

## **Torsional Strengthening of RC Box Beams Using External Prestressing Technique**

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**Abstract:** This paper investigate the behavior of reinforced concrete box beam under pure torsion. The box beam was strengthened experimentally with external prestressing technique (EPT) using two different directions horizontally and vertically. Ten strengthened box beams, with and without web opening, were tested. The study emphasizes prestressing direction and transverse opening dimensions. The failure modes, torsional capacities, rotation, stress in external tendon and strain in internal reinforcements were studied in details. The experimental results indicated that the contribution of EPT strengthening using the horizontal and vertical directions to torsional capacity of box beam, with and without opening, is significant with ratios ranged between 31% to 58% respectively. The contribution was enhanced using the vertical direction. It was found that the presence of transverse openings decrease the torsional capacity. While EPT strengthening at the opening increased torsional capacity compared to beams without opening. A computing procedure is presented to predict the torsional capacities of RC beams under torsion. The calculated results fit well with the experimental one.

**Keywords:** Strengthening, Box Beams, External Prestressing, pure torsion, web openings.

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### **I. Introduction**

Many building and bridge elements are subjected to significant equilibrium torsional moments that may require repair and strengthening. Torsional strengthening of concrete members may be achieved by one of the following methods: (1) increasing the member cross-sectional area, (2) adding transverse reinforcement, (3) using externally bonded steel plates, (4) applying an axial load to the member by External prestressing. External prestressing may be defined as a prestress introduced by tendons located outside a section of a structural member, only connected to the member through deviators and end-anchorage. The advantages of such technique are; smaller sectional areas, ease in inspection of the tendons and in their replacement and low friction losses. This type of prestressing can be applied to both new and existing structures that need to be strengthened due to several reasons such as: changes in use, deficiencies in design or construction phase and structural degradation. Strengthening structural elements subjected to torsion using EPT enable the designer to selectively increase their ductility, and torsional capacity. In an external prestressing system, there is no strain compatibility between the cable and the concrete for whole cross-sections, the increment of cable strain must be evaluated by taking into account the whole structure, rather than performing the calculation at each section, independently (Ali, 2013). Structural performance of damaged girder can be recovered and improved by external post tension. The level of external prestressing force required in strengthening depends directly on the level of damage due to overloading. The effect of damage's levels on flexural rigidity, crack and deflection of the test girder was studied by Phuwadolpaisarn, (2013). Experimental investigations on strengthening of RC beams by external prestressing have been carried out (Manisekar, 2014). It was observed that the ultimate load carrying capacity of strengthened members have increased by 48 % and 17 % for single-draped tendon profile and straight tendon profile respectively.

Naser, (2012) studied the application of external prestressing tendons for strengthening of existing bridges which has been used in many countries since the 1950s and has been found to provide an efficient and economical solution in a wide range of the bridge types and conditions. Shear strengthening, flexural strengthening and torsional strengthening of reinforcement concrete beams using composite materials were studied by several researchers and investigators at several institutions. Especially, studying the strengthening of structural elements using EPT was studied by Atta, (2012), El-Shafiey and Atta, (2012), Jeyasehar, (2008), Matta and et al, (2009) and Algorafi and et al, (2010). The reasons for the lack of research in this area include the specialized nature of the problem and the difficulties in conducting realistic tests and representative analyses. It is also a reason that few practical structures need to be strengthened to increase the strengthening of RC beams especially torsional strengthening. In the design of many concrete structural elements, torsion is significant and has to be considered such as curved structures, members of a space frame, eccentrically loaded beams, ring beams at the bottom of circular tanks, etc. For this reason, many researchers such as

Varghese,(2010),Jing, (2007) and Panchacharam, (2002)have paid more attentions to the studies of torsional capacity and bending torque coupling effect. Some researchers studied RC beams under pure torsion such as Mahaidi, (2007), Varghese, (2010), Abdeldjelil, (2004) and Mohammed, (1983).

The author Bernardo et al, (2012, 2011, and 2009), have many of works about the torsion. Bernardo et al., (2013) presented a study on the plastic behavior and twist capacity of High-Strength Concrete hollow beams under pure torsion. The plastic twist capacity of the beams was studied by using a Plastic Trend Parameter. Bernardo et al, (2012) developed a new computation procedure to predict the overall behaviour of reinforced concrete beams under torsion. They generalized the Space-Truss Analogy, by using the VATM formulation, in order to make it able to predict the entire T- $\theta$  curve of RC beams under torsion, and not only the points corresponding to the ultimate behaviour. Bernardo et al., (2008) developed a simple computation procedure to predict the general behaviour of reinforced concrete beams under torsion. Both plain and hollow normal strength concrete beams are considered. Different theoretical models are used to reflect the actual behaviour of the beams in the various phases of loading. Bernardo et al., (2014) presented the failure mode and the development of cracking analysis in high strength hollow beams under pure torsion up to failure. In modern construction, transverse openings in reinforced concrete beams are often provided for the passage of utility ducts and pipes. The presence of transverse openings will transform simple beam behaviour to more complex behaviour, as they induce a sudden change in the dimension of the beam's cross section. The ultimate strength, shear strength, crack width and stiffness may also be seriously affected. Some researchers studied these parameters such as Mansur, (1999), Mohammed, (1983), Abdeldjelil, (2004) and Ahmed and et al, (2012). The present study is rather unique in addressing torsional strengthening of reinforced concrete (RC) box beams under torsion using external prestressing technique (EPT).

