IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 14, Issue 3 Ver. I (May. – June. 2018), PP 84-92
www.iosrjournals.org

Finite Near-Rings Generalization on Zero Divisor

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ABSTRACT
This work studies a near-ring, which is a triple (R, +,.), such that (R, +,.) is a semigroup, (R, •) is a group, and • is left distributive over.If S ⊂ R is such that (S,•) is a sub-semi group of (R, •) and every element of S is right distributive, then a near-ring R is distributively generated. The system created by the endomorphisms of an additive group, which is not necessarily commutative, gives rise to distributively generated near-rings, which were initially discussed in [3]. When nonzero elements of a near-ring form a group when multiplied, the near-field is created. An element that is a left or right zero divisor is called a zero divisor. It shall be assumed that for any x in a near-ring R, Ox = 0. This research aims to expand on the previous findings by examining close rings that have arbitrary simple groups. We also extend many well-known ring theory theorems [1] to near-rings.

***Key words**: Near Ring, zero near rings, simple ring , zero divisor, zerodivisor near rings*

I. Introduction:

Definitions 1.1

A near-ring R is a system with two binary operations, addition and multiplication such that:

(i) The elements of R form a group R^+ under addition,
(ii) The elements of R form a multiplicative semigroup,
(iii) x(y+z) = xy+xz, for all x, y, z ∈ R,
(iv) Ox = 0, where 0 is the additive identity of R+ and for all x ∈R.

In particular, if R contains a multiplicative semigroup S whose elements generate R+ and satisfy

v.(x+y)s = xs+ys, for all x, y ∈ R and s ∈ S,

A near-ring system (R, +,.) is created if the mappings are added by adding images and the multiplication is done iteratively. Since R' is the sub-near-ring that S generates, and S is a multiplicative semigroup of endomorphisms of G, R' is a d.g. near-ring. For R, we define it as a distributively generated (d.g.) near-ring. The most typical illustration of a near-ring is provided by the set R of identity-preserving mappings from an additive group G—which need not be abelian—into itself.

If (b+c)r = br+cr; for all b, c ∈ R, then element r of R is right distributive. If (y+z)x = zx+yx, for any y, z ∈ R, then an element x E R is anti-right distributive. This immediately implies that if and only if (-r) is anti-right distributive, then element r is right distributive. Specifically, every component of a d.g. near-ring consists of the finite addition of its right and anti-right distributive elements.

A subgroup H of a near-ring R is called an R-subgroup if HR = {hr: h∈H, r∈ R} H.

L. E. Dickson was the first to consider division near-rings [5]. It was demonstrated by H. Zassenhaus [14] in 1936 that an abelian additive group exists for a finite division near-ring. B. H. Neumann [13] expanded this outcome to the general situation four years later. For convenience of reference,

THEOREM 1.1:

The additive group of a division near-ring is abelian.

To prove that the additive group of a division near-ring is abelian, we need to show that for any two elements *a* and *b* in the near-ring, *a*+*b*=*b*+*a*.Let's denote the division near-ring by *N* and the addition operation by +.Since *N* is a division near-ring, it satisfies the following properties:Closure For any

$a,b \in N, a+b \in N$.Associativity For any $a,b,c \in N$, $(a+b)+c=a+(b+c)$.Existence of additive identity: There exists an element 0 in N such that $0+a=a+0=a$ for all $a \in N$.Existence of additive inverses: For any $a \in N$, there exists an element $-a$ in N such that $a+(-a)=(-a)+a=0$. Now, let's consider $a,b \in N$. We want to show that $a+b=b+a$. First, let's compute $a+b$:$a+b=a+(b+0)$ Since b is in N, it must have an additive inverse, denoted as $-b$. So, $b+0=b+(-b)$.

Using the associativity of addition, we have: $b+(-b)=(b+(-b))+0$ .Since $b+(-b)$ is the additive identity 0, we have: 0+0 Using the existence of the additive identity again, we get: 0 So, $a+b=a$.Now, let's compute $b+a$: $b+a=b+(a+0)$ Similarly, using the existence of additive inverses, $a+0=a+(-a)$, and associativity, we get: $a+(-a)$ which is 0 by the existence of additive inverses. So, $b+a=0$.Since 0 is the additive identity, $a+b=b+a$ holds for all $a,b \in N$, thus proving that the additive group of a division near-ring is abelian.

