IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 12, Issue 1 Ver. III (Jan- Feb. 2015), PP 51-57
www.iosrjournals.org

Car Dynamics Using Quarter Model and Passive Suspension, Part III: A Novel Polynomial Hump

Galal Ali Hassaan
(Emeritus Professor, Department of Mechanical Design & Production Engineering, Faculty of Engineering/Cairo University, Giza, Egypt)

Abstract : *A novel polynomial speed hump is presented in this work. The dynamics of a quarter-car model are investigated to reach the conditions of ride comfort when using the novel polynomial hump. The study assumed passive car suspension elements of linear characteristics. It covers car crossing speed between 5 and 30 km/h, polynomial hump of length between 3 and 9 m and height between 60 and 120 mm. The proposed polynomial hump provides smooth kinematics for the sprung-mass of the vehicle including its acceleration. A ride comfort diagram is presented using MATLAB simulation allowing the design of the polynomial hump for any desired hump-crossing speed in the range 6 to 26 km/h and hump length between 3 and 9 m.*
Keywords: *Car dynamics , quarter-car model , Passive suspension system , Novel polynomial hump , Ride comfort, Hump design diagram.*

I. Introduction

There are available limited designs for speed humps. Speed humps are required to generate calm traffics specially in residential areas or because of security reasons. On the other hand speed humps have some drawbacks such as increasing noise levels, limiting the traffic flow capacity and causing ride incomfort. The subject is important to everybody and finds an increasing interest from specialized researchers and authorities.

Weber (1998) pointed out a number of hump designs including circular (Watts), trapezoidal, sinusoidal and combi. He quoted the dimensions of circular and Seminol humps. He used humps with 75-100 mm height and 3.7-9.1 m length [1]. Weber and Braaksma (2000) used Watts speed hump of 3.7 m length and height of 75-100 mm and Seminole hump of 6.7 m length and 75-100 mm length [2]. Parry, March and Cripps (2003) investigated round top humps (75 mm height and 3.7 m length), flat top humps (75 mm height, 1.05 m sides and 5m flat length) and sinusoidal hump (75 mm height and 3.7 m length) [3].

Bjarnason (2004) discussed a Watts hump of 100 mm height and 3.66 m length. He studied the flat top and round top humps and the effect of the crossing speed of the vehicle on the vertical acceleration in g [4]. Tam (2006) pointed out the circular, sinusoidal, parabolic speed humps of 75 mm length and 3.7 m length, flat top humps with flat –tapped sides of 75 mm height, 0.75 m flat sides and top flat of length in the range 2.5 – 6 m [5]. Aghazadeh, Saeedi and Yazdi (2006) used a half-car model to study the ride quality of vehicles crossing three types of speed humps. They studied a round top hump of 3.7 m length and 75 mm height, flat top straight ramps hump of 8 m length, 75 mm height and 6 m top flat length [6].

Greenwood, Nowson and Ahmed (2007) used a round top road hump of height in the range 25-100 mm, flat top load humps with flat top of 2.5 m and height in the range 25-100 mm [7]. Granlund and Brandt (2008) investigated flat top asphalt humps with cobble stone ramps and round top asphalt humps. They covered bus speed from 5 to 45 km/h and put limits for the health risk as a criteria to limit bus speeds [8]. Hessing (2008) presented an optimal road hump of 100 mm height and 3.5 m length [9]. The Broward County Traffic Engineering Division (2010) recommended using a speed hump of 76.2 mm height, 3.6 top and 1.8 m transitions on both rear and front sides within the unincorporated area County [10].

Berthod (2011) stated that a sinusoidal shape is preferred over circular or parabolic shapes because it provides a more gentle transition. He quoted the dimensions of speed hump with and without flat centre [11]. Dedovic and Sekulic (2012) studied the vibration of a bus crossing a flat top hump of 80 mm height and 12 m overall length, and a rounded hump of heights 30 and 50 mm and lengths 0.83 and 0.96 m respectively. They used a full-car model [12]. Yaacob and Hamsa (2013) studied the effect of using road humps in reducing traffic flow in a residential area in Kuala Lumpur. They used two speed humps of heights 80 and 60 mm, length 12 and 11.3 m and width 3.7 and 3.5 m [13]. Hassaan (2014) examined the dynamics of a car crossing a circular hump using a quarter-car model. He investigated the suspension damping effect in the range 1–15 kNs/m and vehicle speed in the range 5-25 km/h. The hump he used has a 100 mm height and 5.2 m length [14]. Hassaan (2015) presented a novel simple harmonic speed hump and studied the car dynamics during crossing this hump using a quarter car model. He covers a car hump crossing speed in the range 5-30 km/h, hump length 3-9 m and hump height 60-120 mm [15].

