IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 12, Issue 3 Ver. II (May - Jun. 2015), PP 116-146
www.iosrjournals.org

Debottlenecking of Bernoulli's apparatus and verification OF Bernoulli's principle

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Abstract : *This thesis aims toDebotteleneck the Bernoulli's apparatus kept in hydraulic machines laboratoryof Mechanical Engineering Department of BIT SINDRI DHANBAD,JHARKHAND,INDIA,which was out of order from more than a decade. Also aims to verify well known Bernoulli's equation with this apparatus.Chapter one gives some insight towards basics of fluid mechanics. Chapter two deals with Bernoulli's theorem and its applications.Chapter three deals with constructional details and experimentation method of Bernoulli's apparatus.Chapter four deals with observations and calculations for verifyingBernoulli'stheorem.Chapter five gives final results which verify Bernoulli's theorem.Chapter six tells about scope for future works.*
Keywords –*Bernoulli's theorem, Debottlenecking of apparatus, Fluid mechanics,Pitot tube,Surfacetension*

I. Introduction

1.1 Fluid

Matter exists in two principal forms: solid and fluid. Fluid is further sub-divided into liquid and gas.Fluid may be defined as a substance which is capable of flowing. [1] A fluid at rest does not offer any resistance to the shear stress, i.e. it deforms continuously as shear stress is applied. It has no definite shape of its own, but conforms to the shape of the containing vessel. A fluid can offer no permanent resistance to shear force and possesses a characteristic ability to flow or change its shape. Flow means that the constituent fluid particles continuously change their positions relative to one another. This concept of fluid flow under the application of a shear stress is illustrated in the following figure. [3]

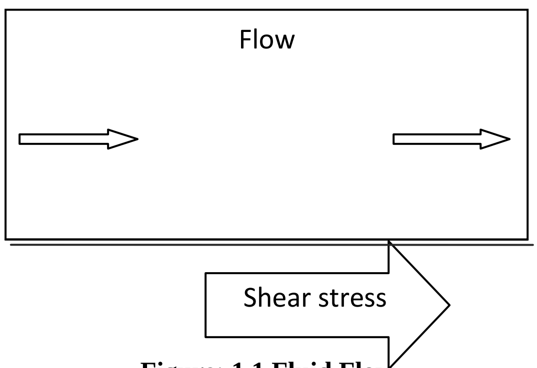


Figure: 1.1 Fluid Flow

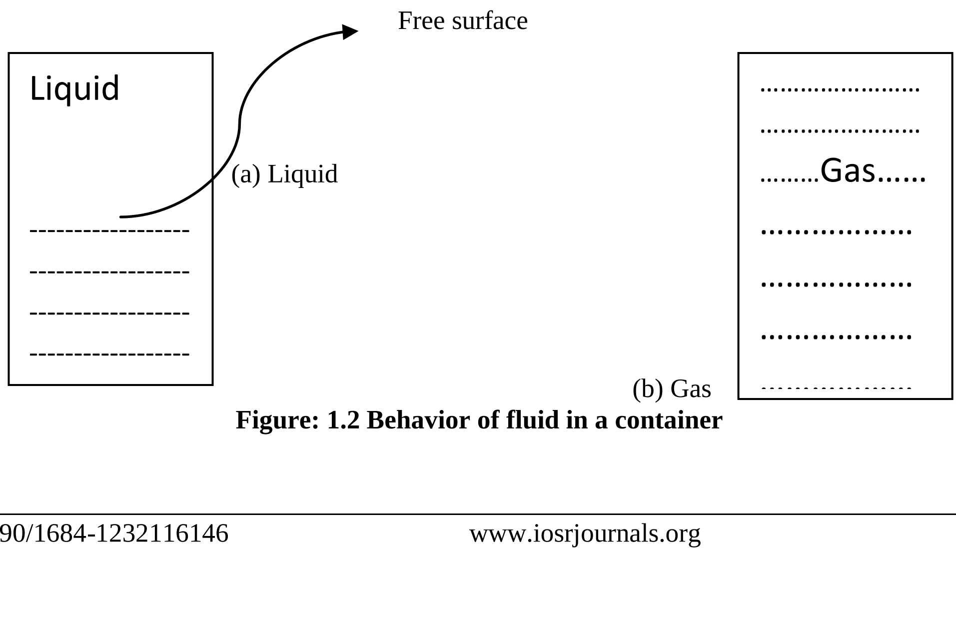


Figure: 1.2 Behavior of fluid in a container

1.2 Fluid Mechanics

It is that branch of science which deals with the study of fluid at rest as well as in motion. It includes the study of all liquids and gases but is generally confined to the study of liquids and those gases only for which the effects due to compressibility may be neglected.