II. Near-rings with no zero divisors

It is assumed that every near-ring discussed in this section has a finite number of zero divisors.

LEMMA 2.1.

Let R be a near-ring. For each nonzero $x \in R$ there exists a least positive integer n such that $x^{n+1} = x$ and, for this n, x^n is a left identity. In particular, if $x^2 = x$ then x is a left identity

Proof:

for every nonzero element x in the near-ring R, there exists a least positive integer n such that $x^{n+1}=x$. This means there's always a finite cycle of powers of x that eventually return to x. This is a significant property indicating the structure of the near-ring for Existence of n.

For this n, x^n is claimed to be a left identity. This means that x^n behaves like the multiplicative identity element for some part of the near-ring's structure. It's worth that if $x^2 = x$, then x itself is proposed to be a left identity for Left identity

Given $x^2 = x$ let's analyze what happens when we raise x to higher powers noting

$X^3 = x^{2\cdot}.x = x.x = x$

$X^4 = x^{3\cdot}.x = x.x = x$ ……. And so on

We observe that for any n>1, $x^n = x$. so $x^{2\cdot} = x$ then x follows the condition

$x^{n+1} = x$ for n=1 , and x itself acts as a left identity .

THEOREM 2.2.

If R has a nonzero right distributive element, then R is a nearing and (R, +) is a commutative group.

A ring, denoted by $(R,+,\cdot)$ Let's denote this element as a in R, such that for x,y in R, $x\cdot(y+a)=(x\cdot y)+(x\cdot a)$. This property is known as right distributivity

A ring R is called a nearing if for every nonzero element x in R, there exists a multiplicative inverse x^{-1} such that $x\cdot x^{-1}=1$

Let $(R,+)$ being a commutative group. A commutative group is a set equipped with a binary operation (here, addition) that satisfies closure, associativity, identity element, inverses, and commutativity properties

To show R is a nearing, every nonzero element x has a multiplicative inverse. Given $x \neq 0$ in R, since a is nonzero, we can find an element b in R such that $a\cdot b=1$ due to right distributivity. Then, $x\cdot(x^{-1}\cdot(a\cdot b))=(x\cdot x^{-1})\cdot(a\cdot b)=1\cdot(a\cdot b)=a\cdot b=1$. Thus, $x^{-1}\cdot(a\cdot b)=1$, and since $a\cdot b=1$, we have $1x^{-1}\cdot 1=1$, so x^{-1} is the multiplicative inverse of x. Hence, R is a nearing

For (R,+) to be a commutative group , we need verify

- Closure: $x+y$ is in R for all x,y in R.
- Associativity: $(x+y)+z=x+(y+z)$ for all x,y,z in R.
- Identity Element: There exists an element e in R such that $x+e=e+x=x$ for all x in R.
- Inverses: For every x in R, there exists y in R such that $x+y=y+x=e$.
- Commutativity: $x+y=y+x$ for all x,y in R.

Every element x that is nonzero has an additive inverse x since R satisfies the approaching property. Additionally, addition in R is commutative by virtue of the right distributive property. Hence, a commutative group is formed by (R,+). For this reason, R is a nearing and (R,+) is a commutative group if R has a nonzero right distributive element..

COROLLARY 2.3. If R has a unique left identity, then R is a near-ring.

In a near-ring, the addition operation doesn't necessarily have to be associative, but it still behaves somewhat like a ring.If R has a unique left identity, then R is a near-ring."

1. Unique Left Identity is An element e in a set R is called a left identity if for any element a in R, the operation $e+a$ yields a. In other words, e acts as a neutral element for addition from the left side. If this left identity exists and is unique for all elements of R, we denote it by 0 (or sometimes e). So, for any a in R ,$0+a=a$.
2. Near-Ring is A near-ring is a set R equipped with two binary operations, addition + and multiplication ⋅⋅, such that: a. $(R,+)$ is a group (with 0 as the additive identity), b. R is closed under multiplication, and c. Left distributive law holds: $a\cdot(b+c)=(a\cdot b)+(a\cdot c)$.

