IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 5 Ver. III (Sep. - Oct. 2017), PP 85-87
www.iosrjournals.org

On The Products of K -Pell Number And K-Pell Lucas Number

*1 Ashwini Panwar, 2 Kiran Sisodiya, 3 G.P.S. Rathore
[1] *School of Studies in Mathematics, Vikram University Ujjain, India*
[2] *School of Studies in Mathematics, Vikram University Ujjain, India*
[3] *Department of Mathematical Science, College of Horticulture, Mandsaur, India*
*Corresponding Author: *1 Ashwini Panwar,*

Abstract: *In this paper some products of k-Pell number and k-Pell-Lucas number are investigated .It also present generalized identities on the product of k-Pell and k-Pell-Lucas numbers to establish connection formulas between them with the help of Binet`s formula.*

Date of Submission: 18-10-2017 Date of acceptance: 03-11-2017

I. Introduction

Past several years have witnessed serious and sincere devotion of the scholars towards the study of the Fibonacci sequence, a well known sequence of numbers. A considerable amount of research work has been done in this regard and many papers have also been published. All of these exhibit eminence of Fibonacci sequence such as the work of Hoggatt in [7] and Vorobiov in [1], among others also relating with Fibonacci sequence in Falcon and Plaza [6]. The Fibonacci sequence belongs to group of sequences which are defined recursively .The sequence of Pell, Pell-Lucas and Modified Pell number also fall in this category.

For any positive real number k, The k-Pell sequence $\{P_{k,n}\}$ [2] defined as

$$P_{k,0} = 0, P_{k,1} = 1,$$
$$P_{k,n+1} = 2P_{k,n} + kP_{k,n-1};\ n \geq 1 \quad [1]$$

The k-Pell-Lucas sequence $\{Q_{k,n}\}$[4] defined as

$$Q_{k,0} = Q_{k,1} = 2,$$
$$Q_{k,n+1} = 2Q_{k,n} + Q_{k,n-1};\ n \geq 1 \quad [2]$$

The Binet`s formula [3] for k-Pell sequence and k-Pell-Lucas sequence are given by

$$P_{k,n} = \frac{r_1^n - r_2^n}{r_1 - r_2}, \quad [3]$$
$$Q_{k,n} = r_1^n + r_2^n \quad [4]$$

Where $r_1 = 1 + \sqrt{1+k}$ and $r_2 = 1 - \sqrt{1+k}$ are the roots of characteristic equation of the sequences $\{P_{k,n}\}$,$\{Q_{k,n}\}$ respectively.

And also

$$r_1 + r_2 = 2,$$
$$r_1 r_2 = -k,$$
$$r_1 - r_2 = 2.$$

For k = 1, we obtain the silver ratio which is related with the Pell number. Silver ratio is the limiting ratio of consecutive Pell numbers.

2. Product of k-Pell Numbers and k-Pell-Lucas Numbers

Theorem 2.1. $P_{k,2n}.Q_{k,2n} = P_{k,4n}$**, where** $n \geq 1$**.**

Proof.

$$P_{k,2n}.Q_{k,2n} = \left[\frac{r_1^{2n} - r_2^{2n}}{r_1 - r_2}\right].[r_1^{2n} + r_2^{2n}]$$
$$= \frac{r_1^{4n} + (r_1.r_2)^{2n} - (r_1.r_2)^{2n} - r_2^{4n}}{r_1 - r_2}$$
$$= \frac{r_1^{4n} - r_2^{4n}}{r_1 - r_2}$$
$$= P_{k,4n}. \quad [5]$$

Theorem 2.2. $P_{k,2n}.Q_{k,2n+2} = P_{k,4n+2} - 2(k)^{2n}$**, where** $n \geq 1$**.**

Proof.

$P_{k,2n}.Q_{k,2n+2} = \left[\frac{r_1^{2n}-r_2^{2n}}{r_1-r_2}\right].[r_1^{2n+2} + r_2^{2n+2}]$
$= \frac{r_1^{4n+2}+r_1^{2n}.r_2^{2n+2}-r_2^{2n}.r_1^{2n+2}-r_2^{4n+2}}{r_1-r_2}$
$= \frac{r_1^{4n+2}-r_2^{4n+2}}{r_1-r_2} - (r_1.r_2)^{2n}.\left[\frac{r_1^2-r_2^2}{r_1-r_2}\right]$
$= P_{k,4n+2} - (-k)^{2n}.2$
$= P_{k,4n+2} - 2(k)^{2n}.$ [6]

Theorem 2.3. $\mathbf{P_{k,2n}.Q_{k,2n+1} = P_{k,4n+1} - (k)^{2n}}$**, where** $\mathbf{n \geq 1}$.
Proof.
$P_{k,2n}.Q_{k,2n+1} = \left[\frac{r_1^{2n}-r_2^{2n}}{r_1-r_2}\right].[r_1^{2n+1} + r_2^{2n+1}]$
$= \frac{r_1^{4n+1}+r_1^{2n}.r_2^{2n+1}-r_2^{2n}.r_1^{2n+1}-r_2^{4n+1}}{r_1-r_2}$
$= \frac{r_1^{4n+1}-r_2^{4n+1}}{r_1-r_2} + (r_1.r_2)^{2n}.\left[\frac{r_2-r_1}{r_1-r_2}\right]$
$= P_{k,4n+2} - (-k)^{2n}$
$= P_{k,4n+2} - (k)^{2n}.$ [7]