## **II. Experimental Work**

### **2.1. Test specimens and strengthening schemes**

The experimental work consisted of ten identical RC single cell square box beams having cross section of 500mm x 500mm. The wall thickness is 100mm and the overall span of the beam was 2600mm while the loaded span was 2300mm. The concrete dimensions of beams were chosen similar to previous works such as Algorafi and et al, 2010 and Mahaidia, 2007. The beam section was designed according to the Egyptian code, ECCS 203-2001. All the tested specimens were reinforced with four 12mm diameter longitudinal bars in both bottom and top flanges in addition to two 12mm diameter longitudinal bars in each side. Diameter of 8mm stirrups spaced by 167mm was used throughout the beam span. The reinforcement was used on outer parameter of walls because wall thickness exceeded 1/6 box section width according to Egyptian code, ECCS 203-2001. The cell of size 300x300 mm was concentric with the beam section. The cell extended for 1000mm measured from each side of the center line of the beam. Beyond the cell, whole beam section (500x500mm) was solid concrete. Tested specimens were divided into two groups; each group consisted of five specimens. The first group casted without transverse opening while the second group casted with transverse opening. Figure 1 shows details of the specimens without openings and with openings. For second group, the opening sizes were 150x150mm, 150x300mm, and 150x450mm. The openings were made in two opposite side walls. Main reinforcement (stirrups) was almost kept the same except the stirrups intersecting with opening were discontinued as shown in figure 1. For each group, one specimen was control beam while the other four beams were strengthened using external prestressing technique (EPT). For the first group, one specimen was loaded up to 70% of ultimate torsional moment (first crack of control beam) then repaired using longitudinal EPT. Three strengthening techniques were used. The first was done by using external prestressing longitudinally tendons. The second strengthening technique was vertical external tendons at the middle of the beam to apply post tension force around the opening. The last strengthening technique was similar to the previous technique with the difference that post tension force was spreaded uniformly along the whole span. Three strengthening techniques were shown in figure 2. Table 1 shows the test matrix and specimens notation.

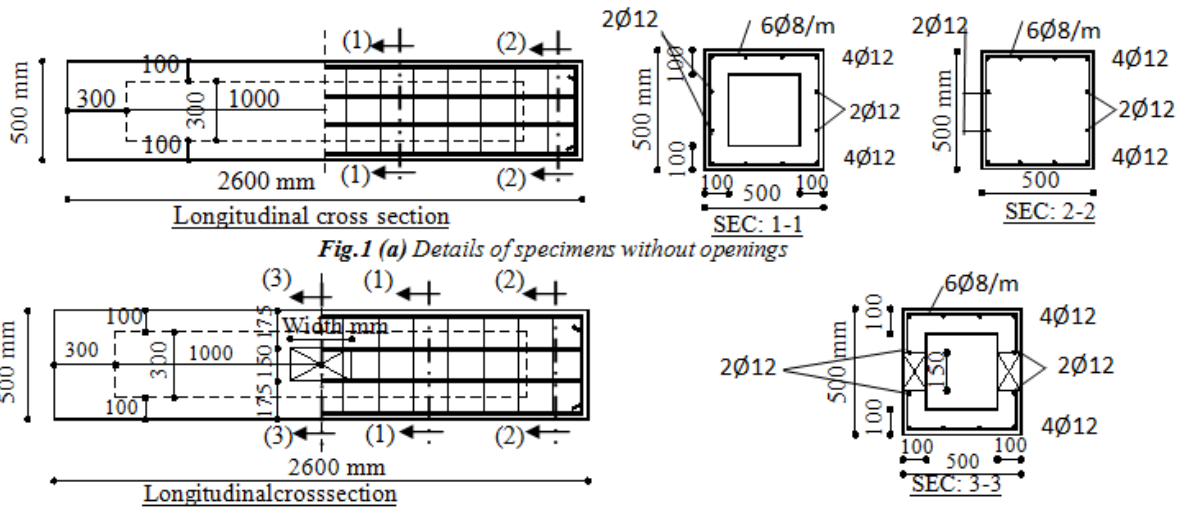


Fig. 1 (a) Details of specimens without openings

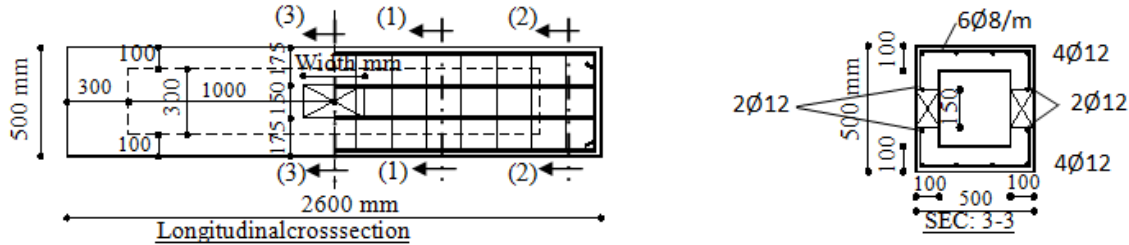


Fig. 1 (b) Details of specimens with openings.  
Fig. 1 Details of specimens.

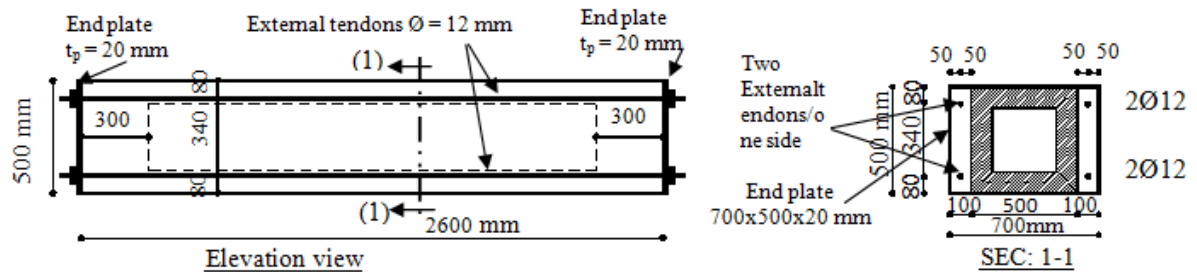


Fig. 2 (a) Strengthening technique (1)

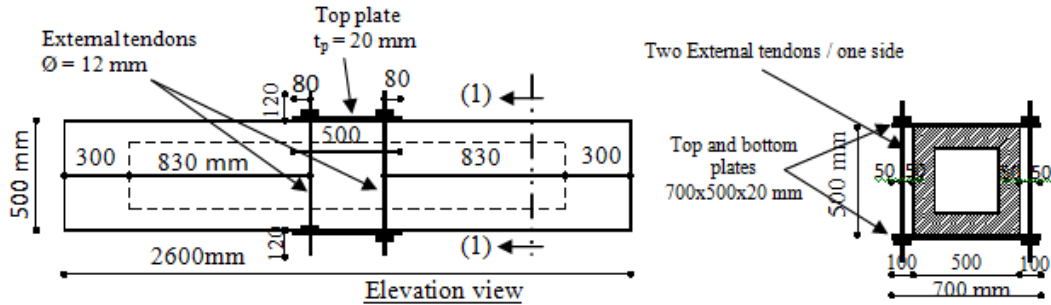


Fig. 2 (b) Strengthening technique (2)