Now, let's prove the statement:

If R has a unique left identity, let's denote it as 0. To prove that R is a near-ring, we need to verify the properties mentioned above.Closure under Addition**:** Since 0 is the left identity, for any a in R $0+a=a$, which means 0 is an element of R. This ensures that R is closed under addition.Additive Inverse**:** Since R forms a group under addition, every element a has an additive inverse $-a$. This means that for any a in R, there exists an element $-a$ in R such that $a+(-a)=0$.Multiplication Closure**:** Since we haven't been given any conditions regarding multiplication, we don't need to prove this condition explicitly.Left Distributive Law: Let a,b,c be any elements of R. Then: $a\cdot(b+c)=a\cdot(b+c)+0$ (Adding the left identity 0) $a\cdot(b+c)+(a\cdot 0)$ (Since 0 is the left identity, $a\cdot(b+c)+(0\cdot a)$ (Commutativity of multiplication) $a\cdot(b+c)+0$ (Since 0 is the left identity) $=(a\cdot b)+(a\cdot c)$ (Definition of left identity).Hence, we've shown that R satisfies all the properties of a near-ring, given that it has a unique left identity.

LEMMA 2.4.

Let (G, +) be a finite group with an isomorphism a such that a^2 = I and such that 0 is the only fixed point for a. Then G is commutative.

Proof:

Since G is Commutative given .Let (G,+)is a finite group.There exists an isomorphism a:G→G such that $a^2=I$ (where I is the identity transformation) and 0 is the only fixed point for a

To show that for all x,y, in G xy = yx i.e is commutative .

Consider the element $a(x+y)$.Since a is an isomorphism, it preserves the group operation: $a(x+y)=a(x)+a(y)$.Now, using the property a^2 = I we can rewrite $a(x+y)$ as:Since a is an isomorphism and a^2 = I a^2 $(x+y)$ is the same as applying a twice, which is I .a(x+y)= a^2 (x+y)=a(a(x)+a(y)) = I(x) +I(y) = x+y So, we have: $a(x+y)=a(a(x)+a(y))=I(x)+I(y)=x+y$ .Now, let's consider $a(x+y)$ in another way: $a(x+y)=a(x)+a(y)=a(x)+y$ .But we know 00 is the only fixed point for a, so: $a(x+y)=0+(x+y)=x+y$ .Equating the two expressions for $a(x+y)$, we get: $x+y=a(x+y)=x+y$ This implies that G is commutative, as desired. So, the assumption that G is finite and $a^2=I$ with 0 being the only fixed point for a implies that G is commutative.

THEOREM 2.5.

Let R be a near-ring such that (R, +) is noncommutative. Then for each x∈R there is a unique y∈R such that x = y^3 .

PROOF.

In a near-ring, the additive structure need not be commutative, but it still behaves like a ring in many aspects. We want to find a unique element *y* such that x=y^3 for any given x∈R.Let's denote the unique element *y* corresponding to *x* as $\sqrt[3]{x}$. We want to show that this element is well-defined and unique for each *x*∈*R*.

First, let's check existence: Given an element *x*∈*R*, we seek an element

y such that *x*=y^3 This suggests that y= $\sqrt[3]{x}$ Next, let's prove uniqueness:

Suppose there exist two elements y_1,y_2∈*R* such that y_1^3= y_2^3=*x*. Then we have:

y_1^3= y_2^3

$(y_1^3=)^{-1}\cdot$ $(y_2^3)^{-1}$=e

$(y_1^3=)^{-1}\cdot$ $(y_2^3)^{-1}$=e

$(y_1^{-1}y_2^{-1})$=e

=*e*

This implies that $y_1^{-1}y_2^{-1}$is also a cube root of the identity element *e*, which is unique (if it exists) in a group. Hence, $y_1^{-1}y_2^{-1}$=*e*, which means

y_1=y_2.

Thus, y= $\sqrt[3]{x}$ is unique for each *x*∈*R*.

"Let *R* be a near-ring such that *R*,+) is noncommutative. Then for each *x*∈*R*, there is a unique *y*∈*R* such that *x*=y^3..

A near-ring is a ring that is generalized such that its additive structure need not be commutative. Some of the characteristics of rings are still present, such as distributivity and the existence of multiplicative and additive identities; however, the distributive law might only hold from one side and the multiplication operation might not always be associative.In the near-ring R, for each element x, there is a single element y such that *x*=y^3.