In general the study of fluid mechanics may be divided into three categories, viz. fluid statics is the study of fluids at rest. Fluid kinematics is the study of fluids in motion without considering the forces responsible for the fluid motion. Fluid dynamics is the study of fluid in motion with forces causing flow being considered.

1.3 Development Of Fluid Mechanics

The beginning and development of the science of fluid mechanics dates back to the times when the ancient races had their irrigation systems, the Greeks their hydraulic mysteries, the Romans their methods of water supply and disposal, the middles ages their wind mills and water wheels. It is quite evident from the excavations of Egyptian ruins and Indus Valley Civilization that the concepts of fluid flow and flow resistance, which form the basis of irrigation, drainage and navigation systems, were known to be the man who lived at that time about 4000 years ago.

Through their sustained and continued efforts, a host of research workers contributed so extensively to the subject that by the end of 19^{th} century all the essential tools of hydraulics were at hand; the principles of continuity, momentum and energy; the Bernoulli's theorem; resistance formulae for pipes and open channels; manometers, Pitot tubes and current meters; wind tunnels and whirling arms; model techniques; and Froude and Reynolds laws of similarity; and the equations of motion of Euler, Navier-Stoke and Reynold. Historically the development of fluid mechanics has been influenced by two bodies of scientific knowledge: empirical hydraulics and classical hydrodynamics. **Hydraulics** is an applied science that deals with practical problems of flow of water and is essentially based on empirical formulae deduced from laboratory experiments. However, neither hydraulics not classical hydrodynamics could provide a scientific support to the rapidly developing field of aeronautics: the former because of its strong empirical slant with little regard for reason, and the latter because of its very limited contact with reality. The solution to the dilemma was provided by LudwiegPrandtl in 1904 who proposed that flow around immersed bodies be approximately by boundary zone of viscous influence and a surrounding zone of irrotational frictionless motion. This approach has a tremendous effect upon understanding of the motion of real fluids and eventually permitted analysis of lifting vanes, control surfaces and propellers. [3]

1.**4 Significance Of Fluid Machanics**

The subject of fluid mechanics encompasses a great many fascination areas

Like:

→ Design of a wide range of hydraulic structures (dams, canals, weirs etc.) and machinery (pumps, turbine and fluid couplings)
→ Design of a complex network of pumping and pipelines for transporting liquids; flow of water through pipes and its distribution to domestic service lines.
→ Fluidic control devices; both pneumatic and hydraulic
→ Design and analysis of gas turbines, rocket engines, conventional and supersonic aircrafts
→ Power generation from conventional methods such as hydroelectric, steam and gas turbines, to newer ones involving magneto fluid dynamics.
→ Methods and devices or the measurement of various parameters, e.g., the pressure and velocity of a fluid at rest or in motion.
→ Study of man's environment in the subjects like meteorology, oceanography and geology
→ Human circulatory system, i.e. flow of blood in veins and the pumping action of heart.[4]

1.5 Types Of Fluid

Fluids are classified into two types, viz. ideal fluids and real fluids. Ideal fluids are those fluids which have no viscosity, surface tension and compressibility. These fluids are imaginary and do not exist in nature. No resistance is encountered to these fluids, as they move. The concept of ideal fluids is used to simplify the mathematical analysis of fluid flow problems. Real fluids posses' viscosity, surface tension and compressibility and these fluids are actually available. As such, certain resistance is always encountered to these fluids when they move. Water and air though real fluids have very low viscosity and, therefore, they are treated as ideal fluids for all practical purposes without any appreciable error. [3]

Real fluids are further of two types, i.e. Newtonian and non-Newtonian fluids. Newtonian fluids are those fluids which obey Newton's law of viscosity, i.e. for Newtonian fluids are is linear relationship between shear stress and velocity gradient, e.g. water, air, etc.

Non-Newtonian fluids are those fluids which do not obey Newton's law of viscosity. The viscous behavior of non-Newtonian fluids may be given by the power law equation of the type,

$$\tau = k\left(\frac{du}{dy}\right)^n \qquad [1.1]$$

Where k is called consistency index and n is called flow behavior index.