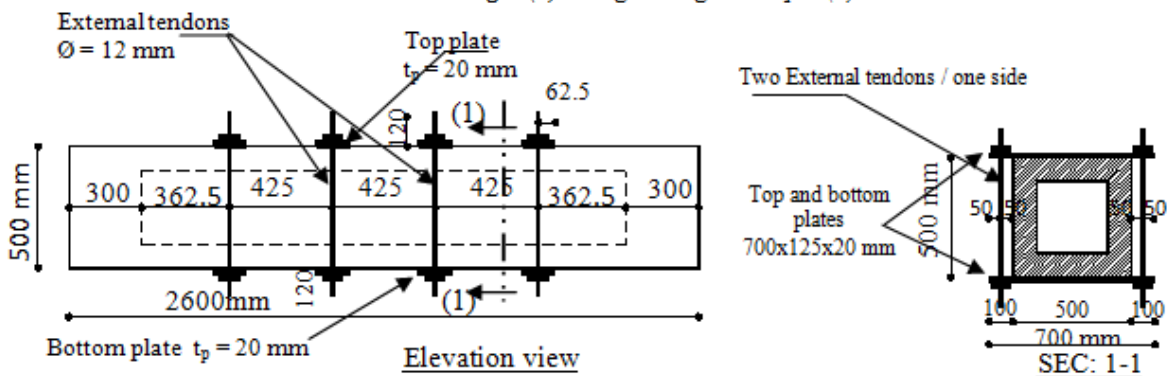


Fig. 2 (c) Strengthening technique (3)

Fig. 2 Configuration of strengthening techniques

**Table 1** Retrofit Scheme of Specimens

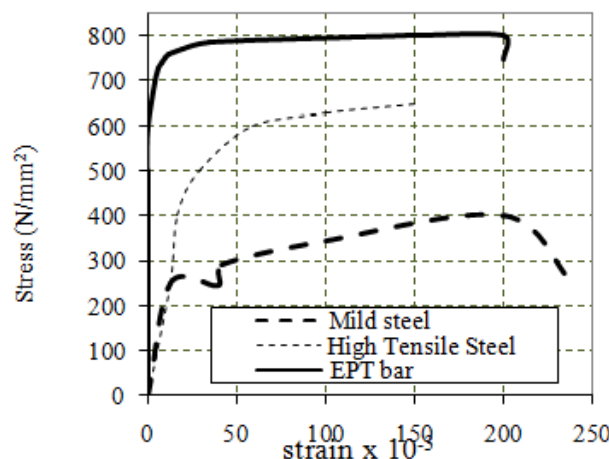
Group	Opening	Specimen notation	Dimension of opening		Retrofit scheme	Strengthening technique
			Depth (mm)	Width (mm)		
GI	without openings	GI-C	N/A	N/A	Control beam	N/A
		GI-1	N/A	N/A	Strengthened by longitudinal (EPT).	1 <sup>st</sup>
		GI-2	N/A	N/A	Strengthened by vertical. EPT at mid span	2 <sup>nd</sup>
		GI-3	N/A	N/A	Strengthened by vertical. EPT along span.	3 <sup>rd</sup>
		GI-4	N/A	N/A	Repairing at 70% of ultimate load by longitudinal EPT.	1 <sup>st</sup>
GII	with openings	GII-C	150	150	Control beam	N/A
		GII-1	150	150	Strengthened by longitudinal EPT	1 <sup>st</sup>
		GII-2	150	150	Strengthened by vertical EPT at mid span	2 <sup>nd</sup>
		GII-3	150	300	Strengthened by vertical EPT at mid span	2 <sup>nd</sup>
		GII-4	150	450	Strengthened by vertical EPT at mid span	2 <sup>nd</sup>

**2.2 Material properties**

Concrete with compressive cube strength of 35 N/mm<sup>2</sup> was used for casting of all specimens. The concrete mix consisted of fine aggregate (1500 Kg/m<sup>3</sup>), well-graded coarse aggregate (10-20mm) of 1250 Kg/m<sup>3</sup>, ordinary Portland cement (350 Kg/m<sup>3</sup>), water cement ration of 0.6 and admixture. In order to attain acceptable level of workability of the fresh concrete, super plasticizer (Sikament-163M) was used with ratio 1 ltr/100kg of cement weight. Stirrups were made from mild steel have yield stress ( $f_{yv}$ ) of 250 N/mm<sup>2</sup> while longitudinal bars were made from high tensile steel have yield stress ( $f_{yl}$ ) of 400 N/mm<sup>2</sup>. Bars of diameter 12mm were used for external prestressing. The direct tensile test was conducted for each bar type and the results were shown in figure 3 and listed in table 2.

**Table 2** The Test Results of EPT bars

Description	0.2% yield Strength MPa	Tensile strength MPa	Elongation %	Reduction of Area %
Stainless steel bright bars	728	800	20.0	68.0

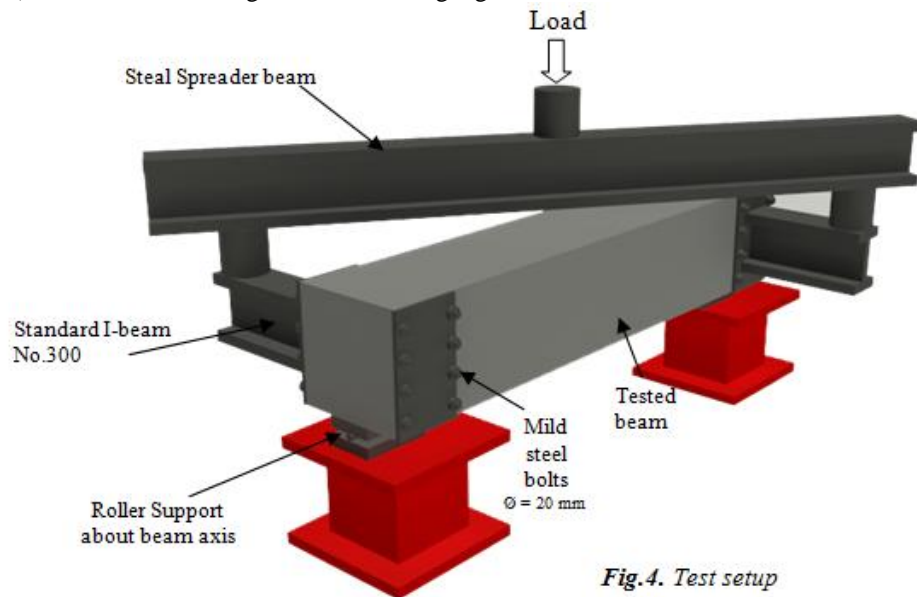


**Fig.3.** Stress strain curves

**2.3 Test setup and instrumentations**

All beams were tested under pure torsion as shown schematically in figure 4. The load was applied using a hydraulic actuator of 450 kN capacity. A load cell was attached to the loading actuator to record the applied load at 3 seconds. The load was applied through a diagonally placed steel spreader beam at the end of the

