IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 12, Issue 3 Ver. II (May - Jun. 2015), PP 66-71
www.iosrjournals.org

Modified formulas for bucking length factor for rigid steel frame structures

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Abstract: *In most current codes for design of steel structures, specifications for the design of compression members utilize the effective length factor K. This parameter is employed to facilitate the design of frame members by transforming an end-restrained compression member to an equivalent pinned-ended member. The effective length factor is obtained either by solving the exact equations using a numerical iterative solution which may be computationally expensive or by using a pair of alignment charts for the two-cases of braced and sway frames. The accuracy of the solution using the second approach depends on the size of the charts and the reader's sharpness of vision. To eliminate these approximations, simple equations for determining the effective length factor as a function of the rotational resistant at the column ends (GA, GB) are required. Similar equations are available in the French design rules for steel structures since 1966, and are also included in the 1978 European recommendations. In this paper, modifications to the French design rules equations for effective length factors are presented using multiple regressions for a tabulated exact values corresponding to different practical values of the rotational resistance at column ends (GA, GB). The investigated equations are more accurate than the current French rules equations recommended in steel codes of several countries. Comparisons between the numerical results of the equations developed in this study and those obtained by current equations with those obtained by exact solutions are given also in this paper.*
Keywords: *Effective length; steel column; multiple regressions; new formula; braced frame; sway frame*

I. Introduction

The design of compression members such as steel columns and frames starts with the evaluation of the elastic rotational resistance at both ends of the column (GA, GB), from which the effective length factor (K) is determined. The exact mathematical equations for braced and sway rigid frames were given by Barakat and Chen, 1990. These equations may be computationally expensive. An alternative approach to determine these parameter is could be by using a pair of alignment charts for braced and sway frames, which was originally developed by Julian and Lawrence, and presented in detail by Kavanagh (1962). These charts represent the graphical solutions of the mathematically exact equations which are commonly used in most design codes (e.g., Manual of American institute of steel construction (LRFD and ASD), 1989 and the Egyptian code of practice for steel constructions (LRFD and ASD), 2008. The accuracy of the alignment charts depends essentially on the size of the chart and on the reader's sharpness of vision. Also, having to read K-factors from an alignment chart in the middle of a numerical computation, in spreadsheet for instance prevents full automation and can be a source of errors.

Obviously, it would be convenient to have simple mathematical equations instead of the charts which are commonly used in most codes of steel constructions. The American Institute does publish equations but their lack of accuracy may be why they seem not to be used in steel design. Mathematical relations are available in the French design rule for steel structures since 1966, and are also included in the 1978 European recommendations (see e.g., Dumonteil, 1992).

In this paper, a modification to the French rule equations is developed to achieve more accurate closed form expressions for the determination of the effective length factors as a function of the rotational resistance at column ends. The presented equations are more practical since they can be easily coded within the confines of a spreadsheet cell or within any mathematical software, such as Matlab, Maple or Mathematica.

II. Background For Exact And Approximate Equations

Consider a steel column AB elastically restrained at both ends. The rotational restraint at one end, A for instance, is presented by restraint factor GA, expressing the relative stiffness of all the columns connected at A to that of all the beams framing into A, given as:

$$G_A = \frac{\sum(I_C / L_C)}{\sum(I_b / L_b)} \quad (1)$$

In the European Recommendation 1978, two different factors βA and βB are used (rather than GA and GB as in French Rules). The definition of β differs from that of G, given as:

$$\beta = \frac{\sum(I_b / L_b)}{\sum(I_b / L_b) + \sum(I_C / L_C)} \quad (2)$$

The mathematical expression relating G and β is thus given as follows:

$$\beta = 1/(1+G) \quad (3)$$

The Europeans tend to prefer using β than using βG since β =0 implies a hinged end and β =1 indicates a fixed end. Obviously, the K-factor will be the same if the same elements are introduced in G and β in both expressions.

1. Braced Steel Frames

Braced steel frames are frame structures in which the side sway is effectively prevented as shown in Figure (1-a), and, therefore, the K-factor less than or equal to 1.0. The side sway prevented alignment chart is the graphic solution of the following mathematical equation:

$$\frac{G_A G_B}{4}(\pi / K)^2 + \left(\frac{G_A + G_B}{2}\right)\left(1 - \frac{\pi / K}{\tan(\pi / K)}\right) + 2\frac{\tan(\pi / 2K)}{\pi / K} = 1 \quad (4)$$

This equation is mathematically exact, in that certain physical assumptions are exactly translated in mathematical terms. Whether these assumptions can be reasonably extended to a specific structure is a matter for the designer to decide.

