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The Thick Orthotropic Plates Analysis Methods, Part I: A Review

Parham Mohajerani
Lecturer, Department of Civil Engineering, Payame Noor University, Esfahan, Iran

Abstract: *As usage of plates, especially thick plates, are increased in structural and mechanical projects, the need for more precise plate analysis method increased, too. Designers look for a plate analysis method which can result in an exact stress-strain relationship with consideration of thickness effects, shear deformation and compressive deformation of plate across the thickness. This paper reviews the process of thick plate analysis method developments by describing a number of main plate analytical methods. Weak and strong points of each method are explained for each method.*
Keywords: *Exact 3D solution, Orthotropy,State Space method, Thick plate, Plate analysis*

I. Introduction

The Plates are straight and plane surface structures whose thickness is slight compared to other dimensions geometrically. The Classical Plate Theory (CLPT) limitation for the solution of flexural problems associated with composite laminates have been investigate by several research workers [1-4]. In many researches, it has been shown that the effect of transvers shear deformation is important. Mindlin's plate theory [5] which allows for this type of defamation, has been used as a basis for obtaining more accurate solutions [6]. Classical Plate theory is based on its use of conventional isotropic thin plate theory and transverse shear deformation and rotatory inertia effects neglected in the laminate. Hence, this analytical method yields only average through thickness values for the in-plane stress and gives inadequate values about the essential inter-laminar shear stress, which can be the cause for delamination fact. Therefore, researchers try to use three-dimensional equilibrium that takes into account three-dimensional variation of stresses and strains. This idea helps them to provide more accurate response of laminated composite structures [7, 8].

Method of Initial Function (MIF) [9], Finite Strip Method [10], State Space Method [11, 12] and Finite Element Method [13] are four main methods have been used to solve composite laminate plates structures.

Method of Initial Function (MIF) is one of the most accurate method in elastic analysis of orthotropic composite material which proposed by VasiliĭZakharovichVlasov (1955). V.Z, Vlasov offered his method to solve problems in theory of elasticity. Bahar, Das and Rao introduced the state space and used matrix method in solving progress of MIF [11, 12].

In addition, plate thickness can affect stress-strain results and displacement outputs. Classical Plate theory and some other solutions (i.e. Reissner and Ambartsumyan) have not been mentioned the effect of z variation in their solution. For instance, Ressiner, Vlasov and Mindlin assumed different functions for compressive deformation, but none of them has no relationship to coordinate z. In 1989, Sundara Raja Iyengar and Pandya used a sixth-order governing equation to analyze uniformly loaded simply supported square plates for various thickness and material properties [14] and they inserted thickness factor in their solution.

Exact 3-D solution is the most accurate method based on state space method which is involved with matrix method. The problem of bending of simply supported thick orthotropic rectangular plates and laminates was solved by Rao and Sirinivas[2]. Then, Fan and Ye (1990) have derived solutions in the form of double Fourier series. Jiarang Fan used State Space method for solving thick orthotropic structures [15] with different boundary conditions. Wu and Wardenier in 1998 proposed exact elastic solution for anisotropic thick rectangular plates with four edges simply supported [16].

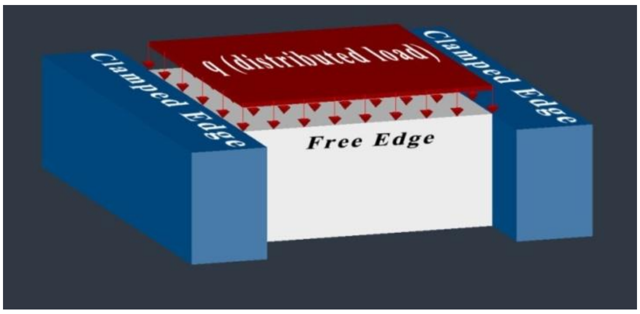

Fig. 1 Load & boundary conditions and general geometry of problem.

II. Elastic Solutions for Anisotropic Simply supported Rectangular Plates

There are different methods for elastic analysis of anisotropic plates. In some methods, compressive deformation is neglected. In some others, researchers got this deformation linear or cubic function. For example, Vlasov assumed this deformation has a cubic function and Hencky in 1947 assumed a linear function for compressive deformation.