two steel arms. These arms were fixed at the end parts of each tested beam. Horizontal movements of a vertical side of the beam were measured using LVDT. Strains of the EPT bars and internal reinforcements (stirrups and longitudinal bars) were measured using electrical strain gauges.



### III. Results And Discussion

#### 3.1. Cracks patterns

The progress of cracks provided useful information regarding the failure mechanism of tested specimens. First crack of all specimens occurred at mid span and increased gradually. When the torque moment was increased, cracks appeared on each side and finally took the spiral shape. At failure, extensive parallel cracks at 400-450 on each side along the span of the beam were observed as shown in Figure 5. For specimens without opening, group (GI), the cracks of strengthened beam (GI-2) spread with smaller number and develop more slowly in strengthening zone because the EPT prevented the increment of the crack's width. Also the failure position of two specimens (GI-C) and (GI-1) took place at mid span because section at the two ends of the beam was solid with high torsional resistance. For specimen (GI-3), due to uniform distributing of EPT along the beam, the cracks distributed between external prestressing bars and not continuously around the specimen with increasing in cracks number. The location of the failure occurred at the distance between last external prestressing bar and the solid end due to good strengthening configuration along the beam. For specimen (GI-2), which strengthened at mid span, the cracks spread in the unstrengthened zone and the failure occurred at the same zone. That may be described as a result of EPT.

For specimens with opening, group (GII), it was noticed that the first crack for all specimens took place at the corner of the opening. Both specimen GII-C and specimen GII-1, were strengthened longitudinally, and had the same crack pattern and the same mode of failure. This is due to the weakness at the opening section is the reason behind such behaviour. The effect of EPT is subjected to only on increasing the normal force value. That led only to increase the value of failure load. For specimens (GII-2, GII-3), that were strengthened vertically at opening, the failure was observed outside the opening area as a result of the existing of EPT at vertical direction near the ends of the opening. This type of strengthening decreased the spread of the cracks from corners and transfer the failure to be outside the opening zone. The failure appears inside the opening zone in GII-4 while it appears outside the opening zone in GII-2. That difference is due to the of small opening size of sample (GII-2) and large opening size of sample (GII-4) (Ahmed and Fayyadh, 2012).



(a) Group (GI)(b) Group (GII)

**Fig. 5** the crack patterns and failure modes for tested specimens.

### **3.2. Torsional capacity and rotation capacity**

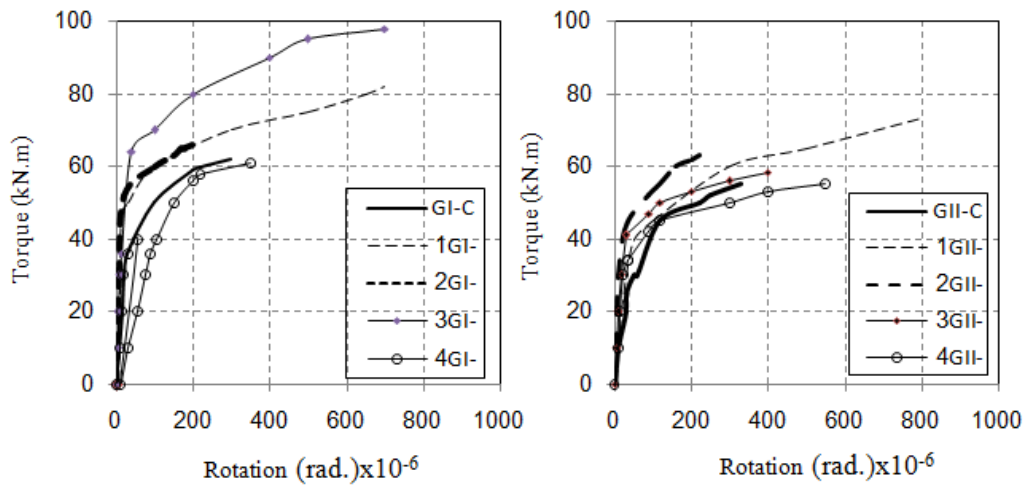
Table 3 lists the cracking torsional moment, ultimate torsional moment, cracking twist angle, ultimate twist angle and toughness for all the tested beams. The results illustrate that EPT strengthening is more effective in improving the crack torque and torsional capacity. Using longitudinal EPT strengthening (GI-1, GII-1) resulted in about 33% increase of ultimate torque. Using vertical EPT strengthening along the span of the beam

(GI-3) improved the torsional capacity up to 58%. The prestressing compression force acted against principal tensile stress (Tarek El-Shafiey and Ahmed Atta, 2012) which increase of ultimate torque in specimen GI-1, specimen GII-1, and specimen GI-3. EPT repairing (GI-4) was slightly affected the crack torque and torsional capacity. The presence of openings seems to be more effective in decreasing the crack torque and torsional capacity because the opening decreased the section rigidity. Hence it became the weakness position through the whole beam. In a comparison with the reference beam (GI-C), the crack torque and torsional capacity of specimen (GII-C), with opening 150mmx150mm, are decreased by 15% and 11% respectively. However, compared to the reference beam (GI-C), the torsional capacity of specimens (GII-3, GII-4) is decreased by 12% and 17% respectively.