For the transcendental Eq. 4, which can only be solved by numerical methods. The French Rules propose the following approximate solution:

$$K = \frac{3\,G_A G_B + 1.4(G_A + G_B) + 0.64}{3\,G_A G_B + 2.0(G_A + G_B) + 1.28} \quad (5)$$

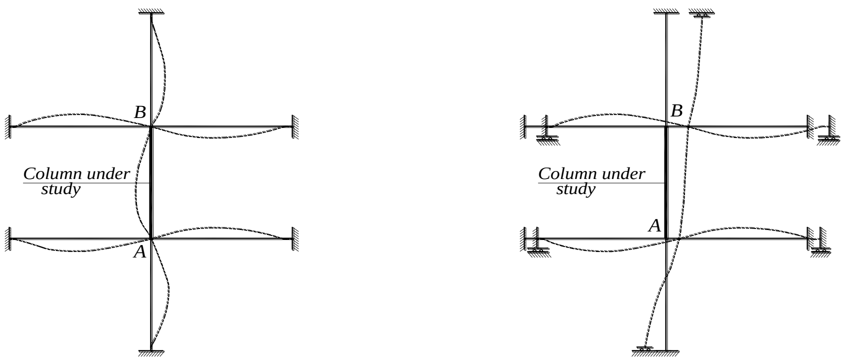
2. Sway Steel Frames

If a rigid frame depends solely on frame action to resist lateral forces, its side sway is permitted as shown in Figure (1-b). In this case, the K-factor is never smaller than 1.0. The mathematical relation for the permitted sway case is given as follows:

$$\frac{G_A G_B (\pi / K)^2 - 36}{6(G_A + G_B)} = \frac{\pi / K}{\tan(\pi / K)} \quad (6)$$

Although Eq. (6) is simpler than Eq. (4), this equation, however, cannot be solved in a closed form. The French Rules recommend the following approximate solution:

$$K = \sqrt{\frac{1.6\,G_A G_B + 4.0(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \quad (7)$$



(a) Braced frame (b) Sway frame

Fig. 1 Braced and sway frames

III. Theoretical Formulas

In this section, firstly, forms of the French rules equations are assumed as follows:

For braced frames,

$$K=\frac{aG_A G_B+b\,(G_A+G_B)+c}{dG_A G_B+e(G_A+G_B)+f} \quad (8)$$

For sway frames,

$$K=\left(\frac{gG_A G_B+h(G_A+G_B)+i}{G_A+G_B+j}\right)^k \quad (9)$$

The parameters a, b,c …k are obtained by applying multiple regression analyses using the following procedure:

- 300 pairs of different practical values of the rotational resistance at column ends (GA, GB) are selected and the corresponding K values for sway and braced framework are used to fit the assumed equations, which can be tested.
- The exact values of K factor were obtained by trial and error applying equations (Eqs. 4, 6) which are solved numerically for both prevented and permitted sway end conditions. The resulting values are approximated to the nearest higher integer numbers
- Using computer software, multiple regressions analyses based on the least-squares fitting method are developed for each suggested formula to obtain the parameters which give the least standard error (less than that corresponding to the current French Rules equations).

Subsequently, simple accurate mathematical expressions are investigated to determine the effective length factor K for braced and sway end conditions as described below.

In case of braced frames; the equation for the effective length factor K is given by:

$$K=\frac{3G_A G_B+1.4(G_A+G_B)+0.695}{3G_A G_B+2(G_A+G_B)+1.39} \quad (10)$$

Where $0 \leq G_A \leq 100, \quad 0 \leq G_B \leq 100$

In case of sway frames; to get good results, two equations of effective length factor are investigated according to the domain of the rotational resistance at column ends (GA, GB) as follows

$$K=\left(\frac{0.97G_A G_B+3.3(G_A+G_B)+6.7}{G_A+G_B+6.9}\right)^{0.6} \quad (11)$$

Where

$0 \leq G_A \leq 10, \quad 0 \leq G_B \leq 10$

and,

$$K=\left(\frac{1.4G_A G_B+3.7(G_A+G_B)+6.15}{G_A+G_B+6.45}\right)^{0.52} \quad (12)$$

Where

$10 \prec G_A \leq 100 \quad \text{or} \quad 10 \prec G_B \leq 100$

IV. Accuracy Of Present Equations

The accuracy that we can readily measure is of course the mathematical accuracy, that is, the comparison of the results given by the obtained formals to those obtained by solving the corresponding exact equations. First, take a look to the accuracy of the most common alignment charts and the French Rules equations.

The accuracy of the alignment charts depends essentially on the size of the charts and the reader's sharpness of vision. This accuracy may be about five percent in small charts. In the other hand, the French Rules,1962indicate that Eq. (5), used for braced frames, has an accuracy of -0.50 percent to +1.50 percent while Eq. (7) used for sway frames, is accurate within two percent.