Reissner got compressive deformation is a cubic function across the thickness. Mindlin assumed that function of compressive deformation is sine function. However, the deflection in z direction has no relationship to coordinate z.

Method of initial function (MIF) was one of the first methods that used to investigate the bending problem of rectangular isotropic material by Sundara Raja Iyengar. Bahar, Das and Rao used the state space and matrix method to the MIF [11]. Iyengar and pandya were the first researchers who mentioned this idea that compressive deformation in anisotropic material related to z coordinate and Pagano [17] had the same idea about the effect of z coordinate in compressive deformation value through the thickness. However, his calculation was limited. Wu and Wardenier (1997) removed the limitation in Pagano's solution for anisotropic rectangular simply supported plate and they provided six-order differential equation for estimating of transverse displacement w in simply supported rectangular tick plate.

2.1 Ambartsumyan's Theory

Ambartsumyan (1969) introduced quadratic function for shear stress variation across the thickness. However, in his theory, compressive deformation was z invariant. He presented an elastic solution for bending of hinged orthotropic rectangular plate. Based on Eq. (1) presented in his analytical method, the compressive deformation is not z dependent. f_{mn}in Eq. (1) is an unknown coefficient and it can be calculated based on Eq. (2). Ambartsumyan, based on Navier solution (1820), used Eq. (3) to represent the load function which has a double Fourier series function. a_{mn} as a load function factor for uniformly distributed load was proposed in form of Eq. (4).

Due to z invariant of f_{mn} and a_{mn}, the transverse deformation is constant based on Ambartsumyan solution.

$$\omega = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (1)$$

$$f_{mn} = \frac{\Delta_{1mn}}{\Delta_{0mn}} a_{mn} \quad (2)$$

$$F(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (3)$$

$$a_{mn} = \frac{4}{ab}\int_0^a\int_0^b f(x,y)\, sin\frac{m\pi x}{a} sin\frac{n\pi y}{b} dx\, dy \quad (4)$$

$$\Delta_{0mn} = \frac{h^3}{12}\left[D_{11}\frac{\pi^4 m^4}{a^4} + 2(D_{12}+2D_{66})\frac{\pi^4 m^4 n^2}{a^2 b^2} + D_{22}\frac{\pi^4 n^4}{b^4}\right] + \frac{h^2}{10}\left[\left(D_{11}\frac{\pi^2 m^2}{a^2} + D_{66}\frac{\pi^2 n^2}{b^2}\right)\left(D_{22}\frac{\pi^2 n^2}{b^2} + D_{66}\frac{\pi^2 m^2}{a^2}\right) - (D_{12}+D_{66})^2 \frac{\pi^4 m^2 n^2}{a^2 b^2}\right](a_{44}\frac{\pi^2 m^2}{a^2} + a_{55}\frac{\pi^2 n^2}{b^2}) \quad (5)$$

$$\Delta_{1mn}=\frac{12}{h^3}\left\{\frac{h^6}{144}+\frac{h^5}{120}\left[a_{55}\left(D_{11}\frac{\pi^2m^2}{a^2}+D_{66}\frac{\pi^2n^2}{b^2}\right)+a_{44}\left(D_{22}\frac{\pi^2n^2}{b^2}+D_{66}\frac{\pi^2m^2}{a^2}\right)\right]\right.$$
$$-\frac{h^4}{100}a_{44}a_{55}\left[(D_{12}+D_{66})\frac{\pi^4m^2n^2}{a^2b^2}-\left(D_{11}\frac{\pi^2m^2}{a^2}+D_{66}\frac{\pi^2n^2}{b^2}\right)\right.$$
$$\left.\left.\times\left(D_{22}\frac{\pi^2n^2}{b^2}+D_{66}\frac{\pi^2m^2}{a^2}\right)\right]\right\}$$
$$+\frac{h^2}{10}\{A_1\left[\frac{h^3}{12}\frac{\pi^2m^2}{a^2}+a_{44}\frac{h^2}{10}\frac{\pi^2m^2}{a^2}\left(D_{22}\frac{\pi^2n^2}{b^2}+D_{66}\frac{\pi^2m^2}{a^2}\right)-a_{55}\frac{h^2}{10}(D_{12}+D_{66})\frac{\pi^4m^2n^2}{a^2b^2}\right]$$
$$+A_2[\frac{h^3}{12}\frac{\pi^2n^2}{b^2}+a_{55}\frac{h^2}{10}\frac{\pi^2n^2}{b^2}\left(D_{11}\frac{\pi^2m^2}{a^2}+D_{66}\frac{\pi^2n^2}{b^2}\right)-a_{44}\frac{h^2}{10}(D_{12}+D_{66})\frac{\pi^4m^2n^2}{a^2b^2}\} \quad (6)$$