Figure 6 shows the relationship between torque and angle of rotations for all beams. According to the results obtained from table 3 and Fig. 6, the EPT strengthening can obviously change the deformation capacity of box beams. For all specimens, test results indicated that the angle of rotation was approximately constant at about  $10 \times 10^{-6}$  radian until occurrence of the first crack then increased gradually until failure. The longitudinal and vertical strengthening technique had a simple effect on the crack angle of rotation as the test results indicated. At the same level of torque, the rotation of strengthened beams was smaller than the rotation of control beams. The reason was the restraint due to prestressing force. The curves which are shown in figure 6 indicated that at a given value of rotation, the strengthened beams carried additional torque than control beams. Compared to the control beam (GI-C), the ultimate twist angle of (GI-1) is increased by 133%, because the tendons were placed horizontally outside the beam and that allow to free rotation of the beam. Moreover for beam GI-2, the ultimate twist angle is decreased by 40%. For beams GI-3 and GI-4, the ultimate twist angle is increased by 133% and 17% respectively. The increase in specimens GI-3 was due to uniform distribution of tendons along beam length. On the other hand, the decrease in specimens GI-2 may be due to high rigidity of two plates at mid span. For specimen GI-2, the maximum angle of rotation was smaller than the maximum angle of rotation of GI-3 because the plates of strengthening placed at mid span restrained the rotation and divided the span into two parts. Transverse openings have effect on the rotation. For two control beams the opening increased the maximum angle of rotation by 10% because the weakness results from opening. For the two beams strengthened longitudinally (GI-1, GII-1) the opening increased the maximum angle of rotation by 14% due to the weakness of the opening cross section. The beams strengthened vertically (GI-2 and GII-2) the rotation was less than the control beams due to the restriction of rotation resulted from the existing high rigidity prestressing plates.

**Table 3** The Experimental Test Results

specimen	Cracking torque (kN.m)	Crack twist angle (rad.)x10 <sup>-6</sup>	Ultimate torque (kN.m)	Ultimate twist angle (rad.) x10 <sup>-6</sup>	Increase of ultimate torque (%)	Toughness (kN.m.rad. x10 <sup>-4</sup> )
GI-C	36	10	62.3	300	-	156
GI-1	46.8	10	82.5	750	33	450
GI-2	52	12	66.4	200	6.5	116
GI-3	63.7	26	98	700	58	581
GI-4	36.1	15	61	350	-	146
GII-C	30.5	9.8	55.5	340	-	140
GII-1	40	13	73.1	800	31.7	452
GII-2	43.65	8	62.8	220	13.2	113
GII-3	40.5	8.5	58.4	400	-12	197
GII-4	33.75	9	55	550	-17.2	267



(a) Group GI (b) Group GII  
 Fig. 6 Torque - rotation curves.

### 3.3. Ductility

The toughness may be considered as a ductility indicator (Meng, 2007). The toughness was measured as the area under the torque - rotation curve for each beam. In table 3, it is noticed that toughness of the strengthened beams horizontally (GI-1 and GII-1) is higher than reference beams (GI-C and GII-C). This might be because the location of tendons did not provide any restraint against rotation of the beam. The beam (GII-2), with opening 150x150mm and with vertical strengthening at the mid span, provided the lowest toughness because the strengthening was concentrated at mid span. This led to decrease in the movement of the beam. However, (GI-3), without opening and with vertical strengthening along beam length, provided the highest toughness due to the small size of the plates compared with GI-2. It can be seen from table 3 that of specimens with openings (GII-3 and GII-4) have higher toughness than reference specimen without opening (GI-2) due to presence of openings.

### 3.4 Reinforcement strains

#### 3.4.1 Longitudinal bar strain in specimens of group GI

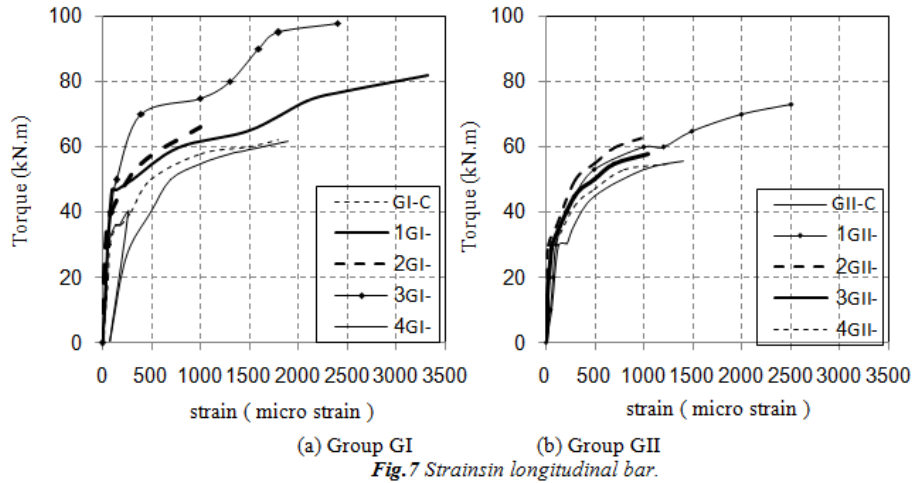
Figure 7 shows the torque versus strain in longitudinal bar for tested beams without opening. The curve of GI specimens showed that the strain in longitudinal bar at cracking torsional moment was about 4-15% of the maximum strain. For specimens GI-1, the yield strain in longitudinal bar took place at about 90% of torsional moment capacity. The test results indicated that the strain in longitudinal reinforcement was affected by EPT. For all the tested beams, up to cracking, the strains were nearly equal at different levels of torque. After cracking, the strains in bars of strengthened beams were smaller than unstrengthened beams at the same level of torque. The maximum strain in longitudinal bar in specimen (GI-1) was higher than the maximum strain in longitudinal bar in specimen (GI-C) by about 83%. As the torsional capacity for the strengthened beam (GI-1) increases, it generates higher force in the longitudinal direction than that generated in the control beam. In specimen GI-3, the increase of the torsional capacity was 33% compared to GI-C. By comparing specimen GI-3, strengthened vertically along span, with specimen GI-2, strengthened vertically at mid span, it can be seen that, at the same level of torque, the strain of GI-3 was smaller than GI-2 after cracking due to the uniform distribution of vertical strengthening along span of GI-3 compared to concentrated prestressing at mid span in GI-2. For specimen GI-4, that was repaired using longitudinal EPT after loading up 70%, the slope of curve after repairing process, and reloading, was smaller than slope before unloading because the torsional rigidity of the beam decreased after occurrence of cracks. After cracking torque, the strain of the specimen GI-4 showed higher value than the control specimen GI-C at the same level of torque as the torsional rigidity decreased. Although the ultimate strain in GI-1 was 83% higher than GI-C, the increase in ultimate strain of GI-4 was only 10%. This may be related to the loss of torsional rigidity due to preloading as shown in figure 7.

