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# Numerical Solution for the Design of a Traditional Aerospike Nozzle using Method of Characteristics

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***Abstract:*** *In this paper we develop a computer code which uses the Method of Characteristics and the Stream Function to define the traditional aerospike nozzle contour for isentropic, inviscid, irrotational supersonic flows of any working fluid for any user-defined exit Mach number. The contour obtained is compared to theoretical isentropic area ratios for the selected fluid and desired exit Mach number. The accuracy of the nozzle to produce the desired exit Mach number is also checked. The flow field of the nozzle created by the code is independently checked with the commercial Computational Fluid Dynamics (CFD) code ANSYS-FLUENT. ANSYS- FLUENT predictions are used to verify the isentropic flow assumption and that the working fluid reached the user-defined desired exit Mach number. Good agreement in area ratio and exit Mach number is going to be achieved, verifying that the code is accurate.*

***Key words:*** *Supersonic, Method of Characteristics, Stream Function, Backward Characteristic, Isentropic, Prandtl- Meyer expansion angle.*

**Nomenclature:**
$r$ - *Radial coordinate*
$\phi$ - *Angular coordinate*
$u$ - *x direction component velocity*
$v$ - *r direction component velocity*
$w$ - *$\phi$ direction component velocity*
$\theta$ - *Flow Direction*
$\alpha$ - *Mach Angle*
$\upsilon$ - *Prandtl-Meyer Expansion angle*
β - *radius defining the arc of the expansion region*
$\Psi$ - *Stream Function*
*C-* - *Right running Characteristic*
*C+* -*Left running Characteristic*

## I. Introduction

Although the burn characteristics of the fuel are an important part of the analysis of a rocket, the rocket's efficiency is primarily dependent upon the nozzle's ability to convert the thermal energy of the fluid to kinetic energy. The main nozzle wall contour plays a critical role in this conversion. It is also important to ensure shocks do not occur within the nozzle. Shocks in the nozzle will disrupt the supersonic flow and will create large losses during the conversion of thermal energy to kinetic energy. The wall contour of the nozzle is the defining factor in whether shocks will or will not form within the nozzle. The pressure ratio between the chamber and the exit plane of the nozzle dictate the maximum potential Mach number reached by the working fluid. There are many configurations of supersonic nozzles that will achieve the necessary conversion of thermal energy to kinetic energy to create a rocket's thrust and one among them is the Aerospike nozzle. For aerospike nozzles, the flow is bounded on one side by a wall whereas in an annular nozzle the flow is bounded by a wall contour on all sides. Figure 2.1 illustrates a generic Aerospike nozzle configuration.

It has been found that for maximum thrust, the flow direction of the fluid under sonic conditions should be offset from the axisymmetric line by an angle equal to $\nu_{exit}$ the Prandtl- Meyer expansion angle associated with the desired exit Mach number of the nozzle.

## II. Method Of Characteristics Approach

The traditional aerospike nozzle allows the expansion of the flow to happen completely externally.

**Traditional or Minimum-Length Aerospike Nozzle**

Greer, 1960, describes a method which uses geometry and the isentropic area ratio equation to define the contour of the aerospike nozzles. First, before we discuss the method, it is important to note that the angle the direction of the flow at the throat makes with the nozzle's axisymmetric line at the beginning of the traditional expansion is equal to the Prandtl-Meyer expansion angle for the user-defined desired exit Mach number. Using this angle as the sonic flow direction, the Prandtl-Meyer expansion fan centered at the tip of the cowl located at the end of the sonic line furthest from the nozzle's axisymmetric line can be stepped through by a user-defined Prandtl-Meyer expansion angle increment. For each Prandtl-Meyer expansion angle stepped through, its associated Mach number can be calculated. Using the Mach number and geometry, the length of the line from the tip of the cowl is known from the isentropic area ratio equation. From geometric manipulation and the flow properties of an expansion fan, the slope of the line emanating from the tip of the cowl can be calculated. Since the tip of the cowl can be geometrically set by the designer, the points located on the nozzle's contour can be calculated using trigonometry. Greer non-dimensionalized the calculation by dividing the length of the lines emanating from the tip by the length associated with the desired exit Mach number. The calculations are stepped through until the desired exit Mach number is obtained. It is also important to note that the flow properties along the lines emanating from the tip of the cowl are assumed to be constant. This is important because the curved nature of the characteristics is not taken into account for the calculation of axisymmetric nozzles introducing errors. The points on the contour are then connected byline segments to make the aerospike's contour. This method is accurate when comparing the exit to throat area ratio since the isentropic area ratio is used in defining the contour. The contour becomes smoother as the number of points defining the contour increase, aka the Prandtl-Meyer expansion angle increment decreases.

