,1,2,3,.....$

By lemma 2.4 $u_n \to x$, $v_n \to y$ (where u_n and v_n are as in Lemma 2.4)

and $D((x,y),(u_n,v_n)) = d(x,u_n).d(y,v_n) \to 1$ as $n \to \infty$

$\therefore (u_n,v_n)) \to (x,y)$ as $n \to \infty$ (2.5.1)

Similarly $D((x^*,y^*),(u_n,v_n)) \to 1$

$\therefore u_n \to x^*, v_n \to y^*$ as $n \to \infty$

$\therefore (u_n,v_n) \to (x^*,y^*)$ as $n \to \infty$ (2.5.2)

$\therefore$ from (2.5.1) and (2.5.2), $(x,y) = (x^*,y^*)$

$\therefore (x,y)$ is unique coupled fixed point of S.

Theorem 2.6. In addition to the hypothesis of theorem 2.5, suppose that every pair of elements of X has an upper bound or lower bound in X. Suppose (x,y) is a coupled fixed point of S, then $x = y$.

Proof:

$\underline{Case I:}$ Suppose x is comparable to y.

Then $x = S(x,y)$ is comparable to $y = S(y,x)$ and $\lambda \in [0,1)$,

We get $d(x,y) = d(S(x,y), S(y,x))$

$\leq max\{d(x,y), d(y,x)\}^{\lambda}$

$= \{d(x,y)\}^{\lambda} \qquad (\because \lambda \in [0,1))$

$\therefore d(x,y) = 1$

$\therefore x = y$

<u>*CaseII*</u> : If x is not comparable to y .

Suppose there exists an upper bound $u \in X$ comparable to x and y .

Then $x \preceq u$ and $y \preceq u$ and (x, y) is coupled fixed point of S .

Write $u_1 = S(u, y)$ and $u_1' = S(y,u)$

Then $(x, y) \preceq (u_1, u_1')$

Further $d(x,u_1) = d(S(x, y), S(u, y)) \le max\{d(x,u), d(y, y)\}^{\lambda}$

$$= \{d(x,u)\}^{\lambda}.$$

And $d(y,u_1') = d(S(y, x), S(y,u)) \le max\{d(y, y), d(x,u)\}^{\lambda}$

$$= \{d(x,u)\}^{\lambda}.$$

$$\therefore d(S(x, y), S(u_1,u_1')) \le max\{d(x,u_1), d(y,u_1')\}^{\lambda}$$
$$\le max\{\{d(x,u)\}^{\lambda}, \{d(x,u)\}^{\lambda}\}^{\lambda}$$
$$= \{d(x,u)\}^{\lambda^2}.$$

By induction it can be shown that

$$d((x, y), (u_n,u_n')) \le \{d(x,u)\}^{\lambda^n}$$

Also by induction it can be shown that $d(u_n,u_n') \le \{d(u, y)\}^{\lambda^n}$

Further $d(u_n,u_n') = \{d(u, y)\}^{\lambda^n}$

$\therefore u_n \to x, u_n' \to y$ and $d(u_n,u_n') \to 1$ as $n \to \infty$

$\therefore \{u_n\}, \{u_n'\}$ have the same limit.

$\therefore x = y.$

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