where ω is transverse deformation, f_{mn} is transverse deformation coefficient, a is rectangular length in x direction and b denotes rectangular length in y direction. Δ_{1mn}, Δ_{0mn} are related to transverse deformation coefficient factors. $F(x, y)$ is load function and a_{mn} denoting its factor. $a_{mn}$for uniform distributed load of q is equal to $\frac{16q_0}{m.n.\pi^2}$.

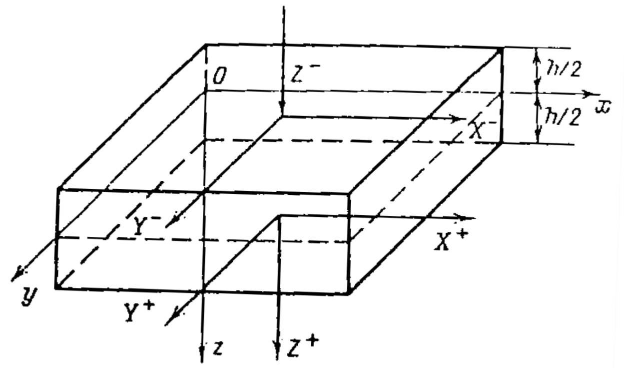

Fig. 2 Ambartsumyan's solution plate geometry.

2.2 Kirchhoff-Love Theory

Kirchhoff-Love theory is also popular as "Classic Plate Theory ". Gustavo Kirchhoff has worked on stress and deformation determination in thin plates. In 1888, Augustus Edward Hough Love, the British mathematician, worked on theory of elasticity. He used Kirchhoff assumptions and tried to develop his idea.

Kirchhoff thin plate theory was based on Euler-Bernoulli beam theory. He assumed, straight line normal to the mid-surface remain straight after deformation. In addition, he assumed straight line normal to the mid-surface remain normal to the mid-surface after bending deformation. It means that effect of shear stress was neglected in his theory. He also assumed the thickness of plate does not change during a deformation [18]. Three dimensional solutions proved that, this assumption is wrong.

Kirchhoff and love changed a 3-D plate into 2-D plate of middle surface plane. This assumption makes elastic plate analysis fallowed by the plane stress solution of plate bending problem. However, it leads to removing of z in their solution. Their solution is not z variant for deformation across the plate thickness.

$$D\nabla^4 w = q(x, y) \quad (7)$$

$$\nabla^4 \equiv \left(\frac{\partial^4}{\partial x^4}+2\,\frac{\partial^4}{\partial x^2\,\partial y^2}+\frac{\partial^4}{\partial y^4}\right) \quad (8)$$

where w is transverse deflection; ∇ is the biharmonic differential operator ($\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ rectangular co-ordinates); $D = Eh^3/12(1 - \upsilon^2)$, the flexural rigidity; E is Young's modulus; h is plate thickness; υ is Poisson's ratio. Exclude such complicating effects as orthotropy, in-plane forces, variable thickness, large deflections, shear deformation and rotary inertia, and non-homogeneity [19].

2.3 Mindlin & Reissner Plate Theories

Raymond Mindlin and Eric Reissner extended the Kirchhoff-love plate theory. They tried to make a better approximation for solving bending of elastic plates. In 1945, Reissner proposed his idea that shear deformation can effect on the bending of elastic plates. Reissner assumed that the bending stress is linear while shear stress is parabolic through the thickness. It means that the normal to the mid-surface remains straight but not necessarily perpendicular to it.