The geometry of a traditional aerospike nozzle is give in the figure below

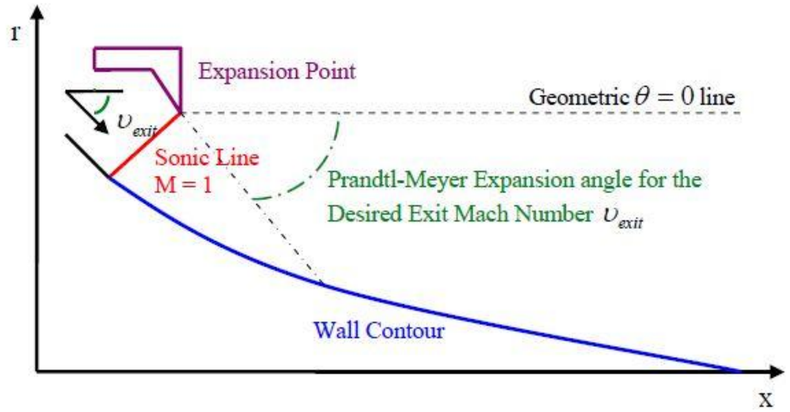


**Figure 2.1:** Generic Geometry of a Traditional Aerospike Nozzle

## III. Design Methodology

### 3.1 Discretizing the Characteristic and Compatibility Equations

To implement the characteristic and compatibility equations into a computer code for designing supersonic nozzle contours, the equations for axisymmetric, irrotational, inviscid flow developed in Appendix A must be discretized with boundary conditions defined and applied. The first step in designing a computer code is to discretize the characteristic and compatibility equations. They are rewritten below

$$\left(\frac{dr}{dx}\right)_{char} = \tan(\theta \mp \alpha) \qquad \text{----3.1}$$

$$d(\theta + \alpha) = \frac{1}{\sqrt{M^2-1} - \cot\theta}\frac{dr}{r} \qquad \text{----3.2(a)}$$

(along c_characteristic)

$$d(\theta - \alpha) = \frac{1}{\sqrt{M^2-1} + \cot\theta}\frac{dr}{r} \qquad \text{----3.2(b)}$$

(along $c_+$ characteristic)

Equation 3.1 can be split to illustrate the two separate C- and C+ characteristic equations.
They are written below

$$\left(\frac{dr}{dx}\right)_{c_-} = \tan(\theta - \alpha) \qquad \text{----3.3 (a)}$$

$$\left(\frac{dr}{dx}\right)_{c_+} = \tan(\theta + \alpha) \qquad \text{----3.3(b)}$$

Using the Forward Difference Technique and rearranging equations 3.3a and b yields

$$r_{i+1} - \tan(\theta_i - \alpha_i).x_{i+1} = r_i - \tan(\theta_i - \alpha_i).x_i \qquad \text{----3.4(a)}$$

(along c_characteristic)

$$r_{i+1} - \tan(\theta_i + \alpha_i).x_{i+1} = r_{i+1} - \tan(\theta_i + \alpha_i).x_{i+1} \qquad \text{----3.4(b)}$$

(along $c_+$ characteristic)

Note that all variables with subscript i are known quantities and variables with subscript i+1 are unknown quantities. Equations 3.4a and 3.4b are the discretized characteristic equations that will define the location in the x-r space where the C- and C+ characteristics curves intersect. This collection of points is called the Characteristic Net.