Examples: Milk, blood, liquid cement, concentrated solution of sugar, etc.
A fluid ,in which shear stress is more than the yield stress and shear stress is proportional to the rate of shear strain (or velocity gradient) , is known as ideal plastic fluid.

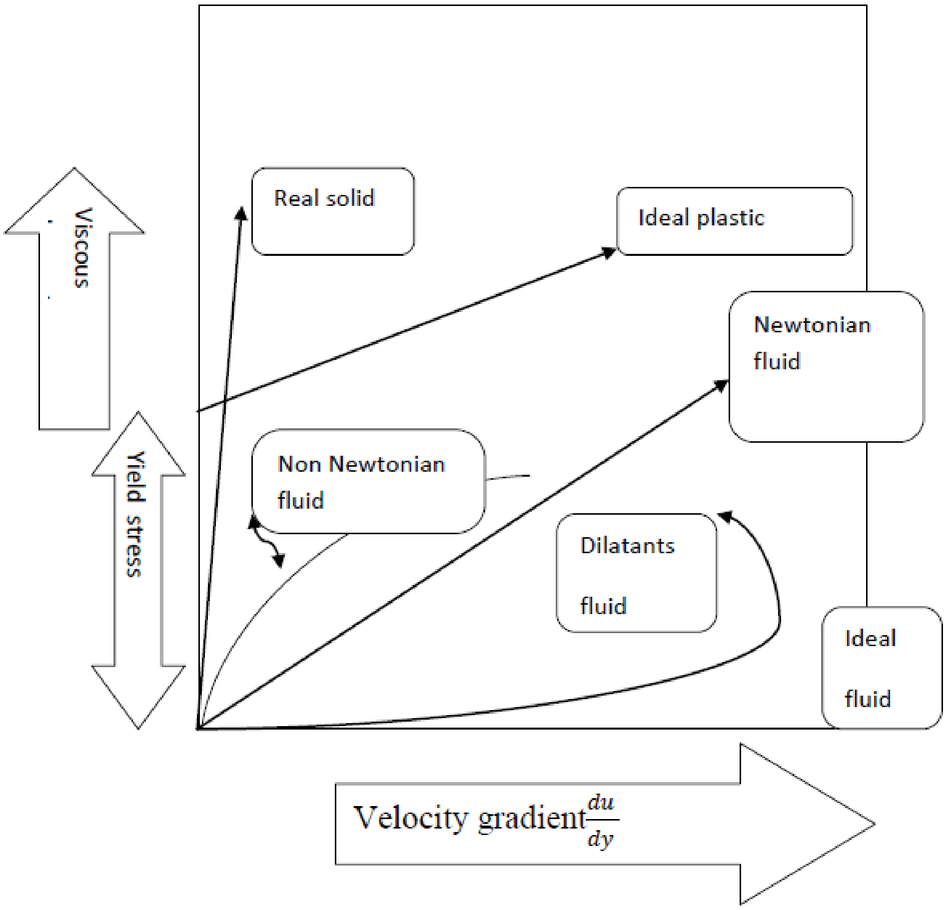


Figure: 1.3 Variation of shear stress with velocity gradient (time rate of deformation)

1.6 Fluid Properties
Mass Density:
Mass density of a fluid is the mass which it possesses per unit volume at a standard temperature and pressure (STP). It is denoted by the symbol ρ (rho) and is also known as specific mass. Therefore, mass density. [2]

$$\rho = \frac{m}{\forall} \qquad [1.2]$$

Where m is the mass of fluid having volume ∀
The unit of mass density in SI system is kg/m^3 and its value at standard temperature and atmospheric pressure (STP) for water at 4°C is 1000 kg/m^3 and for air at 20°C is 1.24 kg/m^3. [2]

Specific weight:

Specific weight of a fluid is its weight per unit volume at STP and is denoted by γ (gamma). Specific weight is also known as weight density or unit weight. Therefore, specific weight $\gamma = \frac{W}{\forall}$ [1.3] Where w is the weight of fluid having volume ∀ .
SI unit of specific weight is N/m^3 and its value for water at STP is 9810 N/m^3. The specific weight of a fluid changes from one place to another depending

upon the changes in the gravitational acceleration, g. The mass density and specific weight are related to each other as

$$\gamma = \rho g \qquad [1.4]$$

Specific gravity: Specific gravity of a fluid is the ratio of specific weight (or mass density) of the fluid (γ_S) to the specific weight (or Mass density) of a standard fluid. For liquids, the standard fluid is taken as water at 4℃, and, for gases, the standard fluid is air or hydrogen at some specified temperature and pressure. Specific gravity is also known as relative density and may be denoted by G or S. Thus,

$$G = \gamma_S / \gamma_W \qquad [1.5]$$

By knowing the specific gravity of any fluid, its specific weight can be calculated.