Mindlin assumed that compressive deformation across the plate thickness has linear variation. However in his theory, the plate thickness does not change after deformation [5]. Mindlin ignored the normal stress through the thickness and this assumption leads to plane-stress condition problem. In contrast, Reissner accounts this normal stress in his plate theory [20]. Both Mindlin and Reissner theories independently proposed plate theories that incorporate the effect of transvers shear deformation for analysing "thick plate" (Wang et al., 2001). As it has mentioned before, Reissner is the first researcher who proposed effect of transverse shear deformation on the bending of elastic plates. In Eqs. (9) and (10), the displacement in n and s directions (n= x direction, s= y direction) contain of shear term. These two formulas indicate that, due to the effect of shear deformation, normal and tangential line elements in middle surface do not remain perpendicular to the linear element which was before deformation perpendicular to the middle surface [21].

$$\frac{\partial w}{\partial n} - \frac{12(1+\upsilon)}{5hE} V_n = -\overline{U}_{n'} \tag{9}$$

$$\frac{\partial w}{\partial s} - \frac{12(1+\upsilon)}{5hE} V_s = -\overline{U}_{s'} \tag{10}$$

where:
$\overline{u_{n'}}$: Displacement in direction n [22]
$\overline{u_{s'}}$: Displacement in direction s (y)
w : Displacement component normal to the plane of the plate

Reissner proposed this idea that w is biharmonic function. It means, he assumed w has a forth order partial differentiation function.

2.4 Viktor Valentinovich Vlasov Method

V.V. Vlasov used Method of Initial Function (MIF) to solve displacement function for simply supported plate [23]. He analyzed plate bending behavior based on MIF for homogenous and non-homogenous material. As it has mentioned in Eqs. (11) and (12), w is the displacement through the thickness and it has biharmonic function. However, he proved this equation based on *z= ± h/2*. By setting *z= ±h/2*, the results shows, the displacement across the thickness is not z variant. He proposed Eq. (11) to calculate w in the case of homogenous plate bending condition:

$$w = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \frac{\nu}{1-\nu} z^2 (c_3 + 3c_4 x) \tag{11}$$

and Eq. (12) for non-homogenous case defined as:

$$w = \frac{q}{24D}\{x(a^3 - 2ax^2 + x^3) + \frac{3}{10}\frac{8-3\upsilon}{1-\upsilon} h^2 x(a-x) + z^2\left[\frac{3(5-3\upsilon)}{10(1-\upsilon)} h^2 + \frac{6\upsilon}{1-\upsilon} x(x-a) - \frac{1+\upsilon}{1-\upsilon} z^2\right] \tag{12}$$

where:
$z = \pm\frac{h}{2}$
h : Plate thickness
a : a ×a rectangular plate

In Eqs. (11) and (12), z can be set as h/2 or –h/2. This means that, V. V. Vlasov believed that vertical displacement across the thickness is constant.in both cases (h/2 or –h/2), because of even power of z variable, the value of w at top and bottom of plate at specific x and y are similar. Similarity in the value of top and bottom surface is not acceptable because Wu and Wardenier proved that displacement of top is more than bottom for simply supported anisotropic thick plates.

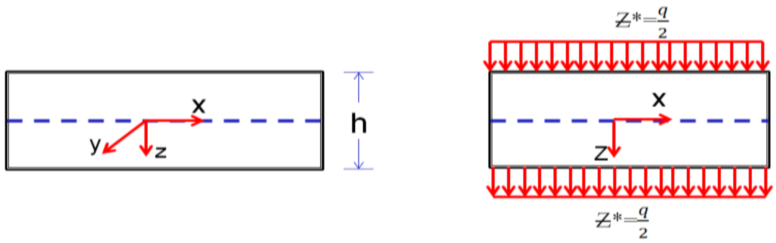

Fig. 3 Vlasov solution geometry.

2.5 Pagano Solution

Pagano (1969) worked on exact elastic response of rectangular, pinned edge laminates consisting of any number of orthotropic or isotropic layers [24]. On the other hand, he has presented further evidence regarding the range of validity and limitation of CLPT. He came up with six-order partial differentiation (Eq. 13), which is z variant. He proved his idea by using Cardano's mathematical method to solve the differential equation. However, Pagano assumed H factor in Cardano's method is always negative for all material (Eq. 16). This assumption is incorrect and Wu and Wardenier (1997) proved H can be any real number. The sign of H related to the combination of geometric properties and material properties.