Equation 3.2a and 3.2b, the compatibility equations, can also be discretized. Using the Forward Difference Technique and rearranging gives

$$(\theta_{i+1} + \nu_{i+1}) = (\theta_i + \nu_i) + \frac{1}{\sqrt{M^2-1} - \cot\theta}\frac{r_{i+1} - r_i}{r_i} \qquad \text{----.3.5(a)}$$

(along c_characteristic)

$$(\theta_{i+1} - \nu_{i+1}) = (\theta_i - \nu_i) - \frac{1}{\sqrt{M^2-1} - \cot\theta}\frac{r_{i+1} - r_i}{r_i} \qquad \text{----3.5(b)}$$

(along $c_+$ characteristic)

Since the Prandtl-Meyer expansion angle is known for any given point on the expansion arc,a root finding routine can be employed to solve for the Mach number associated with the Prandtl-Meyer expansion angle and user-defined ratio of specific heats for the working fluid.

$$\nu = \sqrt{\frac{\gamma-1}{\gamma+1}}\tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M_i^2 - 1)}\right) - \tan^{-1}\left(\sqrt{M_i^2 - 1}\right) \qquad \text{----3.6}$$

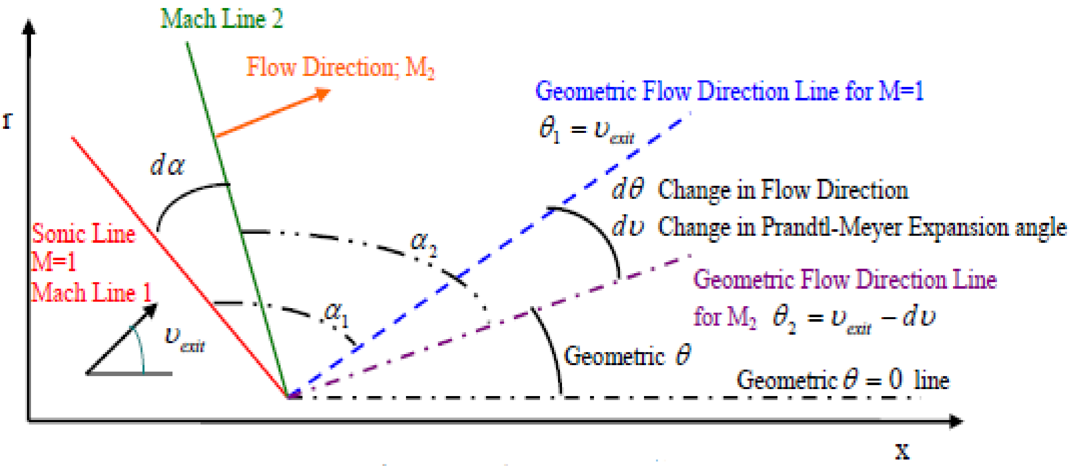


**Figure 3.1.1:** Geometric Relationship between Mach Lines and Flow Direction

Looking at Figure 3.1.1, it can be see that the blue-dashed line indicates the direction of the flow at the sonic line given by the relationship

$$\theta_{sonic\ line} = v_{exit} \qquad \text{----3.7}$$

The purple-dot-dashed line indicates the direction of the flow after the flow has past through a characteristic line (Mach wave) with a change in Prandtl-Meyer expansion angle of dθ and in turn, a change in flow direction of dθ = dv . Since the flow at the throat is sonic, i.e. a Mach number equal to one, equation 3.6 shows that the Prandtl-Meyer expansion angle at the throat is equal to zero. If the incremental change in Prandtl-Meyer expansion angle, dθ , is known for each Mach wave, the Mach number along each Mach wave could be calculated by using a root finding routine as previously described previously for equation 3.6. Once the Mach number is calculated, the Mach angle á for each Mach wave can be calculated. Now that Mach angle is now known, the geometric angle the characteristic makes with the x-axis can be calculated from Expansion Fan Theory using

$$\sigma_{mach\ line} = \alpha_{mach\ line} + (v_{exit} - dv) \qquad \text{----3.8}$$

If we assume that the location of the expansion point is known combined with the knowledge of the calculated geometric angle the characteristic makes with the x-axis, an equation approximating the characteristic emanating from the expansion point can be obtained.

Assuming that the expansion point is located at (0, 0), the characteristic equation for an aerospike nozzle becomes

$$r_{machline} = \tan(\sigma_{mach\ line}).x \qquad \text{----3.9}$$

Notice that except for the variable r, the analysis does not stipulate that this is an axisymmetric solution. As with the previously discussed annular nozzle, the Stream Function can be utilized to calculate the wall contour of the aerospike nozzle. The axisymmetric Stream Function solved for and discretized ensures that the solution is axisymmetric.