Specific Volume:

Specific volume is the volume per unit mass of fluid. Thus, it is the reciprocal of specific mass. The term 'specific volume' is most commonly used in the study of compressible fluids. [9]

Viscosity: Viscosity is primarily due to cohesion and molecular momentum exchange between the fluid layers. It is defined as "the property of a fluid by virtue of which the fluid offers resistance to the movement of one layer of fluid over an adjacent layer and, as the fluid flows, this effect appears as shear stress acting between the moving layers of fluid". The fast moving upper layer exerts a shear stress on the lower slow moving layer in the positive direction of flow. Similarly, the lower layer exerts a shear stress on the upper moving fast layer in the negative direction of flow. According to Newton, shear stress (τ) acting between the fluid layers is proportional to spatial change of velocity normal to flow, i.e. shear stress,

$\tau \alpha \frac{du}{dy}$ or

$$\tau = \mu \frac{du}{dy} \qquad [1.6]$$

Where μ is constant of proportionality and is called coefficient of viscosity or dynamic viscosity or simply viscosity of the fluid. The term (du/dy) is called velocity gradient at right angle to the direction of flow.
Equation [1.6] is known as **Newton's law of viscosity**. The unit of μ in different systems are as follows

Table no: 01

System	Units of μ
SI	Ns/m^2 = Pa.s
MKS	$Kg(f).s/m^2$
CGS	$Dyn.s/cm^2$

The unit $dyne/cm^2$ is also called poise (P) and 1P = 1/10 Pa.s. A poise is relatively large unit, hence the unit centipoises (cP) is generally used and 1 cP = 0.01 P. Viscosity of water and air at 20℃ and at standard atmospheric pressure are 1.0 cP and 0.0181 cP respectively.

Kinematic viscosity:

Kinematic viscosity is the ratio of dynamic viscosity (μ) to the mass density (ρ) and is denoted by the symbol v (Greek 'nu')

$$V = \frac{\mu}{\rho} \qquad [1.7]$$

In SI units, v is expressed as m^2/s. In CGS system, it is expressed as cm^2/s is also termed stoke and 1 stoke = 1 $cm^2/s = 10^{-4} m^2/s$. Also, 1 centistokes = 0.01 stoke.

Surface tension and capillarity:

Liquids have characteristics properties of cohesion and adhesion. Surface tension is due to cohesion, whereas capillarity is due to both cohesion and adhesion.

Surface tension:

It is the property of a liquid by virtue of which the free surface of liquid behaves like a thin stretched membrane. It is a force required to maintain unit length of free surface in equilibrium and may be denoted by σ (Greek sigma). In SI units, surface tension is expressed as N/m. surface tension values are generally quoted for liquids when they are in contact with air, e.g. σ for air water interface (at 20℃) is equal to 0.0636 N/m and, for air-mercury interface, it is equal to 0.4944N/m.

Capillarity:

It is the phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid. It is also known as meniscus effect. The rise of liquid surface is known as capillary rise and the fall of liquid surface as capillary depression. Capillary rise will take place when adhesion is more than cohesion. It has been observed that for tubes of diameters greater than 5mm the capillary rise or fall is negligible. Hence, in order to avoid a correction for capillarity effects, diameter of tubes use in manometers for measuring pressure should be more than 5mm. [8]

Compressibility:

Compressibility of a fluid is expressed as reciprocal of bulk modules of elasticity. In case of liquids, effects of compressibility are neglected, however in some special cases such as rapid closure of valve (as in water hammer phenomenon), where changes of pressure are either very large or sudden, it is necessary to consider the effects of compressibility. [7]

1.7 **Basic Law's Of Fluid Mechanics**

(a) Pascal's law

*It states that the intensity of pressure at a point in a static fluid is*equal in all directions. This is proved as:

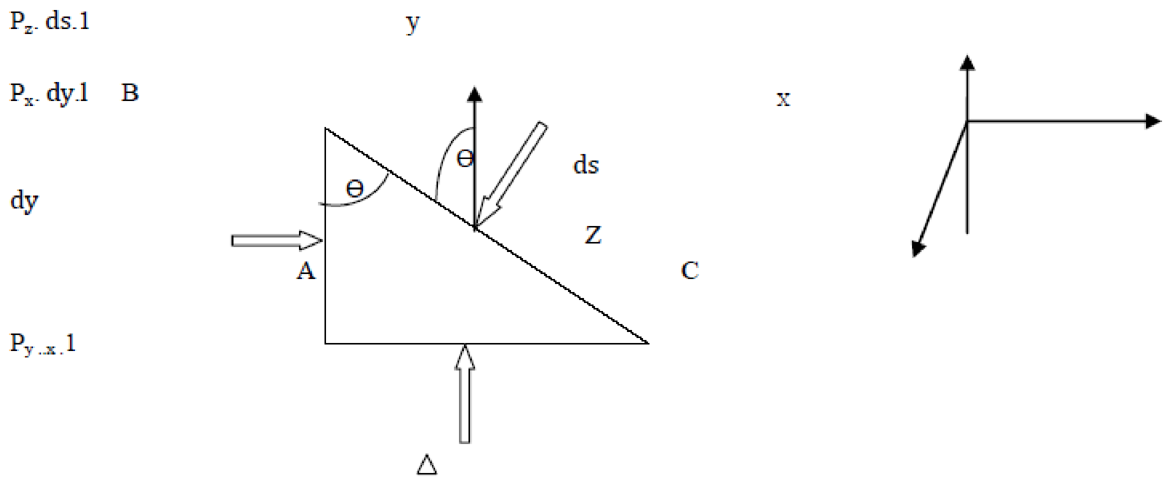


Figure: 1.4Forces on a fluid element

We consider an arbitrary fluid element of wedge shape in a fluid mass at rest. Let the width of the element is unity and P_x ,P_y .P_z are the pressure or intensity of pressures acting on the face AB, AC, and BC respectively. Let $\angle ABC = \theta$ then the force acting on the element are :

1. Pressure force normal to the surface
2. Weight of element in the vertical direction.

The forces on the faces are:

Force on the face AB = $P_x \times$ Area of face AB = $P_x \times dy \times 1$

Similarly force on the face AC = $P_y \times \Delta x \times 1$

Force on the face BC = $P_z \times ds \times 1$

Weight of element $= \frac{AB \times AC}{2} \times 1 \times w$

Where, w = weight density of fluid.

Resolving the forces in x-direction, we have

$P_x \times dy \times 1 - P_z\ (ds \times 1) \sin(90^0 - \theta) = 0$

or$P_x \times dy \times 1 - P_z\ (ds \times 1) \cos\ \theta = 0$

But from Fig.1.4 ,$ds \cos\theta$ = AB = dy

$\therefore P_x \times dy \times 1 - P_z dy \times 1 = 0$

Or $P_x = P_z$ [1.8]

Similarly, resolving the forces in y- direction, we get

$P_y \times dx \times 1 - P_z (ds \times 1) \cos(90^0 - \theta) - \frac{dx \times dy}{2} \times 1 \times w = 0$

or$P_y \times dx - P_z ds \sin\theta - \frac{dxdy}{2} \times 1 \times w = 0$
But $d_s \sin\theta = dx$ and also the element is very small and hence weight is negligible.
$\therefore \quad P_y dx - P_z dx = 0$
$or P_y = P_z$ [1.9]

From above equations we have,
$p_x = p_y = p_z$ [1.10]

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions. [2]

(b) Mass Conversation i.e. Continuity Equation

The equation based on the principle of conversation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid flow per second is constant. Consider two cross-section of a pipe. [2]

Let,
V_1 = Average velocity at cross-section 1-1
ρ_1 = Density at section 1-1
A_1 = Area of pipe at section 1-1
And V_2, ρ_2, A_2 are corresponding values at section 2-2
Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$
Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