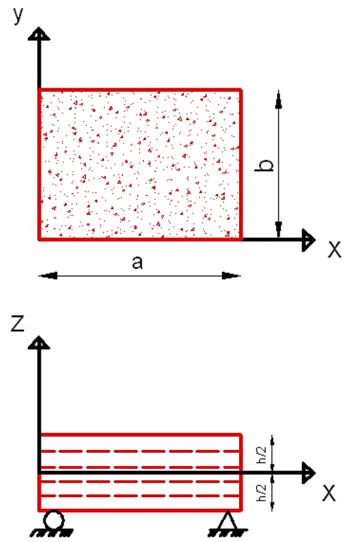

Fig. 4 Pagano's solution geometry.

$$-As^6 + Bs^4 + Cs^2 + D = 0 \tag{13}$$

$$\gamma^3 + d\gamma + f = 0 \tag{14}$$

$$\gamma = s^2 - \frac{B}{3A} \tag{15}$$

$$H = \frac{f^2}{4} + \frac{d^3}{27} \tag{16}$$

where:

S: $\frac{\partial w}{\partial z}$

A, B, C and D: coefficients that consist of elastic coefficient and load

f, d: Cardano's method coefficient

This equations is true for simply supported rectangular plates [24].

Another idea which Pagano insisted on is related to convergence of the elasticity solution to classical plate theory. He got this general idea that convergence of the elasticity solution to CLPT is more rapid for the stress components than the plate deflection. Pagano believed that this observation could be a good consideration in selecting the form of plate theory required in the solution of specific boundary value problem [24].

2.6 K.T.Sundara Raja Iyengar and S.K.Pandya Solution

K.T.Sundara Raja Iyengar and S.K.Pandya (1983) worked on problem of analyzing orthotropic rectangular plate based on initial function method. As it has mentioned, Vlasov introduced this method of initial function. In this method, there is not any assumption that was made regarding the distribution of stress and displacement. In 1975, Bahar proposed "A State Space approach to elasticity "to solve the initial function results. Iyengar and Pandya used this approach in order to formulate the general problem of orthotropic rectangular thick plate analysis [14].

In addition, they used Navier solution for uniformly distributed load on a simply supported square plate. In State Space method, the value of stress and displacement are related to the value of stress and displacement in a plate where z = 0 (Eq. 20). Pandya used the equilibrium equations as:

$$\sigma_x = C_{11}\varepsilon_x + C_{12}\varepsilon_y + C_{13}\varepsilon_z \tag{17}$$

$$\sigma_y = C_{12}\varepsilon_x + C_{22}\varepsilon_y + C_{23}\varepsilon_z$$
$$\sigma_z = C_{13}\varepsilon_x + C_{23}\varepsilon_y + C_{33}\varepsilon_z$$
$$\tau_{yz} = G_{yz}\gamma_{yz}$$
$$\tau_{xz} = G_{xz}\gamma_{xz}$$
$$\tau_{xy} = G_{xy}\gamma_{xy}$$

By elimination of σ_x ,σ_y,τ_{xy} and the strains and using the strain-displacement and equilibrium equations we can get :

$$\frac{\partial}{\partial z}\begin{Bmatrix} U \\ V \\ Z \\ X \\ Y \\ W \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & a_{11} & 0 & -\alpha \\ 0 & 0 & 0 & 0 & a_{22} & -\beta \\ 0 & 0 & 0 & -\alpha & -\beta & 0 \\ -C_2\alpha^2 - C_6\beta^2 & -(C_3 + C_6)\alpha\beta & C_1\alpha & 0 & 0 & 0 \\ -(C_3 + C_6)\alpha\beta & -C_6\alpha^2 - C_4\beta^2 & C_5\beta & 0 & 0 & 0 \\ C_1\alpha & C_5\beta & C_{10} & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} U \\ V \\ Z \\ X \\ Y \\ W \end{Bmatrix} \quad (18)$$

$$\frac{\partial}{\partial z}\{S\} = [D]\{S\} \quad (19)$$

$$\{S\} = \left[e^{[D]z}\right]\{S_0\} \quad (20)$$

$$[L] = e^{[D]z} = [I] + z[D] + \frac{z^2}{2}[D]^2 + \cdots \quad (21)$$

where:
$\{S\}$: state vector
$\{S_0\}$: The value of state vector on initial plane (z=0)
[L]: operator matrix of mapping functions
[D]: System Matrix
$C_1 ... C_{10}$:Mechanical constants [15]
U, V and W : displacement in x, y and z direction

As shown in Fig. 5, the results of MIF and Reissner's theory are the same in case of σ_{zz} across the thickness. They also proved that the maximum deflection variation across the plate thickness is not constant. Fig.6 clearly reveals that the top loaded surface of plate has higher deflection compared to the lower one.