The next step is to define an initial streamline condition. This is done by assuming a throat length of one. This also allows for a non-dimensional calculation scheme. Assuming the length of the sonic line is 1, solving mach angle for the sonic condition of M = 1, solving equation 3.9 for the slope of the characteristic defining the sonic line and assuming that the location of the expansion point is (0, 0), the location of the initial point on the wall contour can be solved for from geometry by

$$x_1 = 1.0\cos\left(\frac{\pi}{2} + v_{exit}\right) + x_{expansion\ \ point} \qquad \text{----3.10(a)}$$

$$r_1 = 1.0\sin\left(\frac{\pi}{2} - v_{exit}\right) + r_{expansion\ \ point} \qquad \text{----3.10(b)}$$

The final condition needed to solve for in order to calculate the rest of the wall contour points satisfying the Stream Function is the geometric flow direction along each characteristic. According to Angelino, the flow direction at the sonic line should be equal to the Prandtl- Meyer expansion angle with respect to the desired exit Mach number to obtain maximum thrust, see equation 3.7.

Since the characteristics are straight lines and no other characteristics intersect them, the flow along the characteristics exhibit a constant Prandtl-Meyer expansion angle and flow direction equal to their values at the expansion point associated with their respective characteristics. According to Figure 2.1, this means the flow direction angle is decreasing at the same rate as the Prandtl-Meyer expansion angle is increasing.

$$\theta = v_{exit} - v \qquad \text{----3.11}$$

Sweeping through the expansion fan by an incremental change in Prandtl-Meyer expansion angle, dv, all variables for the characteristics are known using equations 3.8 through 3.12. Since each characteristic's equation is defined throughout the expansion fan, a similar method described above for calculating the points satisfying the Stream Function for the annular nozzle can be utilized to find the points defining the wall contour of the aerospike nozzle. The calculation is stepped through the expansion fan by a user-defined incremental change in Prandtl-Meyer expansion angle until the direction of the flow, θ, is equal to zero.

In order to achieve a better thrust to weight ratio, the aerospike nozzle can be truncated. The truncation is based on a user-defined percentage of the total length of as if the flow was allowed to reach its final flow direction of 0 radians. The truncated nozzle's contour points are the same as the ideal length nozzles. The truncated nozzle's contour ends when its x-component equals the user-defined percentage of the x-component of the last contour point of the ideal length.

## IV. Results and Discussion

This section discusses the checks performed to verify the accuracy of the code developed and the plot of the nozzle contour is shown in figure 4.1.0. A combination of theoretical and CFD simulations were employed to verify the accuracy of the code for all the rocket nozzle configurations. This section highlights the general trends in the nozzle for the various checks performed.

### 4.1 Theoretical Accuracy of Computer Code

The first check of accuracy for the program was comparing the desired exit Mach number with the exit Mach number calculated by the program. Table 4.1.1 below shows the percent difference between the desired and computer calculated exit Mach numbers. Table 4.1.1 also shows how the code becomes more accurate as a smaller change in Prandtl-Meyer expansion angle is used during calculations.

Since the equations were based on isentropic flow theory, the accuracy of the code was also checked by calculating the exit to throat area ratio using equation 4.1 substituting in the user- defined ratio of specific heats and computer calculated exit Mach number. This yields the theoretical area ratio for the Mach number actually calculated by the program.

$$\frac{A_{exit}}{A_{throat}} = \sqrt{\frac{1}{Ma^2{}_{exit}} \cdot \left[\frac{2}{\gamma+1} \cdot \left(1+\frac{\gamma-1}{2} \cdot Ma^2{}_{exit}\right)\right]^{\frac{\gamma+1}{\gamma-1}}} \qquad \text{----4.1}$$

The theoretical and computer calculated isentropic area ratios for the desired exit Mach number were also compared for a user-defined ratio of specific heats in Table 4.1.1.