II. Analysis

2.1 Quarter Car Model

A quarter-car model consists of the wheel and its attachments, the tire (of parameters m_1 for mass, k_1 for stiffness and c_1 for damping coefficient), the suspension elements (of parameters k_2 for stiffness and c_2 for the damping coefficient) and quarter the chassis and its rigidly connected parts (of mass m_2) .

The dynamic system is a standard two degree of freedom one having the dynamic motions:

y: The exciting motion at the ground.

x_1: The axle motion.

x_2: The chassis motion.

The driver and passenger seats are assumed rigidly connected to the chassis not to increase the degree of freedom of the dynamic system.

The parameters of the quarter-car model according to Florin, Ioan-Cosmin and Liliana are considered in this analysis except for the suspension damping coefficient c_2. Their parameter are given in Table 1 except the damping coefficient of the suspension which is set by the author [16].

Table 1: Quarte-car model parameters [16].

Parameters	Description	Value
k_1 (kN/m)	Tire stiffness	135
c_1 (kNs/m)	Tire damping coefficient	1.4
m_1 (g)	Un-sprung mass	49.8
k_2 (kN/m)	Suspension stiffness	5.7
c_2 (kNs/m)	Suspension damping coefficient	15
m_2 (g)	Sprung mass	466.5

2.2 Model Input

The input to the quarter-car model is a novel hump which is expected to provide better kinematics to the car-chassis when crossing the hump. This is what I call 'polynomial hump'. It has the characteristics:

- Starting and ending by zero vertical displacement.
- Starting and ending by zero vertical velocity.
- Starting and ending by zero vertical acceleration.
- Possibility of starting and ending by zero vertical jerk.
- Providing the maximum height at mid-span of the hump.

The polynomial hump profile is generated using MATLAB for any desired characteristics and maximum height. Fig.1 shows scaled polynomial humps for the heights 60, 80,100 and 120 mm.

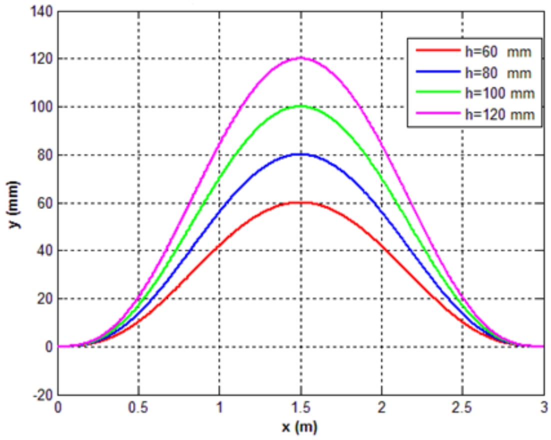

Fig.1 Polynomial hump.

A polynomial hump is characterized by its dimensions:

Height : h (maximum y)

Length : L (maximum x)

In the x-y plane a polynomial hump has the mathematical model:

$$y = \alpha_1 x^6+\alpha_2 x^5+\alpha_3 x^4+\alpha_4 x^3+\alpha_5 x^2+\alpha_6 x+\alpha_7 \quad (1)$$

where: x = horizontal displacement from the hump starting point
α = polynomial coefficient (7 values)
x and y are the coordinates of any point on the hump from the hump starting point.

The polynomial coefficients are evaluated to satisfy the following seven boundary conditions:

$y = 0$	at	$x = 0$	(initial displacement)
$y = 0$	at	$x = L$	
$y' = 0$	at	$x = 0$	(initial velocity)
$y' = 0$	at	$x = L$	
$y'' = 0$	at	$x = 0$	(initial acceleration)
$y'' = 0$	at	$x = L$	
$y = h$	at	$x = 0.5L$	

2.3 Mathematical Model of the Quarter-car

Writing the differential equation of the unsprung and sprung masses of the quarter-car model yields the following two equations:

$$m_1x_1'' + (c_1+c_2)x_1' - c_2x_2' + (k_1+k_2)x_1 - k_2x_2 = k_1y + c_1y' \quad (2)$$

$$m_2x_2'' - c_2x_1' + c_2x_2' - k_1x_1 + k_2x_2 = 0 \quad (3)$$

The state model of the dynamic system is driven from Eqs.2 and 3 as follows:

- State variables: z_1, z_2, z_3 and z_4. The state variables are related to the masses displacements x_1, x_2 and velocities x_1', x_2' as:

$$z_1 = x_1 \quad , \quad z_2 = x_1'$$
$$z_3 = x_2 \quad , \quad z_4 = x_2' \quad (4)$$

Output variable:
The output variable of the quarter-car model is the sprung mass motion, x_2. It is related to the state variables through:

$$x_2 = z_3 \quad (5)$$

State model:
Combining Eqs.2, 3 and 4 gives the state model of the quarter-car model as:

$$z_1' = z_2 \quad (6)$$
$$z_2' = (1/m_1)\,\{k_1y + c_1y' - (c_1+c_2)z_2 - (k_1+k_2)z_1 + k_2z_3\} \quad (7)$$
$$z_3' = z_4 \quad (11)$$
$$z_4' = (1/m_2)\,\{c_2z_2 - c_2z_4 + k_2z_1 - k_2z_3\} \quad (12)$$

III. Quarter-Car Model Dynamics

- The state model of this dynamic problem is linear since the suspension parameters are assumed constant (linear characteristics).
- MATLAB is used to solve this problem using its command "ODE45" [17,18].
- The sprung mass motion is excited by the polynomial hump displacement only, i.e. zero initial conditions are set in the solution comment.
- The car speed is changed in the range: 5 to 30 km/h when crossing the hump with 2.5 km/h increment.
- The height of the polynomial hump is changed in the range: 60 to 120 mm with 20 mm increment.
- The length of the polynomial hump is changed in the range: 3 to 9 m with 1 m increment.
- The purpose of this research was to emphasise the effect of the dimensions of the polynomial hump on the sprung mass displacement and the ride comfort in terms of the maximum sprung-mass acceleration in m/s^2.

3.1 Sprung-mass Displacement

The displacement of the sprung-mass as generated by MATLAB for a car velocity of 10 km/h, hump length of 3 m, besides the system parameters in Table I and hump height in the range 60-120 mm is shown in Fig.2.

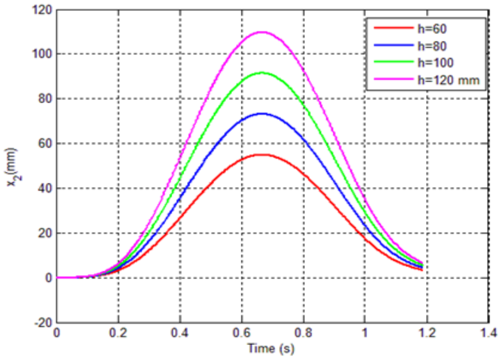

Fig.2 Sprung-mass displacement for V = 10 km/h and L = 3 m.

3.2 Sprung-mass Maximum and Minimum Displacements

- As clear from all the sprung-mass response of the quarter model as shown in Fig.2, the displacement reaches a maximum value then drops to a zero value as the car crosses the hump.
- The maximum and minimum values of x_2 are calculated in the same code generating the kinematics of the sprung mass using the commands 'max' and 'min'.
- The dynamic response of the sprung mass for all the conditions of height h, length L and car speed V does not show any minimum value less than zero.
- The maximum displacements of the sprung-mass depend on the hump dimensions for a specific car speed.
- Fig.3 illustrates graphically this relation as generated by MATLAB for a polynomial hump of 3 m length.

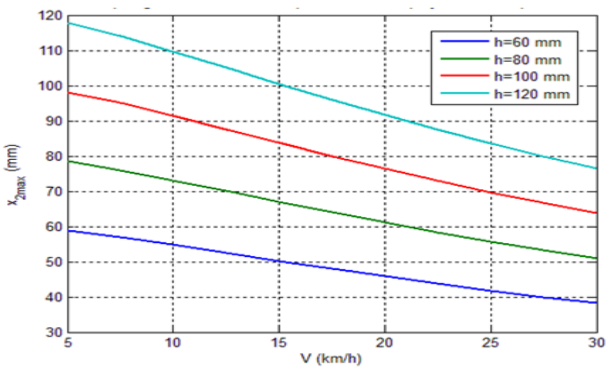

Fig.3 Sprung-mass maximum displacement for L = 3 m.