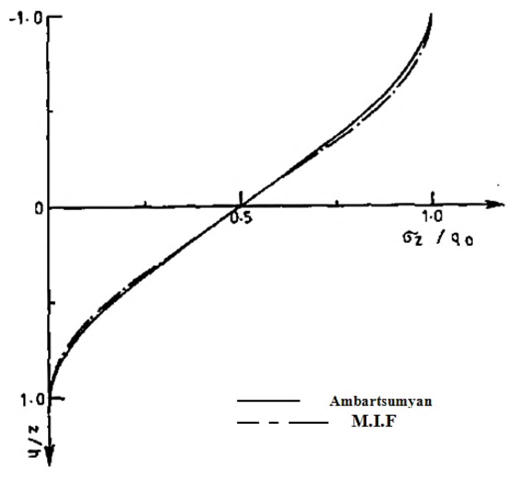

Fig. 5 Variation of σ_z across thethickness [14].

($\frac{a}{h}$ =2.5, 5,10,20,40 and 80 ;$\frac{E_x}{E_y}$= 1, 3, 10 and 40; $E_z = E_Y$; $G_{yz} = 0.5 \times E_y$; $G_{xz} = G_{xy} = 0.6 \times E_y$; $\mu_{xy} = \mu_{xz} = \mu_{yz} = 0.25$)

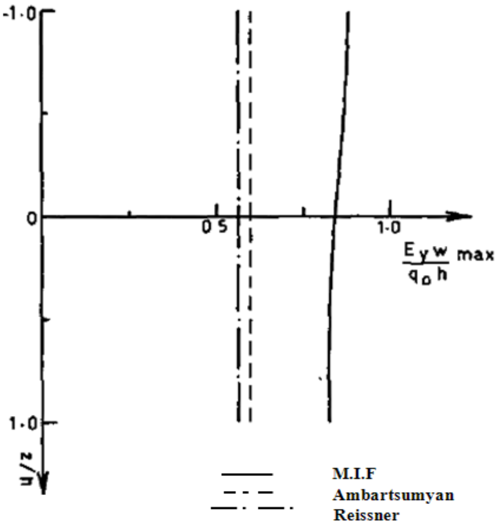

Fig. 6 Variation of w_{max} across the thickness [14].

$$\left(\frac{a}{h} = 2.5;\ \frac{E_x}{E_y} = 1, 3, 10 \text{ and } 40;\ E_z = E_Y;\ G_{yz} = 0.5 \times E_y;\ G_{xz} = G_{xy} = 0.6 \times E_y;\ \mu_{xy} = \mu_{xz} = \mu_{yz} = 0.25\right)$$

2.7 Wu and Wardenier

In 1997, they used state space method to solve the problem of anisotropic thick rectangular plates in bending (Fig. 7). Wu and Wardenier used State Space method for getting the result for simply supported thick rectangular plate. They found a graph which shows the variation of displacement across the thickness of plate (Fig. 8). They compared their own calculations results with Ambartsumyan and Reissner's theories results [25].

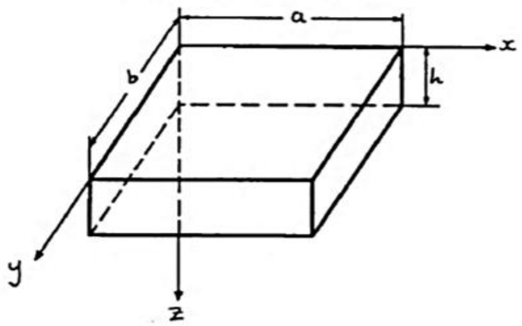

Fig. 7 Plate Geometry[16].