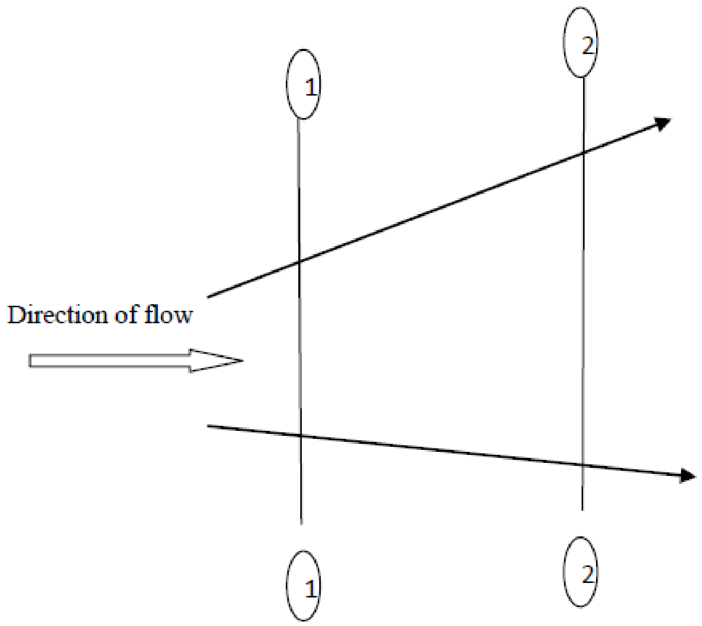


Figure 1.5 Fluid flowing through a pipe

According to law of conservation of mass rate of flow at section 2-2 is equal to rate of flow at section 1-1
or$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ [1.11]
Equation [1.10] $p_x = p_y = p_z$ is applicable to the compressible as well as incompressible fluids and is called Continuity Equation. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation [1.11] reduces to

$$A_1 V_1 = A_2 V_2 \quad [1.12]$$

(c) Bernoulli's Theorem.

It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are:

Pressure head $= \frac{P}{\rho_g} = \frac{P}{w}$

Kinetic head $= \frac{V^2}{2g}$

Datum head = z

Thus mathematically, Bernoulli's theorem is written as

$$\frac{P}{W} + \frac{V^2}{2g} + z = \text{Constant.} \qquad [1.13]$$

Thus; 'In a steady flow system of frictionless incompressible fluid , the sum of velocity, pressure, and elevation heads remains constant at every section'. Thus it is however, on the assumption that energy is neither added to nor taken away by some external agency.

(d)Momentum Equation
It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force action on a fluid mass 'm' is given by the Newton's second law of motion.

$$F = m\times a$$

Where, a is the acceleration acting in the same direction as force F.
But,

$$a = \frac{dv}{dt}$$

$$\therefore \quad F = m\frac{dv}{dt}$$

$$= \frac{d(mv)}{dt} \text{ \{m is constant and can be taken inside the differential\}}$$

$$\therefore \quad F = \frac{d(mv)}{dt} \qquad [1.14]$$

Equation [1.14] is known as the momentum principle. Equation (1.14) can be written as

$$F.\, d\, t = d\,(m\, v) \qquad [1.15]$$

Which is known as the impulse-momentum equation and states that the impulse of a force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum d(m v) in the direction of force.

II. Bernoulli's Theorem

2.1 Fluid Dynamics

Fluid dynamics is the study of fluid motion that involves force of action and reaction. i.e. forces which cause acceleration and forces which resist acceleration. Dynamics of fluid motion is essentially governed by the Euler's equation (momentum principles) and Bernoulli's equation (Energy principle). Derivation of the momentum and energy equations stems from Newton's second law of motion, F = ma. [6]

2.2 Energy And Its Form

Certain terms pertaining to energy are defined here with a view to avoid any miss concepts in the derivation and significance of the energy and momentum equations. Energy represents the capacity to produce a change in the existing conditions. i.e. capacity to exert force through a distance and do work. Energy cannot be seen: its presence can however be felt by observing the properties of the system. When energy is added to or subtracted from a system there occurs a change in one or more characteristics of the system. Energy of a system may be of the forms: [5]

i. Stored energy, i.e. energy contained within the system boundaries. Examples are the potential energy kinetic energy and the internal energy.
ii. Energy in transit, i.e. the energy which crosses the system boundaries. Heat and work represent the energy in transit.