3.3 Sprung-mass Acceleration

The sprung-mass acceleration is the second derivative of its displacement with respect to time.

- The MATLAB command "diff" to differentiate the x2-t response twice producing the acceleration.
- Doing this, it didn't give any useful information.
- The author tried to overcome this pug by fitting an 8th order polynomial to the sprung mass velocity (dx_2/dt) time response, then differentiated this polynomial analytically yielding the sprung-mass acceleration.
- A sample result of this procedure is shown in Fig.4 showing the effect of the dimensions of the polynomial hump on car dynamics when crossing a 3m polynomial hump with 10 km/h speed.

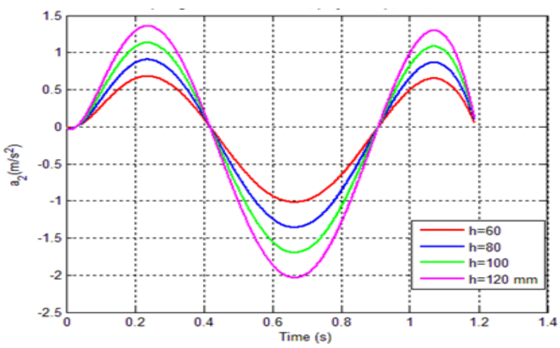

Fig.4 Sprung-mass acceleration at V = 10 km/h and L = 3 m .

- The maximum acceleration of the sprung-mass at certain car speed depends on the hump dimensions.

- There are maximum and minimum acceleration values of the sprung mass.
- The variation of the acceleration is smooth starting by zero acceleration at the beginning of the hump. This expected to give more comfort since the jerk in this case will be zero.
- The maximum sprung mass acceleration is taken as the maximum of the absolute value of the acceleration.
- The ride comfort maximum acceleration is considered as 0.8 m/s^2 according to ISO 2631 [19].
- The effect of car hump crossing speed, and dimensions of the polynomial hump is illustrated in Figs.5 through 8 as generated by MATLAB for a hump length of 3, 4, 5 and 6 m respectively.
- The two horizontal lines represent the 0.8 and 1.6 m/s^2 limits of the uncomfortable acceleration range [19].
- All the curves in Figs.5 through 8 are very smooth and don't need any smoothing process which is a special characteristic of the polynomial hump.

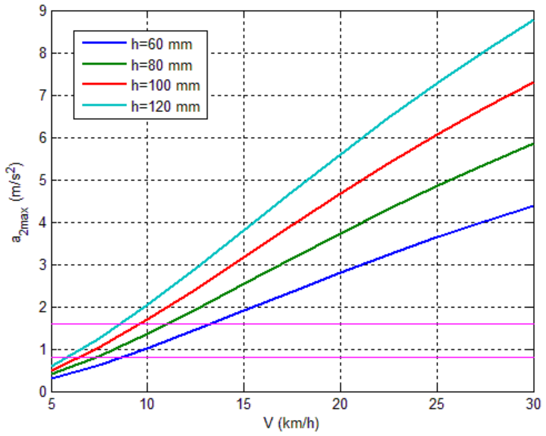

Fig.5 Sprung mass maximum absolute acceleration for L = 3 m.

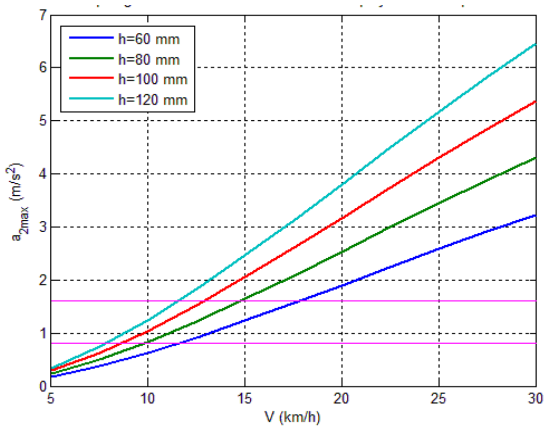

Fig.6 Sprung mass maximum absolute acceleration for L = 4 m.