The percentage of errors for all points considered in the present regressions analyses (about 300 point for each case) indicate that the investigated equations in the present work is accurate within 1.0 percent for both sway and braced frames.

V. Comparison Of The Results

Using a few sample points, tables (1, 2) show the comparison of the effective length factor Kobtained by Equations [8 – 10] of the present work (P.W.), and that obtained by the current French Rules equations with exact values for braced frames and sway frames respectively.

Table 1. Comparison of K-factors obtained by P. W. (year), French Rules(year)and exact results [side sway is prevented]

G_A	G_B	exact Value	P. W.	% Diff.	French Rule [7]	% Diff.
0.1	0.4	0.603	0.604	0.10	0.608	0.88
0.25	0.25	0.611	0.614	0.47	0.619	1.30
0.1	0.9	0.648	0.646	-0.28	0.651	0.42
0.25	0.75	0.672	0.672	0.05	0.677	0.79
0.5	0.5	0.686	0.687	0.17	0.692	0.92
0.1	1.9	0.683	0.682	-0.14	0.685	0.36
0.25	1.75	0.716	0.717	0.18	0.721	0.68
0.5	1.5	0.751	0.752	0.13	0.756	0.62
1	1	0.774	0.774	0.02	0.778	0.49
0.5	4.5	0.792	0.796	0.54	0.798	0.77
1	4	0.840	0.842	0.24	0.844	0.43
2.5	2.5	0.877	0.877	0.05	0.879	0.20
0.5	9.5	0.806	0.812	0.71	0.813	0.88
1	9	0.858	0.862	0.42	0.862	0.52
2.5	7.5	0.913	0.914	0.08	0.914	0.15
5	5	0.930	0.931	0.06	0.931	0.11
50	4	0.952	0.953	0.10	0.953	0.11
50	10	0.977	0.977	0.04	0.977	0.04
100	50	0.994	0.994	0.01	0.994	0.01

Table 2. Comparison of K-factors obtained by P. W., and French Rules with exact results [side sway is permitted]

G_A	G_B	exact value	P. W.	% Diff.	French Rule [7]	% Diff.
0.1	0.4	1.083	1.078	-0.45	1.093	0.96
0.25	0.25	1.083	1.080	-0.29	1.095	1.15

0.1	0.9	1.159	1.158	-0.09	1.170	0.99
0.25	0.75	1.162	1.164	0.21	1.178	1.40
0.5	0.5	1.164	1.169	0.40	1.183	1.65
0.1	1.9	1.286	1.283	-0.22	1.290	0.30
0.25	1.75	1.295	1.297	0.14	1.306	0.84
0.5	1.5	1.307	1.314	0.53	1.326	1.44
1	1	1.317	1.327	0.77	1.342	1.87
0.5	4.5	1.575	1.575	0.03	1.577	0.15
1	4	1.634	1.638	0.23	1.647	0.78
2.5	2.5	1.711	1.716	0.28	1.732	1.23
0.5	9.5	1.777	1.783	0.34	1.774	-0.15
1	9	1.874	1.881	0.36	1.881	0.36
2.5	7.5	2.092	2.093	0.06	2.104	0.59
5	5	2.228	2.222	-0.26	2.236	0.36
50	4	2.949	2.956	0.24	2.973	0.81
50	10	3.948	3.940	-0.21	3.939	-0.22
100	50	7.476	7.513	0.49	7.393	-1.12

It can be noticed that although the present equations are simple, it gave results very close to the exact values and more accurate comparing with the solution by the currentFrench Rules equations. Then, the present equations can be rather used by the designer engineers with sufficient confidence.

Also, as shown in Table (3), the standard error of the obtained formula Eq. (8) is about two-third of that of French Rules Eq. (5) in case of braced frames while in the case of sway frames the standard error of Eqs. (9, 10) has less than one-half of that of French Rules Eq. (7).

Table 3. Comparison of standard error of the obtained equations and French Rules

	Equations		s_e
braced frames	French rule equation	(5)	0.0048
	modified equation	(8)	0.0033
sway frames	French rule equation	(7)	0.0212
	modified equation	(9)	0.0043
	modified equation	(10)	0.0137

VI. Conclusions

In this paper, simple closed form mathematical expressions (modified French rules equations) for the determination of the effective length factor K for steel columns are derived using multiple regressions analyses from the numerical results of the exact solution. The analysis is carried out for a wide range of the rotational resistance at column ends GA, GB (from 0 to 100).

The obtained mathematical expressions are more practical in design purposes for the structural engineer than the graphical charts since they provide a higher accuracy and can be easily coded in the Excel spreadsheet or in mathematical software such as Matlab, Maple and Mathematica.

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