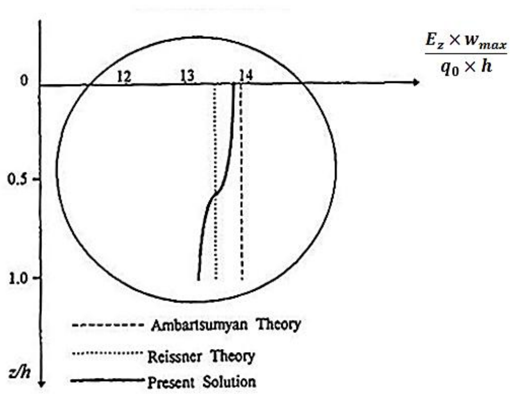

Fig. 8 Maximum transvers displacement across the thickness of the plate (*h/a=0.2* loaded on its upper surface (z=0) by uniform normal loading q_0 [16].

$$\frac{d^6 W_{mn}}{d^6} + A_0 \frac{d^4 W_{mn}}{d^4} + B_0 \frac{d^2 W_{mn}}{d^2} + C_0 W_{mn} = 0 \tag{22}$$

They proved that the variation of maximum transvers displacement across the thickness is nonlinear and they introduced a six-order differential equation for transverse displacement (w) by using State Space method.

2.8 Fan's State Space Solution

In early 90s, Jia-rang Fan believed that traditional theories of thick plate analysis are established based on assumptions which are limited those methods. He believed that some elastic constant had been missed in traditional plate analysis methods. Thus, by using State Space method he solved the exact behavior of thick rectangular plate with simply supported edges. He introduced Eqs. (23-25) as a system matrixes of his solution based on Eq. 19 which has differently introduced by K.T.Sundara Raja Iyengar and S.K.Pandyain 1983.

$$\frac{d}{dz}[U_{mn}(z) V_{mn}(z) Z_{mn}(z) X_{mn}(z) Y_{mn}(z) W_{mn}(z)]^T = \begin{bmatrix} 0 & A_{mn} \\ B_{mn} & 0 \end{bmatrix} [\, U_{mn}(z) V_{mn}(z) Z_{mn}(z) X_{mn}(z) Y_{mn}(z) W_{mn}(z)]^T \tag{23}$$

$$A_{mn} = \begin{bmatrix} C_8 & 0 & -\xi \\ 0 & C_9 & -\eta \\ \xi & \eta & 0 \end{bmatrix} \tag{24}$$

$$B_{mn} = \begin{bmatrix} C_2\xi^2 + C_6\eta^2 & (C_3 + C_6)\xi\eta & C_1\xi \\ (C_3 + C_6)\xi\eta & C_6\xi^2 + C_4\eta^2 & C_5\eta \\ -C_1\xi & -C_5\eta & C_{10} \end{bmatrix} \tag{25}$$

where:

$$\xi = \frac{m\pi}{a} \qquad \eta = \frac{n\pi}{b}$$

Eq.23 shows that stresses and displacements are directly related to coordinate z in State Space solution by Fan and none of elastic constant has been missed in his solution.

III. Conclusion

Many well-known researchers have done researches on elastic bending plate analysis. They all tried to get the real stress-strain relation from their methods. Some of them, such as Kirchhoff and Love, introduced their solution for thin plate bending analysis and they remove plate thickness from their analysis procedure. Then, Ambartsumyan believed that z coordinate should be involved in thick plate analysis and it should be involved in displacement across the thickness. However, the way he used thickness factor (i.e. z coordinate) in his formulas, the z factor removed due to even power of z and the two specific value of z, ±h/2. Thus displacement across the thick ness is not z variant and it is become constant.

Reissner introduced shear deformation effect on bending of elastic plate and Mindlin assumed linear compressive deformation across the plate thickness.

Pagano, Iyengar and Pandya were three researchers who mention z factor in their solutions. Pagano come up with six-order differential equation which has developed by Wu and Wardenier later. Iyengar and Pandya used method of initial function and improved state space method. In 90s, Fan used state space method to find the exact solution of elastic bending of thick plate. He used this method to develop the exact stress-strain relations in bending problem of thick orthotropic plate. He used this method to solve bending behavior of thick plate with different boundary conditions, such as simply supported rectangular plate and four edges clamped boundaries. He did not touch the problem of elastic bending of thick orthotropic plate with symmetric clamped-free edges. Author will developed State Space solution for this specific boundary condition based on this review paper.

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