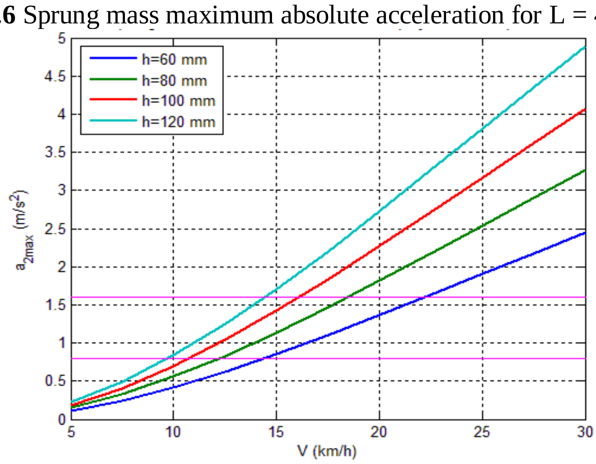

Fig.7 Sprung mass maximum absolute acceleration for L = 5 m.

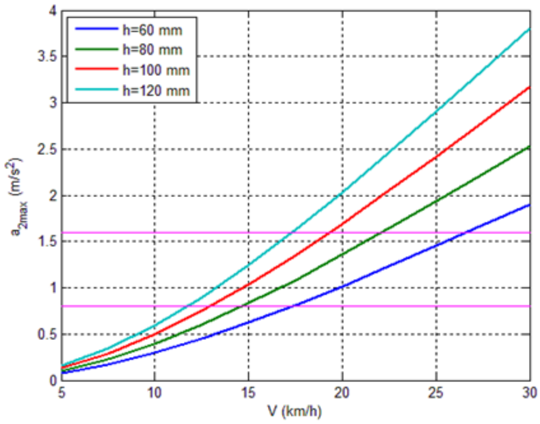

Fig.8 Sprung mass maximum absolute acceleration for L = 6 m.

3.4 Maximum Car Speed for Ride Comfort

According to ISO 2631, the ride comfort range ends with 0.8 m/s^2 [19]. Imposing this limit on the car dynamics of a quadratic-car model when passing a polynomial hump of the dimensions stated in section III gives an estimation for the maximum car speed when passing the hump for accepted ride comfort. This maximum car speed is given graphically in Fig.9 against hump length for hump height in the range: 60 ≤ h ≤ 120 mm.

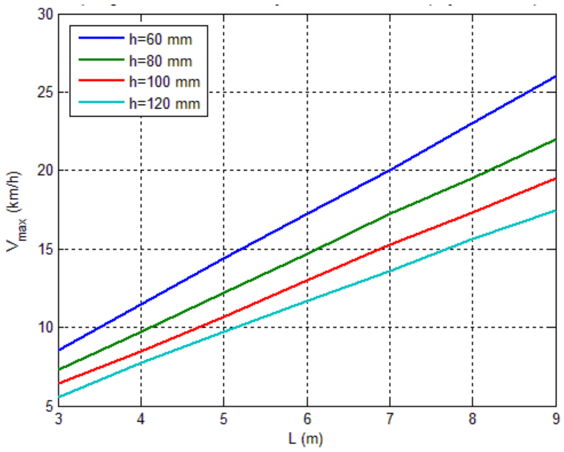

Fig.9 Car maximum speed for ride comfort across the polynomial hump.

The diagram in Fig.9 is zoomed to allow civil engineers to use in their plans to calm traffic without harming the vehicles users or violating the international standards.

IV. Conclusion

- A quarter-car model with passive elements was used in this study to investigate the car dynamics during passing a novel polynomial hump.
- The damping coefficient of the suspension was kept constant at 15 kNs/m.
- Car speed between 5 and 30 km/h was considered during crossing the polynomial hump.
- Hump length between 3 and 9 m was investigated.
- Hump height between 60 and 120 mm was considered.
- The time response of the sprung mass did not show any undershoot.
- The sprung mass acceleration was smooth with zero initial jerk.
- Ride comfort was considered through investigating the maximum sprung mass absolute acceleration during crossing the hump.
- The simulation of the quarter-car model using MATLAB showed that it is possible to reach a car speed of 26 km/h when crossing a polynomial hump if the hump is designed with 9 m length and 60 mm height.
- A diagram was provided allowing the design parameters of the polynomial hump for desired hump crossing speeds in the range 6 to 26 km/h.
- A 25 km/h hump crossing speed can be safely used for a polynomial hump of (8.7 m, 60 mm) hump dimensions.