2.2.1Potential energy and Datum energy is the energy possessed by a fluid body by virtue of its position or location with respect to some arbitrary horizontal plane. Essentially it represents the work necessary to move the fluid, against the gravitational pull of the earth, from a reference elevation to a position y above or below the reference elevation/datum plane. Thus, Potential Energy (P.E.) = m g y

Where, y is positive upward.

For each unit mass of fluid passing a cross-section of the stream tube, the potential energy would be gy. [7]

2.2.2 Kinetic energy is the energy possessed by a fluid body by virtue of its motion. Invoking Newton's second law of motion.

Force = mass x acceleration

$dF = m\frac{dv}{dt}$ [2.1]

multiply both sides by ds, the differential displacement of both force and fluid mass.

$$dF \times ds = mdv\frac{ds}{dt}$$

or , $\delta W = m\, v\, dv$

if the fluid mass accelerates from velocity v_1 to v_2 then

$$W = m\int_1^2 v\, dv = m\,\frac{(v_2^2 - v_1^2)}{2} \quad [2.2]$$

Essentially the kinetic energy represents the work necessary to accelerate the fluid mass from rest to velocity v. Thus,

Kinetic energy K.E. = $\frac{1}{2}mv^2$ [2.3]

For each unit mass of fluid passing a cross-section of stream tube, the kinetic energy would be $\frac{v^2}{2}$.

2.2.3 Internal energy is measure of the energy stored within the fluid mass due to the activity and spacing of the fluid molecules. It is essentially comprises (i) kinetic energy due to molecular agitation and (ii) potential energy due to the attractive and repulsive forces between the molecules or atoms constituting the fluid mass.

2.2.4 Heat and work

Heat is the energy in transit (without transfer of mass) across the boundaries of a fluid system because of temperature difference between the fluid system and its surroundings. Transfer of heat energy is in the direction of lower temperature. Work is the energy in transit (without transfer of mass and with the help of a mechanism) because of a property difference, other than temperature, between the fluid system and surroundings. [4]

Flow rate of displacement energy is a measure of the work required to push a fluid mass across the control surface at the entrance and exit cross-sections.

Consider a one dimensional fluid system focused on the entrance cross-section of the system. Pressure intensity p_1 velocity v_1 are assumed to be uniform at this section of cross sectional area A_1. During an infinitesimal time dt, this section shifts through a distance ds, given by v_1dt. The displacement ds, is so small that any variation in fluid properties can be neglected. Work done during displacement of the fluid mass is called the flow work, and it is prescribed by the relation

Flow work = force x displacement

$= p_1\, A_1\, v_1dt$

$= p_1\, V_1dt$ —

And the rate of which flow work is done on the area is obtained by dividing throughout by dt. Thus,

Flow work = $p_1\, V_1dt = p_1\, v_1 = \frac{P_1}{\rho_1}$ ~~per~~ unit mass ….. [2.4]

Where v_1 and ρ_1 represent the specific volume and density of the mass.

2.3 Euler's Equation

Euler's equation of motion is established by applying Newton's saw of motion to a small element of fluid moving within a stream tube. The element has a mean cross-sectional area dA. Length ds and the centroid of the downstream face lies at a level dy higher than the centroid of the upstream face. Motion of the element is influenced by:

Normal forces due to pressure: let p and (p+dp) be the pressure intensities at the upstream and downstream face respectively. Net pressure force acting on the element in the direction of motion is then given by,

pdA – (p + dp) dA = - dp$dAType$e $equation\ here.$

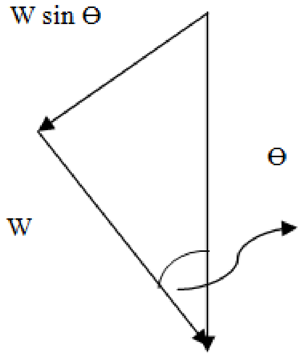


Figure: 2.1 Fluid elements with friction

Tangential force due to viscous shear: If the fluid element has a perimeter dp, then shear force on the element is dFs = τ dp ds

Where τ is the frictional surface force per unit area acting on the walls of the stream tube . The sum of all the shearing forces is the measure of the energy lost due to friction.
Body force such as gravity action in the direction of gravitational field.If ρ is the density of the fluid mass, then the body force equals ρg dA ds. Its component in the direction of motion is

$$= \rho g\ dA\ ds \sin\theta$$

$$= \rho g\ dA\ dy \qquad \text{from figure: 2.1}\left(\because \sin\theta = \frac{dy}{ds}\right)$$