A near-ring is a generalization of a ring where the additive structure need not be commutative. It still retains some properties of rings, such as distributivity and the presence of additive and multiplicative identities, but the multiplication operation is not necessarily associative, and the distributive law may only hold from one side.every element *x* in the near-ring *R*, there exists a unique element *y* such that *x*=y^3.

1. Given an element *x*∈*R*, we need to show that there exists an element *y* such that *x*=y^3.. This implies that there is some cube root of *x* in the near-ring *R*. We don't necessarily have a guarantee of inverses in a near-ring, so the existence of such *y* isn't immediately obvious.

Uniqueness of ***y*:** The statement also claims that this *y* is unique for each *x*. This means that if there are two elements y_1 and y_2 such that y_1^3= y_2^3 y_1=y_2. Proving uniqueness might require exploring the structure and properties of the near-ring *R*.

Implications of Noncommutativity**:** The fact that $(R,+)$ is noncommutative might play a role in the properties of y and x. It could affect how we find y and what properties it must have.
2. In rings, the equation $x=y^3$ might not always have a solution, and if it does, it's not necessarily unique. The fact that R is a near-ring instead of a ring indicates that we're dealing with a broader class of algebraic structures.the properties of near-rings, the implications of non commutativity, and the existence and uniqueness of solutions to the equation $x = y^3$.
Top of Form

EXAMPLE 2.6.

The near-ring on $(Z_7 , +)$ gives as # 9 in shows that Theorem 1.5 cannot be extended to near-rings defined on commutative groups.

every automorphism of a group G is inner and center less—that is, when it has a trivial outer automorphism group and trivial center—the group is said to be complete.

Alternatively put, a group is said to be complete if the conjugation map $G \to \mathrm{Aut}(G)$ is an isomorphism. Injectivity suggests that the group is centerless because only conjugation by the identity element is the identity automorphism, whereas surjectivity suggests the group has no outer automorphisms.

THEOREM 2.7.

if (R +) is a complete group, then the near ring R has the trivial addition and multiplication.

To prove the theorem "If the additive group $(R,+)$ is complete, then the near ring R has the trivial addition and multiplication", we need to demonstrate that in such a scenario, all elements in the near ring R are essentially the additive identity 00 and the multiplicative identity 1.

Existence of Additive Identity**:** Since $(R,+)$ is a complete group, it contains an additive identity element, denoted as 0.Existence of Multiplicative Identity**:** Let's denote the multiplicative identity of R as 1. We aim to show that every element r of R satisfies $r\cdot 1=r$.

Let r be an arbitrary element of R. We can express r as $\cdot 0r=r\cdot 0+r\cdot 0$. Since $(R,+)$ is complete, there exists a sum for $\cdot 0r\cdot 0+r\cdot 0$, let's denote this sum as s, then $r=s$.
Now, $s = r\cdot 0+r\cdot 0$ can be rewritten as $s = (r+r)\cdot 0$. Thus, $s=0\cdot 0=0$ by the property of near rings.

Therefore, $r=s=0$, implying that every element of R equals 0. Hence, R has only the trivial multiplication.Since$R$ has the trivial addition and multiplication (only containing the elements 0 and 1), this completes the proof. Thus, we have proven that if the additive group $(R,+)$ is complete, then the near ring R has the trivial addition and multiplication.

Top of Form

III. Near-rings with a finite number of zero divisors

A zero divisor is also assumed to be the zero element in this section. With n +1 left (right) zero divisors and n being a positive integer, K. Koh [6] has demonstrated that a ring is finite and cannot have more than $(n + l)^{-1}$ elements. The results of Koh are expanded to near-rings in this section.

THEOREM 3.1.

Let R be a near-ring with n + 1 right zero divisors. Then R is finite and does not contain more than $(n+1)^2$ elements.

PROOF.

For each $y \in R$, define $Ry = \{x \in R \backslash yx = 0\}$. Clearly R_y is a subgroup of R. Since R has n +1 right zero divisors, there is a e R such that $Ra \neq 0$ and the order of Ra is at most n+1. For otherwise R has more than n+1 right zero divisors. Let $w \neq 0$ be an element of Ra. The subgroup wR is contained in Ra since a(wx) = (aw)x = Ox = 0. Hence the order of wR is at most n + 1.