**4.1.1. ANSYS-FLUENT SIMULATIONS.**

| **Table 4.1.1** | Code Accuracy check for $\gamma$ =1.4 Ma=3.0 $\beta$=1.0.$r_{throat}r_{throat}$=1.0 (Dimensionless) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $\left[\frac{A_{exit}}{A_{thraot}}\right]_{comp}$ | $\left[\frac{A_{exit}}{A_{thraot}}\right]_{Theory}$ | $\left[\frac{A_{exit}}{A_{thraot}}\right]_{\%error}$ | $Ma_{comp}$ | $Ma_{\%error}$ | $\left[\frac{A_{exit}}{A_{thraot}}\right]_{comp\ ,theory}$ | $\left[\frac{A_{exit}}{A_{thraot}}\right]_{comp\ ,\%erro}$ |
| Δυ=0.05 | | | | | | | |
| **Aerospike** | 4.9828 | 4.2346 | 17.67% | 3.0000 | 0.0% | - | - |
| Δυ=0.025 | | | | | | | |
| **Aerospike** | 4.5875 | 4.2346 | 8.33% | 3.0000 | 0.0% | - | - |
| Δυ=0.01 | | | | | | | |
| **Aerospike** | 4.3716 | 4.2346 | 3.24% | 3.0000 | 0.0% | - | - |
| Δυ=0.005 | | | | | | | |
| **Aerospike** | 4.3025 | 4.2346 | 1.60% | 3.0000 | 0.0% | - | - |
| Δυ=0.0025 | | | | | | | |
| **Aerospike** | 4.2684 | 4.2346 | 0.80% | 3.0000 | 0.0% | - | - |
| Δυ=0.001 | | | | | | | |
| **Aerospike** | 4.2481 | 4.2346 | 0.32% | 3.0000 | 0.05% | - | - |

Using the simulation, typical entropy change contour seen in 100% length traditional aerospike nozzle in Figure 4.1.1. For perspective, the figures used were taken from simulations of an traditional aerospike nozzle designed for an exit Mach number of 3.0.

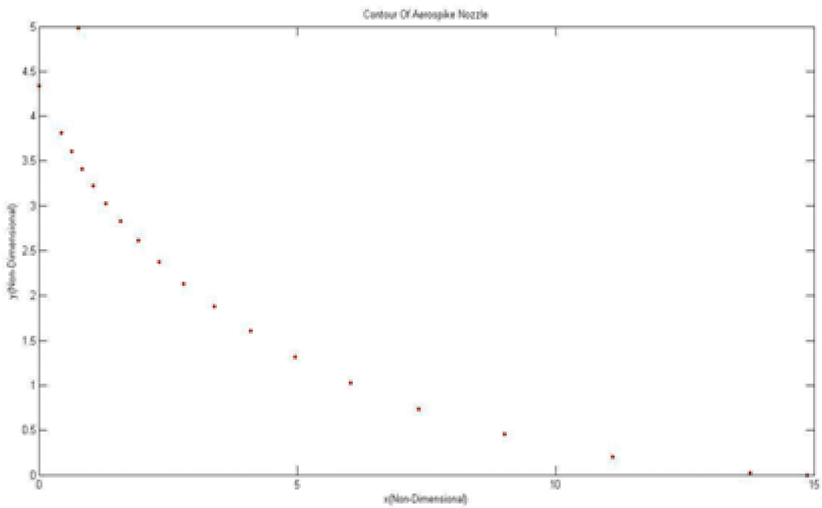


**Figure 4.1.0: Profile of the Traditional Aerospike Nozzle**

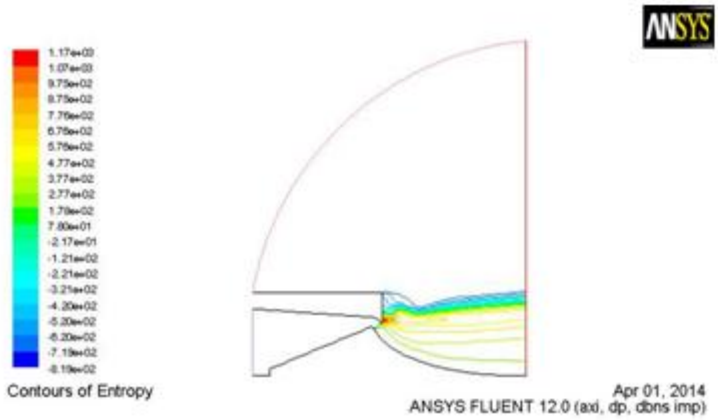


**Figure 4.1.1:** Typical Entropy Change Contours of a 100% Length Traditional Aerospike Nozzle.

As expected, there are significant changes in entropy where the "imaginary" wall(s) of the traditional aerospike nozzle develop along the constant pressure boundaries. Taking a closer look at the plots, it is evident that the main body of the exhaust plume remains at constant entropy validating the isentropic assumption used to develop the computer program.