#### 3.4.2 Longitudinal bar strain in specimens of group GII

Figure 7 shows the torque versus strain in the longitudinal bars for tested beams with openings. Specimens with opening showed the same behavior as specimens without opening. However, the maximum values of strains was smaller because openings decreased ultimate torque then decreased the generated force in the longitudinal direction. For specimens of GII, the maximum strain in longitudinal bar was approximately the same; except for specimen (GII-1), it increased by 60% compared with control beam (GII-C) due to increasing in



ultimate torque by 31% due to EPT. For specimens GII-2, GII-3, GII-4, the maximum strain in longitudinal bar was smaller than specimen GII-C because the strain placed in strengthening zone at mid span.

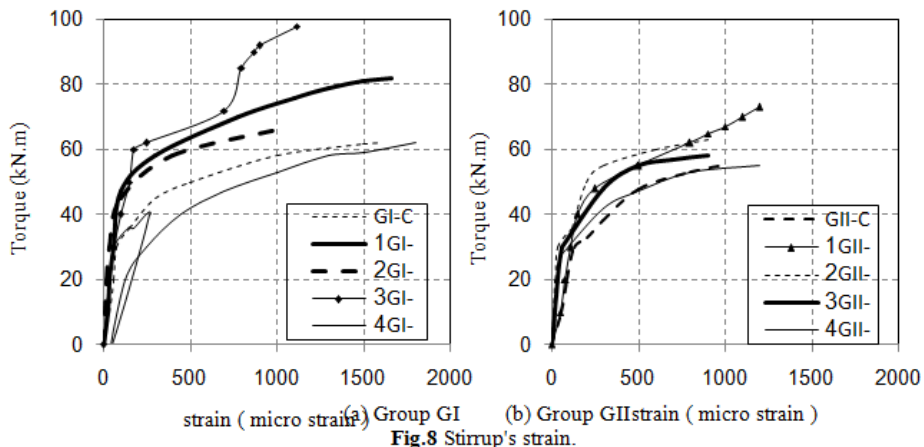


3.4.3 Stirrup strain in specimens of Group (GI)

The variation of the torque with strain in stirrup was shown in figure 8. Both specimens GI-C and GI-1 have the same yield strain at 90% of ultimate torsional moment with different value of yield torsional moment. This may be due to the uniform distribution of the normal forces due to the existing of EPT through the beam length. The maximum strain in stirrup at specimen (GI-1) was higher than the maximum strain in stirrup in specimen (GI-C) nearly by 6% at failure load as shown in Fig. 8. The final torque was increased by 33%. The prestressing force increases the shear strength of the sections. For vertical strengthening, specimens GI-2 and GI-3, the direction of EPT force is parallel to the direction of stirrups, so the maximum strain in stirrup did not exceed the yield strain. It can be noticed that, at the same level of torque, the strain in specimen GI-3 was smaller than GI-2 after cracking due to the uniform distribution of the external tendons. For specimen GI-4, the strain in stirrup has the same trend of longitudinal bar in the same specimen as shown in figure 7. In addition the corresponding strain at ultimate torque was higher than control beam by 13%.

3.4.4 Stirrup strain in specimens of group (GII) at position outside the opening

Figure 8 showed the strain versus relationship for specimens of group GII. For specimens GII-C and GII-1, with opening size 150x150mm, the strain in stirrup of GII-1 was higher than GII-C at the failure moment by 20%, because the longitudinal strengthening was distributed equally along beam sections hence the opening area still the weakest section. Strain in stirrup of specimen GII-2 was smaller than strain in stirrup of specimen GII-1 by 30% because the vertical strengthening concentrated in the opening area where strain gauge fixed at stirrup and transfers the failure cracks outside opening. Specimens GII-2, GII-3 and GII-4 have the same vertical strengthening technique at the opening zone but with different opening size. The maximum strain of GII-4 was higher than GII-3 and GII-2 because the opening size of GII-4 was larger than GII-3 which in turn is larger than GII-2. This might be due to the direct relation between the opening size and the weakness. Hence, the ultimate strain increased. On the other hand the strain in stirrup was the largest in specimen GII-4, with the largest opening size, because the failure cracks were through opening area and strain gauge was placed besides opening.



3.5. Stress in external tendons

Figure 9 shows the average tensile stress in external tendons as a function of the applied torque for strengthened and repaired specimens. The raising rate of the tensile stresses was faster in external tendons after cracking compared to the rate before cracking which reflects the transfer of the force after the first crack from concrete to internal stirrups and external prestressing bars. The average initial stress was 180 MPa equal to 25% of the yield stress of external tendon. The strain in EPT tendons increases to 35% of the yield stress at the end of the test. For specimen GI-3, the shape of the curve was different relative to all specimens. The good distribution of EPT tendons along the specimen led to decreased spread of the cracks in the beam zone, which appears as constant stress until ultimate torque. For specimens GI-1 and GII-1, the final external tendon stress was the smallest in all specimens of the two groups. External tendon length in specimens strengthened longitudinally is higher than specimens strengthened vertically. This led to small strain hence small stress.

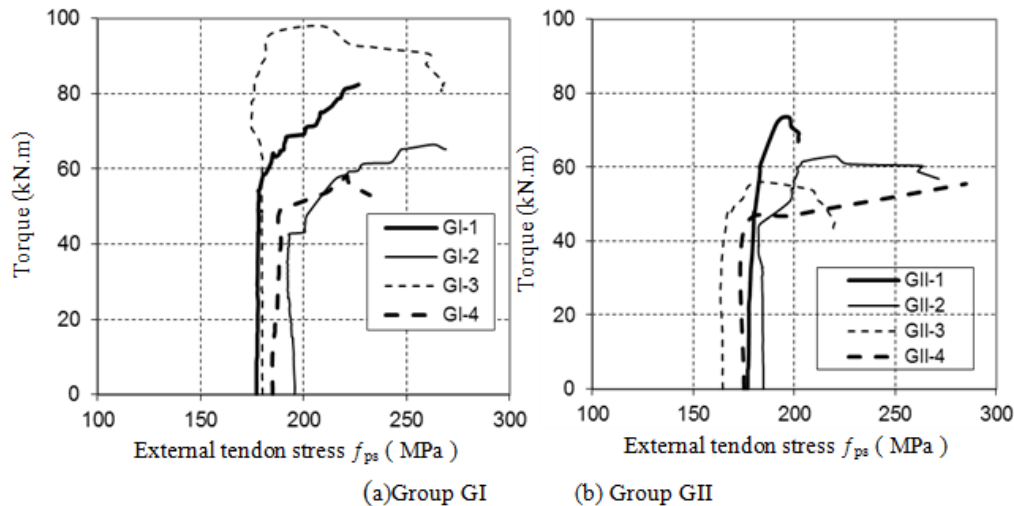


Fig.9 Stress in external tendon.