Theorem 2.4. $\mathbf{P_{k,2n}.Q_{k,2n+3} = P_{k,4n+3} - (k)^{2n}(4 -3k)}$**, where** $\mathbf{n \geq 1}$.
Proof.
$P_{k,2n}.Q_{k,2n+3} = \left[\frac{r_1^{2n}-r_2^{2n}}{r_1-r_2}\right].[r_1^{2n+3} + r_2^{2n+3}]$
$= \frac{r_1^{4n+3}+r_1^{2n}.r_2^{2n+3}-r_2^{2n}.r_1^{2n+3}-r_2^{4n+3}}{r_1-r_2}$
$= \frac{r_1^{4n+3}-r_2^{4n+3}}{r_1-r_2} + (r_1.r_2)^{2n}.\left[\frac{r_2^3-r_1^3}{r_1-r_2}\right]$
$= P_{k,4n+3} + (-k)^{2n}.\left[\frac{(r_2-r_1)(r_2^3+r_1^3+r_1.r_2)}{r_1-r_2}\right]$
$= P_{k,4n+3} - (-k)^{2n}[Q_{k,2} + (-k)]$
$= P_{k,4n+3} - (-k)^{2n}[4 - 2k - k]$
$= P_{k,4n+3} - (-k)^{2n}[4 - 3k].$ [8]

Theorem 2.5. $\mathbf{P_{k,2n-1}.Q_{k,2n+1} = P_{k,4n} + 2(k)^{2n-1}}$**, where** $\mathbf{n \geq 1}$.
Proof.
$P_{k,2n-1}.Q_{k,2n+1} = \left[\frac{r_1^{2n-1}-r_2^{2n-1}}{r_1-r_2}\right].[r_1^{2n+1} + r_2^{2n+1}]$
$= \frac{r_1^{4n}+r_1^{2n-1}.r_2^{2n+1}-r_2^{2n-1}.r_1^{2n+1}-r_2^{4n}}{r_1-r_2}$
$= \frac{r_1^{4n}-r_2^{4n}}{r_1-r_2} + (-k)^{2n}.\left[\frac{r_2^2-r_1^2}{(r_1-r_2)(r_1r_2)}\right]$
$= P_{k,4n} - (k)^{2n}.\left[\frac{r_1+r_2}{(-k)}\right]$
$= P_{k,4n} + 2(k)^{2n-1}.$ [9]

Theorem 2.6. $\mathbf{P_{k,2n+1}.Q_{k,2n} = P_{k,4n+1} + (k)^{2n}}$**, where** $\mathbf{n \geq 1}$.
Proof.
$P_{k,2n+1}.Q_{k,2n} = \left[\frac{r_1^{2n+1}-r_2^{2n+1}}{r_1-r_2}\right].[r_1^{2n} + r_2^{2n}]$
$= \frac{r_1^{4n+1}+r_1^{2n+1}.r_2^{2n}-r_2^{2n+1}.r_1^{2n}-r_2^{4n+1}}{r_1-r_2}$
$= \frac{r_1^{4n+1}-r_2^{4n+1}}{r_1-r_2} + (r_1.r_2)^{2n}.\left[\frac{r_1-r_2}{r_1-r_2}\right]$
$= P_{k,4n+1} + (k)^{2n}.$ [10]

3. Generalized Identities on the Products of k-Pell Number and k-Pell-Lucas Number

Theorem 3.1. $\mathbf{P_{k,m}.Q_{k,n} = P_{k,m+n} - (-k)^m P_{k,n-m}}$**, where** $\boldsymbol{n \geq 1, m \geq 0}$..
Proof.
$P_{k,m}.Q_{k,n} = \left[\frac{r_1^m - r_2^m}{r_1 - r_2}\right].[r_1^n + r_2^n]$
$= \frac{r_1^{m+n}+r_1^m.r_2^n-r_2^m.r_1^n-r_2^{m+n}}{r_1-r_2}$
$= P_{k,m+n} + (r_1.r_2)^m \left[\frac{r_2^{n-m}-r_1^{n-m}}{r_1-r_2}\right]$
$= P_{k,m+n} - (-k)^m P_{k,n-m}.$ [11]

Theorem 3.2. $\mathbf{P_{k,n}.Q_{k,2n+m} = P_{k,3n+m} - (-k)^n P_{k,n+m}}$**, where** $\boldsymbol{n \geq 1, m \geq 0}$.
Proof.

$$P_{k,n}.Q_{k,2n+m} = \left[\frac{r_1^n - r_2^n}{r_1 - r_2}\right].[r_1^{2n+m} + r_2^{2n+m}]$$
$$= \frac{r_1^{3n+m} + r_1^n.r_2^{2n+m} - r_2^n.r_1^{2n+m} - r_2^{3n+m}}{r_1 - r_2}$$
$$= P_{k,3n+m} + (r_1.r_2)^n \left[\frac{r_2^{n+m} - r_1^{n+m}}{r_1 - r_2}\right]$$
$$= P_{k,3n+m} - (-k)^n P_{k,n+m}. \qquad [12]$$

Theorem 3.3. $\mathbf{P_{k,2n+m}.Q_{k,n} = P_{k,3n+m} + (-k)^n P_{k,n+m}}$**, where** $\boldsymbol{n \geq 1, m \geq 0}$.