Using ANSYS-FLUENT to calculate the area-weighted exit Mach number, the 100% traditional aerospike nozzle simulations yield an exit Mach number of 2.3682. Although the results of the simulations for the traditional aerospike nozzles are less encouraging when compared to the desired exit Mach number, this is not all together unexpected. Reflecting back on the calculations used to define the nozzle's contour, only one characteristic equation was used with no consideration taken for the characteristic's curvature or effects the constant pressure boundary may cause. Further investigation is needed to devise a method to incorporate the characteristic's curvature and constant pressure boundary which in turn should improve the accuracy of the code developed. Table 4.1.2 shows Mach number comparisons between the desired, computer calculated and largest contour value for 100% length aerospike nozzle.

After comparing the Mach numbers, it is clear that the code developed is valid, but much less accurate for aerospike nozzles than annular nozzles.

Figure 4.1.2 is the Mach contour of typical 100% length traditional aerospike nozzles, respectively.

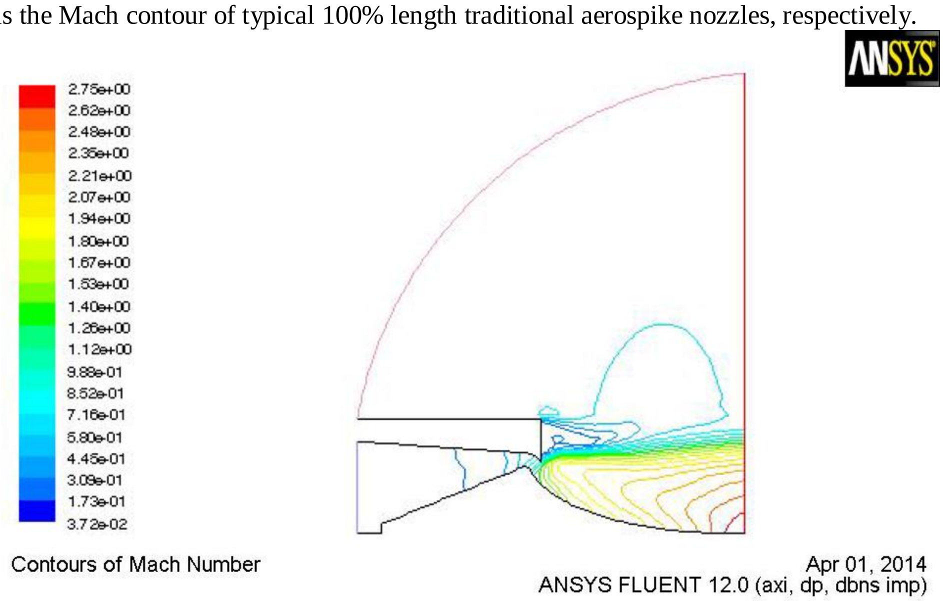


**Figure 4.1.2:** Typical Mach Contours of a 100% Length Traditional Aerospike Nozzle

| **Table 4.1.2** | | **Mach Number Comparisons for 100% Length Aerospike Nozzle** | | |
|---|---|---|---|---|
| | $Ma_{Desired}$ | $Ma_{ComputerCalculated}$ | $Ma_{FluentCalculated}$ | $Ma_{ContourPlot}$ |
| | 3.0 | 3.0000 | 2.3682 | 2.75 |
| Percent Error From $Ma_{Desirerd}$ | | 0.0% | -21.06% | -8.33% |

## V. Conclusion

The code developed proves to be a useful tool in creating annular nozzle contour for isentropic, irrotational, inviscid flow. The program exhibits increasing accuracy in the exit Mach number and exit area ratio as the incremental Prandtl- Meyer expansion angle decreases. This accuracy increase is independent of fluid or desired exit Mach number. The exit Mach number of the nozzles calculated with the program shows good agreement with the ANSYS-FLUENT simulated exit Mach numbers. This independently confirms the accuracy of the program in calculating supersonic nozzle contours for inviscid, isentropic, irrotational supersonic flows.

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