IV. Modification of the torsional capacity equations using ACI 318m-05(ACI) and Egyptian code (EC)

The proposed equations by Egyptian code, 2001(EC) and ACI 318m-05, 2004(ACI) was used for directly evaluating the failure torsional moment. The equations cannot be used for different techniques of strengthening using EPT of beams with and without openings so the following modifications were made to capture the exact values of failure torsional moment in this case.

Egyptian code proposed equations to estimate the failure torque for RC beams without openings under pure torsion as follows: the torsional moment resisted by stirrup only ( $M_s$ ) calculated from Eq. (1), and the torsional moment resisted by concrete only ( $M_c$ ) calculated from Eq.(2).

$$\frac{A_{str}}{S} = \frac{M_s}{2A_o f_{str}} \quad (\text{N, mm}) \quad \text{Eq.(1)}$$

$$q_t = \frac{M_c}{2A_{ot}} \quad (\text{N, mm}) \quad \text{Eq.(2)}$$

where  $q_t$  the shear strength =  $0.24\sqrt{f_{cu}}$ ,  $A_{str}$  is the area of stirrup leg,  $S$  is the spacing between stirrups,  $P_h$  is the perimeter of stirrup,  $A_{oh}$  is the area inside the stirrup,  $t$  is wall thickness of box section equivalent to rectangular section ( $A_{oh}/P_h$ ),  $A_o$  is area enclosed by shear flow path ( $0.85A_{oh}$ ),  $f_{str}$  is yield strength of closed transverse torsional reinforcement.

Egyptian code gives factor  $\Delta$  to determine the shear strength in case the RC beam subjected to axial compression force  $P_u$ . The concrete shear strength ( $q_t$ ) is increased by multiplying by  $\Delta$  where ( $\Delta = 1 + 0.07(P_u/A_g)$  N,mm).

ACI proposed equation to estimate the failure torque for RC beams under pure torsion as follows: the torsional moment resisted by stirrup only ( $M_s$ ) calculated from Eq. (3), and the torsional moment resisted by concrete only ( $M_c$ ) calculated from Eq.(4) for nonprestressed concrete and from Eq.(5) for prestressed concrete.

$$M_s = \frac{2A_o A_{str} f_{str}}{S} \cot \theta \quad (\text{lb, in}) \quad \text{Eq.(3)}$$

$$2\sqrt{f_c} = \frac{M_c}{1.7A_{oh} * t} \quad (\text{lb, in}) \quad \text{Eq.(4)}$$

$$2 \left[ 1 + \frac{N_u}{2000A_g} \right] \sqrt{f_c} = \frac{M_c}{1.7A_{oh} * t} \quad (\text{lb, in}) \quad \text{Eq.(5)}$$

$f_c$  is specified compressive strength of concrete,  $\alpha$  is angle of compression diagonals in truss analogy for torsion,  $N_u$  is factored axial load normal to cross section, and  $A_g$  is gross area of section.

A simple modification carried out on EC and ACI equations to be valid for prediction of final torque of beams with openings strengthened longitudinally. According to experimental work and previous literature review [5], a new factor  $\alpha$  ( $\alpha = 1 - \frac{\sqrt{A_{op}}}{1151}$  mm) based on opening area ( $A_{op}$ ) is added to Eq.(2) to find new formula (Eq.(6)) for evaluating the torque provided by concrete  $M_c$ . same factor  $\alpha$  is added to Eq.(4 and 5) to obtain the following formula Eq.(7 and 8) for evaluating the torque provided by concrete  $M_c$ .

$$M_c = 0.24 \sqrt{f_{cu}} (2A_o * t_e) \Delta \alpha \tag{Eq.(6)}$$

$$2\alpha \sqrt{f_c} = \frac{M_c}{1.7A_{oh} * t} \tag{Eq.(7)}$$

$$2\alpha \left[ 1 + \frac{N_u}{2000A_g} \right] \sqrt{f_c} = \frac{M_c}{1.7A_{oh} * t} \tag{Eq.(8)}$$

To verify the accuracy of modified equations in ACI and EC, the mathematical model is applied on specimens of present study and specimens of previous researches such as Mohammed, 1983 and Hasnat et al., 1988. The results are listed in table 4 and compared to the data obtained from the experimental testing. The results obtained from modified equations were found to be in a very close agreement with the experimental results. Proposed equations using EC provided more accurate results more than ACI in the case of beams strengthened longitudinally using EPT.

**Table 4** Verification of Modified Equations

Reference	specimen	specification	Theoretical Failure Torque (kN.m)		Experimental Failure torque (kN.m)	Theoretical/ Experimental value	
			Egyptian code	ACI code		Egyptian code	ACI code
Current study	GII-1	Box beam of 500x500 cross section with hollow 300x300mm ,with opening 150x150mm and strengthened longitudinally using prestressing process	67.4	57	73	0.92	0.78
	GII-C	Box beam of 500x500 cross section with hollow 300x300mm ,with opening 150x150mm	66	55.4	55.5	1.18	0.99
Mohammed,1983	B1	Solid section of 200x400 mm and with opening 400x200mm	19.1	17.2	22	0.87	0.78
	B2	Solid section of 200x400 mm and with opening 600x200mm	18.5	16.9	19	0.97	0.89
Hasnat et al. 1988	PA11R1L	Solid section of 100x250 mm and with opening of 110 mm diameter and with strengthened longitudinally using prestressing process	5.72	5.52	6.54	0.87	0.84

### V. Conclusions

The torsional behaviour, strain in internal reinforcement, stress in external tendons and mode of failure of RC box beam were investigated. The tested beams are consisted of two groups with and without web openings. Moreover, the beams are strengthened with external prestressing technique (EPT) under pure torsion. The structural behaviour of RC box beams was significantly affected by EPT strengthening as summarized below:

1. The strengthening using EPT enhance the torsional capacity by 58% without affecting the final beam rotation.
2. The strengthening using vertical EPT distributed along the beam section was more effect compared to other techniques.
3. Strengthening using EPT improve ductility behavior
4. Strengthening using longitudinal EPT increase longitudinal bar strain by ratios ranged between 60-80%.