Proof.

$$P_{k,2n+m}.Q_{k,n} = \left[\frac{r_1^{2n+m} - r_2^{2n+m}}{r_1 - r_2}\right].[r_1^n + r_2^n]$$
$$= \frac{r_1^{3n+m} + r_1^{2n+m}.r_2^n - r_2^{2n+m}.r_1^n - r_2^{3n+m}}{r_1 - r_2}$$
$$= P_{k,3n+m} + (r_1.r_2)^n \left[\frac{r_1^{n+m} - r_2^{n+m}}{r_1 - r_2}\right]$$
$$= P_{k,3n+m} + (-k)^n P_{k,n+m}. \qquad [13]$$

Theorem 3.4. $\mathbf{P_{k,2n}.Q_{k,2n+m} = P_{k,4n+m} - (-k)^{2n} P_{k,m}}$**, where** $\boldsymbol{n \geq 1, m \geq 0}$.

Proof.

$$P_{k,2n}.Q_{k,2n+m} = \left[\frac{r_1^{2n} - r_2^{2n}}{r_1 - r_2}\right].[r_1^{2n+m} + r_2^{2n+m}]$$
$$= \frac{r_1^{4n+m} + r_1^{2n}.r_2^{2n+m} - r_2^{2n}.r_1^{2n+m} - r_2^{4n+m}}{r_1 - r_2}$$
$$= P_{k,4n+m} - (r_1.r_2)^{2n} \left[\frac{r_1^m - r_2^m}{r_1 - r_2}\right]$$
$$= P_{k,4n+m} - (-k)^{2n} P_{k,m}.$$
$$= P_{k,4n+m} - (k)^{2n} P_{k,m}. \qquad [14]$$

Theorem 3.5. $\mathbf{P_{k,2n+m}.Q_{k,2n} = P_{k,4n+m} + (k)^{2n} P_{k,m}}$**, where** $\boldsymbol{n \geq 1, m \geq 0}$.

Proof.

$$P_{k,2n+m}.Q_{k,2n} = \left[\frac{r_1^{2n+m} - r_2^{2n+m}}{r_1 - r_2}\right].[r_1^{2n} + r_2^{2n}]$$
$$= \frac{r_1^{4n+m} + r_1^{2n+m}.r_2^{2n} - r_2^{2n+m}.r_1^{2n} - r_2^{4n+m}}{r_1 - r_2}$$
$$= P_{k,4n+m} + (r_1.r_2)^{2n} \left[\frac{r_1^m - r_2^m}{r_1 - r_2}\right]$$
$$= P_{k,4n+m} + (-k)^{2n} P_{k,m}.$$
$$= P_{k,4n+m} + (k)^{2n} P_{k,m}. \qquad [15]$$

II. Conclusion

In this paper we established connection formulas between k-Pell number and k-Pell-Lucas number through use of Binet`s formula.

Acknowledgment

We would like to show our gratitude to our highly esteemed revered teacher Late Dr. B.Singh , Prof. and Ex Head, School of Studies in Mathematics, Vikram University Ujjain (M.P) for sharing his pearls of wisdom with us during the formation of this research paper. Our teacher's invaluable emotional strength and intellectual sharpness which have always anchored us

References

[1]. N.N.Vorobiov, Numberos de Fibonacci, Editora MIR , URSS,1974.
[2]. P.Catarino , "On Some Identities and Generating Function for k-Pell numbers", International Journal of Mathematical Analysis 7(38)(2013),1877-1884.
[3]. P.Catarino , " On Some Identities for k-Fibonacci Sequence" ,International Journal Contemp. Math.Sciences 9(1),2014,37-42
[4]. P.Catarino ,P.Vasco, " On Some Identities and Generating Function for k-Pell-Lucas Sequence",Applied Mathematical Sciences,7(98)(2013),4867-4873.
[5]. P.Catarino ,P.Vasco, " Modified k-Pell- Sequence Some Identities and Ordinary Generating Function",Applied Mathematical Sciences,7(121)(2013),6031-6037.
[6]. S.Falcon,A.Plaza, " on the Fibonacci k-numbers" ,Chaos,Solitions & Fractals ,32(5) (2007),1615-1624.
[7]. V.E.Hoggatt,Fibonacci and Lucas Numbers,A Publication of the Fibonacci Association ,University of Santa Clara ,Santa Clara ,Houghton,Mifflin Company ,1969.
[8]. P.Catarino , "On Some Identities and Generating Function for k-Fibonacci Sequence",International Journal Contemp.Math.Sciences, 9(1)(20144),37-42.]