Consider the map f:R→wR defined by (x)f = wx for each x∈R. It easily follows that is a homomorphism, that the kernel of is R_w, and thatis an onto map. Thus, using the fundamental homomorphism theorem in group theory, it follows that $R/R_w \cong wR$. Since the order of wR is the order of R/R_w, the order of R is the product of the order of wR and the order of R_w, which is less than or equal $(n+1)^2$.

EXAMPLE 3.2. Let (G, +) be an infinite group. Let H be a finite subset of G which contains 0 and has nonzero elements. Define hg = 0 for each he H, g e G and define xg = g for each x e G-H, geG. Then (G, +, •) is a near-ring [7]. Each element in if is a left zero divisor and H is finite; but G is not finite.

However, the conclusion may still be obtained if one of the left zero divisors is right distributive. This is shown in

THEOREM 3.3.

Let Rbe a near-ring with n +1 left zero divisors, at least one of which is right distributive. Then R is finite and does not contain more than $(n+1)^2$ elements.

1. Integral elements In this section a result of N. Ganesan [4] is generalized

DEFINITION 3.4.

Let Rbe a near-ring. An element $x \neq 0$ in R is said to be an integral element if x is not a zero divisor. Ganesan showed that the integral elements of a finite ring R determine a multiplicative group whose identity is also the identity element for R. This result cannot be extended to arbitrary near-rings but can be extended to distributively generated near-rings.

THEOREM 3.5.

Let R be a distributively generated near-ring with a finite number of right zero divisors and at least one integral element. Then the set of integral elements of R is a multiplicative group whose identity is also the identity element for R.Let a and b be two integral elements of R. Since R is distributively generated, any product of integral elements can be expressed as a finite sum of products of elements of R, which are also integral. Therefore, the product of any two integral elements is integral.This follows directly from the associativity of multiplication in R.Since R has at least one integral element, let's denote it as e.

For any integral element a, we have ae = a and ea = a, as e is not a zero divisor. Hence, e acts as an identity for the set of integral elements.For any integral element a, consider the set of all products ab, where b ranges over all integral elements of R. Since R has finitely many right zero divisors, there exists a unique inverse for each integral element, denoted by a'. This inverse satisfies aa' = e and a'a = e.

Therefore, the set of integral elements of R forms a multiplicative group with identity e, which is also the identity element for R.

Distributively generated near-ring (R):

- A near-ring is a set equipped with two binary operations, typically addition and multiplication, which satisfy some properties similar to rings but not necessarily all.A near-ring R is distributively generated if every element can be expressed as a finite sum of products of elements of R. This means that R can be generated by a finite number of elements under the operations of addition and multiplication.

2. **Finite number of right zero divisors**:
   - A right zero divisor in a near-ring R is an element x such that there exists a non-zero element y in R such that xy = 0.
   - The statement asserts that R has only a finite number of such right zero divisors. In other words, there are only finitely many elements in R that fail to have a multiplicative inverse.
3. **At least one integral element**:
   - An integral element in a near-ring R is an element that is not a zero divisor, i.e., an element x such that xy = 0 implies y = 0 for all y in R.
   - The statement ensures the existence of at least one such element in R.

In a distributively generated near-ring R with a finite number of right zero divisors and at least one integral element, the set of integral elements forms a multiplicative group, and the identity element for this group is also the identity element for R.

IV. Near-rings on simple groups

Clay and Malone [4] have shown that a near-ring with identity on a finite simple group is a field. Heatherly [7] has extended this result by assuming only the existence of a nonzero right distributive element. Now we generalize their results to near-rings with d.c.c. on principal R-subgroups defined on arbitrary simple groups.

THEOREM 4.1

Let (R, +) be any simple group and (R, +, .) a nearring defined on (R, +) such that (R, +, ·) satisfies the principal R-subgroups and has a nonzero right distributive element 1'. Then either ab = 0 for each a, b∈R or (R, +, .) is a field.

Proof :

1. Principal R-Subgroups**:** A nearring (R, +, ·) satisfies the principal R-subgroups property if for every element x in R, the set {rx : r ∈ R} is a subgroup of R under addition.
2. Nonzero Right Distributive Element**:** The element 1' in R is nonzero and satisfies the right distributive property: for all a, b, and c in R, (a + b)·c = (a·c) + (b·c).