5. The longitudinal EPT repairing for cracked beams under pure torsion isn't a suitable technique to improve the torsional capacity.
6. Using of vertical EPT technique for strengthening the beams with opening subjected to torsion increased torsional capacity by about 13%.
7. The results of proposed and modified equations of Egyptian code and ACI 318m-05 agreed with the experimental results.

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### **References**

- [1]. A. Ahmed, M.M. Fayyadh, S. Naganathan, and K. Nasharuddin, Reinforced Concrete Beams with Web Openings: A State of The Art Review, *International Journal of Materials and Design*, 40, 2012, 90-102.
- [2]. Abdeldjelil, Reinforced Concrete Box Girders Under Cyclic Torsion, 13<sup>th</sup> World Conference on Earthquake Engineering, 2004, No. 998.
- [3]. A.varghese, Torsional Strengthening of Reinforced Concrete Beams, (structural Engineering Department, Faculty of Engineering, Kerala University, 2010).
- [4]. A. F.Naser, Field Tests of Anchor Beams During Strengthening of Jiamusi Prestressed Concrete Highway Bridge, *Research Journal of Applied Sciences, Engineering and Technology*, 5, 2012, 475-480.
- [5]. A. hasnat, F.wafa , and A.Ali , Prestressed Concrete Beams with Small Opening Under Torsion, *journal of structural engineering*, 114 ,1988 ,1626-1643.
- [6]. A. Atta, Evaluation of The Efficiency OfUHP-SHCC Beams in Flexure Under External Prestressing, *journal of magazine of concrete research*, 64(1),2012, 43-54.
- [7]. A. Atta and T. El-Shafiey, Retrofitting of Reinforced Concrete Beams in Shear Using External Prestressing Technique, *journal of magazine of concrete research*, 64(3), 2012 , 201-211.
- [8]. Building code requirements for structural concrete and commentary (ACI 318m-05, 2004).
- [9]. L.F.A. Bernardo and S.M.R. Lopes, Plastic analysis and twist capacity of high-strength concrete hollow beams under pure torsion, *journal of Engineering Structures*, 49, 2013,190-201.
- [10]. L.F.A. Bernardo, S.M.R. Lopes and J. M. A. Andrade, Modified Variable Angle Truss-Model for torsion in reinforced concrete beams, *journal of Materials and Structures*, 45, 2012, 1877–1902.
- [11]. L.F.A. Bernardo and S.M.R. Lopes, Behaviour of concrete beams under torsion: NSC plain and hollow beams, *journal of Materials and Structures*, 6, 2008, 1143–1167.
- [12]. L.F.A. Bernardo and S.M.R. Lopes, Cracking and failure mode in HSC hollow beams under torsion, *journal of construction and building Materials*, 51, 2014, 163–178.
- [13]. L.F.A. Bernardo, S.M.R. Lopes and J. M. A. Andrade, Softened truss model for reinforced NSC and HSC beams under torsion: A comparative study, *journal of Engineering Structures*, 42, 2012, 278–296.
- [14]. L.F.A. Bernardo and S.M.R. Lopes, Theoretical behavior of HSC sections under torsion, *journal of Engineering Structures*, 33(12), 2011, 3702–3714.
- [15]. L.F.A. Bernardo and S.M.R. Lopes, twist behavior of high-strength concrete hollow beams-formation of plastic hinges along the length, *journal of Engineering Structures*, 31(1), 2009, 138–149.
- [16]. C.A. Jeyasehar and G. Mohankumar, Strengthening by External Prestressing, *journal of Concrete international*, 2008,61-66.
- [17]. Egyptian code for design and construction of concrete structures (ECCS 203, 2001).
- [18]. F. Matta, A. Nanni, A. Abdelrazaq, D. Gremel, and R. Koch, Externally Post-Tensioned Carbon FRP Bar System for Deflection Control, *journal of Construction and Building Materials*, 23(4), 2009 , 1628-1639.
- [19]. M.A. Algorafi, A.A.A. Ali, I. Othman, M.S. Jaafar, and M.P. Anwar, Experimental Study of Externally Prestressed Segmental Beam Under Torsion, *journal of Engineering Structures*, 32(11),2010, 3528-3538.
- [20]. M.A. Mansur, K.H. Tan, Concrete Beams with Openings: Analysis and Design, (Boca Raton, Florida, USA: CRC Press LLC, book,1999)
- [21]. M. Jing, W.Raongjant, and L. Zhongxian, Torsional Strengthening of Reinforced Concrete Box Beams Using Carbon Fiber Reinforced Polymer, *journal of Composite Structures*, 78(2), 2007, 264-270.
- [22]. A. Mohammad, Torsion Tests of R/C Beams with Large Openings, *Journal of Structural Engineering*, 109, 1983,1780-1791.
- [23]. R. Al-Mahaidi, Bond Behaviour of CFRP Reinforcement for Torsional Strengthening of Solid and Box-Section RC Beams, *Journal of Composites: Part B*, 38, 2007 , 720–731.
- [24]. R. Manisekar, Experimental Investigations on Strengthening of RC Beams by External Prestressing, *Asian journal of civil engineering (BHRC)* 15(3), 2014 , 350-363.
- [25]. J. Ali, A Numerical Model of Externally Prestressed Concrete Beam, *International Journal of Scientific & Engineering Research* 4(5),2013 , 229-232.
- [26]. S. Panchacharam, Torsional Behavior of Reinforced Concrete Beams Strengthened with FRP Composites, First FIB Congress, Osaka, Japan,2002 ,13-19.
- [27]. T. Phuwadolpaisarn, Relations Between Structural Damage and Level of External Prestressing Force on The Flexural Behavior of Post-Tensioned Prestressed Concrete Beams, *International Transaction Journal of Engineering, Management, & Applied Sciences & Technologies*, 4(4), 2013 , 241-251.