- A 20 km/h hump crossing speed can be safely used for a polynomial hump of (7 m, 60 mm), (8.2 m, 80 mm) hump dimensions.
- A 15 km/h hump crossing speed can be safely used for a polynomial hump of (5.2 m, 60 mm), (6.1 m, 80 mm), (6.9 m, 100 mm), (7.7 m, 120 mm) hump dimensions.
- A 10 km/h hump crossing speed can be safely used for a polynomial hump of (3.5 m, 60 mm), (4.1 m, 80 mm), (4.7 m, 100 mm), (5.2 m, 120 mm) hump dimensions.

References

[1]. P. Weber, Twards Canadian standard for the geometric design of speed humps, M. Engineering Thesis, Department of Civil and Environmental Engineering, Carleton University, Ottawa, Canada, April 1998.
[2]. P. Weber and J. Braaksma, Towards a North American geometric standard for speed humps, ITE Journal, January, 30-34, 2000.
[3]. I. Parry, F. Marsh and N. Cripps, Investigating the use of simulation model nonlinear (SIMON) for the visual testing of road humps, WP 2003, 1-18, 2003.
[4]. S. Bjarnason, Round top and flat top humps, Thesis, Department of Technology and Society Traffic Engineering, Lund Institute of Technology, Lund University, 2004.
[5]. A. Tam, Traffic calming to hump, bump or dump ?, The Engineer, 29, 38-49, March-May 2006.
[6]. B. Aghazadeh, K. Saeedi and A. Yazdi, Simulation of ride quality of vehicles crossing road humps, Proceedings of the International Conference on Modeling and Simulation, 28-30 August, Konya, Turkey, Paper No.A066, 2006.
[7]. N. Greenwood, P. Nowson and T. Ahmed, Review of travel calming in Merton, November, 233-276, 2007 (www.merton.gov.uk).
[8]. J. Granlund and A. Brandt, Bus drivers exposure to mechanical shocks due to speed humps, 2008 IMAC XXVI Conference & Exhibition on Structural Dynamics, 10 pages, 2008.
[9]. J. Hessling, Analysis and synthesis of speed limiting road humps, Second Meeting on Analysis of Dynamic Measurements, London 18/11, 2008.
[10]. Broward County Traffic Engineering Division, Speed humps specifications, February, 2010.
[11]. C. Berthod, Traffic calming, speed humps and speed cushions, 2011 Annual Conference of the Transportation Association of Canada, Edmonton, Canada, 2011.
[12]. V. Dedovic and D. Sekulic, Effect of shock vibrations due to speed control humps to the health of city bus passengers using oscillatory model with six DOF, International Conference on Traffic and Transport Engineering, Belgrade, November 29-30, 633-642, 2012.
[13]. N. Yaacob and A. Hamsa, The effect of road hump in reducing of motorcars in a residential area in Kuala Lumpur, journal of Designed Built Environments, 13, 1-13, 2013.
[14]. G. A. Hassaan, Car dynamics using quarter model and passive suspensions: Part I: Effect of suspension damping and car speeds, International Journal of Computer Techniques, 1 (2), 1-9, 2014.
[15]. G. A. Hassaan, Car dynamics using quarter model and passive suspension, Part II: A novel simple harmonic hump, Journal of Mechanical and Civil Engineering , 12 (1), 93-100, 2015.
[16]. Florin, M. Ioan-Cosmin and P. Liliana, Passive suspension modeling using MATLAB, quarter-car model, input signal step type, New Technologies and Products in Machine Manufacturing Technologies, 258-263, January 2013.
[17]. M. Hatch, Vibration simulation using MATLAB and ANSYS, CRC Press, 2000.
[18]. R. Dukkipati, Solving vibration analysis problems using MATLAB, New Age International, 2007.
[19]. Y. Marjanen, Validation and improvement of the ISO 2361-1 (1997) standard method for evaluating discomfort from whole-body vibration in a multi-axis environment, Ph.D. Thesis, Loughborough University, January, 2010.

Dedication

- I dedicate this work to the sole of late Prof. Ahmed Ezzat Professor of System Dynamics at the Department of Mechanical Design & Production, Faculty of Engineering, Cairo University in the 1960's.
- Prof. Ezzat taught me System dynamics courses between 1968 and 1970.
- He was the reason to love this specialization and join his department in 1970 as teaching assistant and conduct research in this field up to now.

Biography

Galal Ali Hassaan

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations , Mechanism
- Synthesis and History of Mechanical Engineering.
- Published 10's of research papers in international journals and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.