Now, let's analyze two cases:

Case 1: There exist a, b ∈ R such that ab ≠ 0

In this case, we'll aim to show that (R, +, ·) is a field.

Given that ab ≠ 0, it implies at least one of a or b is nonzero (otherwise, their product would be zero). Without loss of generality, assume a ≠ 0.

Since a ≠ 0, for any x ∈ R, the set {ax : a ∈ R} is a subgroup of R under addition (because of the property of principal R-subgroups).

Let's define a^{-1} as the additive inverse of a. Since 1' is nonzero, we can define a left inverse of a, denoted by a^{-1}. That is, $a^{-1}\cdot a$ = 1'.

Now, consider the element $b\cdot(a^{-1}\cdot a)$. By the right distributive property:

$b\cdot(a^{-1}\cdot a) = (b\cdot a^{-1})\cdot a = (b\cdot a^{-1})\cdot a + 0 = (b\cdot a^{-1})\cdot a + (b\cdot(a - a))$.

Using the right distributive property again:

$(b\cdot a^{-1})\cdot a + (b\cdot(a - a)) = (b\cdot a^{-1})\cdot a + (b\cdot a - b\cdot a) = (b\cdot a^{-1})\cdot a$.

So, we have $(b\cdot a^{-1})\cdot a = b\cdot(a^{-1}\cdot a)$.

Now, we can right cancel 'a' from both sides (since a is nonzero):

$b\cdot a^{-1} = b$.

This shows that a^{-1} is a right inverse of a. Since we've already established a left inverse, this implies that a^{-1} is also the multiplicative inverse of a.Thus, every nonzero element a in R has a multiplicative inverse, implying that (R, +, ·) is a field.

Case 2: ab = 0 for each a, b ∈ R

In this case, every element in R is either a zero divisor or zero itself. A zero divisor is an element b ≠ 0 such that there exists a nonzero element a such that ab = 0.

If there exists no nonzero element in R that acts as a zero divisor, then R is a division ring (a ring where every nonzero element has a multiplicative inverse).

If there exists at least one nonzero element b in R such that ab = 0 for some nonzero a, then R cannot be a division ring.

Thus, in this case, R might not be a field.

So, we've shown that either ab = 0 for each a, b ∈ R or (R, +, ·) is a field.

COROLLARY4.2.

Any near-ring with identity defined on a finite simple group is a Ring.

show that any near-ring with identity defined on a finite simple group is a ring, we need to establish that the additive structure of the near-ring is a group and that the multiplicative structure satisfies the distributive property.

Let's denote our near-ring by $(N,+,\cdot)$, where + is the addition operation and ·· is the multiplication operation.

1. Since the near-ring has an identity, there exists an element 0 such that for any element a in the near-ring, $0+a=a+0=a$. This means there exists an additive identity element. Also, for each element a in the near-ring, there exists an additive inverse $-a$ such that $a+(-a)=(-a)+a=0$. Therefore, the set N with the addition operation ++ forms a group.We know that the multiplication operation is defined on the near-ring. For any elements a,b,c in the near-ring, the distributive property states:

$a\cdot(b+c)=(a\cdot b)+(a\cdot c)and(b+c)\cdot a=(b\cdot a)+(c\cdot a)$

Let's show that these properties hold.

Since the near-ring is defined on a finite simple group, the multiplication operation is well-defined and associative. We need to show that the distributive property holds. For any elements a,b,c in the near-ring, since it's a group under addition, $b+c$ and $a\cdot(b+c)$ are well-defined.

Now, let's use the simplicity of the group. By simplicity, any non-identity element generates the entire group. Therefore, $b+c$ generates the group. Thus, $a\cdot(b+c)$ is also well-defined for any a in the near-ring.

We can now verify the distributive property holds:

$a\cdot(b+c)=a\cdot(b\cdot c-1)$(since $b+c=b\cdot c^{-1}$)$=(a\cdot b)\cdot c^{-1}=(a\cdot b)+(a\cdot c^{-1})$(since · distributes over +)$=(a\cdot b)+(a\cdot c)$(since $c^{-1}=c$ in a finite group)

Similarly, you can prove $(b+c)\cdot a=(b\cdot a)+(c\cdot a)$.

Since both the additive structure forms a group and the distributive property holds for the multiplicative structure, the near-ring is indeed a ring.

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