The resultant force in the direction of motion must equal the product of mass
Acceleration in that direction. That is

$$-dp dA - \rho g\ dA\ dy - \tau\ dp\ ds = \rho\ dA\ ds\ a_s \qquad [2.5]$$

It may be recalled that the velocity of an elementary fluid particle along a streamline is a function of position and time,

$$u = f(s,t)$$

$$du = \frac{\delta u}{\delta s} ds + \frac{\delta u}{\delta t} dt$$

$$\text{or} \quad du / dt = \frac{\delta u}{\delta s}\frac{ds}{dt} + \frac{\delta u}{\delta t}$$

$$a_s = u\frac{\delta u}{\delta s} + \frac{\delta u}{\delta t} \qquad [2.6]$$

In a steady flow the changes are with respect to position only; so $\frac{\delta u}{\delta t}$ = 0 and the partial differentials become the total differentials. Evidently for a steady flow, the acceleration of fluid element along a streamline equals, $a_s = u \frac{du}{ds}$. Substituting this result in equation [2.5], we obtain

$$-dp dA - \rho g\ dA\ dy - \tau\ dP\ ds = \rho\ dA\ u\ du \qquad [2.7]$$

Dividing throughout by the fluid mass

$$u\frac{du}{ds} + \frac{1}{\rho}\frac{dp}{ds} + g\frac{dy}{ds} = -\frac{\tau}{\rho}\frac{dP}{dA} \qquad [2.8]$$

Which is Euler's Equation of motion. Here

(i) The term $u\frac{du}{ds}$ is measure of convective acceleration experienced by the fluid as it moves from a region of one velocity to another region of different velocity; evidently it represents a change in kinetic energy.
(ii) The term $\frac{1}{\rho}\frac{dp}{ds}$ represents the force per unit mass caused by the pressure distribution.
(iii) The term $g\frac{dy}{ds}$ represents the force per unit mass resulting from gravitation pull.
(iv) The term $\frac{\tau}{\rho}\frac{dP}{ds}$ prescribes the force per unit mass caused by friction.

For ideal fluids, τ = 0 and, therefore, equation [2.8] reduces to:

$$u\ du + \frac{dp}{\rho} + g\ dy = 0$$

$\frac{1}{2}d(u^2) + \frac{dp}{\rho} + g\,dy = 0$ [2.9]

Euler's equations [2.8] and [2.9] have been set up by considering the flow within a stream tube and as such apply to the flow within a stream tube or along a streamline because as dA goes to zero, the stream tube becomes a streamline.

2.4 Bernoulli's Theorem: Integration Of Euler's Equation For One Dimensional Flow

Bernoulli's equation relates to velocity, pressure and elevation changes of a fluid in motion. The equation is obtained when the Euler's equation is integrated along the streamline for a constant density (incompressible) fluid. Integration of Euler's equation: [2]

$$\frac{1}{2}\,d(V^2) + \frac{dp}{\rho} + g\,dy = 0\ gives$$

$$\int \frac{1}{2}\,d(V^2) + \int \frac{dp}{\rho} + g \int dy = constant$$

We assume ρ to be constant,

$\frac{p}{\rho} + \frac{V^2}{2} + gy = \text{constant}$ [2.10]

Equation [2.10] is one of the most useful tools of fluid mechanics is known as Bernoulli's equation in honor of the Swiss mathematician Daniel Bernoulli.

The constant of integration (called the Bernoulli's constant) varies from one streamline to another but remains constant along a streamline in steady, frictionless, incompressible flow. Each term has the dimension $(L/T)^2$

m^2/s^2 = Nm / kg or units of Nm/kg and such represents the energy per unit kilogram mass. Evidently the energy per unit mass of a fluid is constant along a streamline for steady, incompressible flow of non-viscous fluid. Dividing equation [2.10] by g and using the relation w = ρg, we get

$\frac{p}{\rho g} + \frac{V^2}{2g} + y = \text{Constant}$

$\frac{p}{w} + \frac{V^2}{2g} + y = \text{Constant}$ [2.11]

This is the form of Bernoulli's equation commonly used by hydraulic engineers.

$\frac{p}{\rho g} + \frac{V^2}{2g} + y = H = \text{Constant}$

⇩ ⇩ ⇩ ⇩

Pressure head Velocity head datum head Total head

Whilst working with Bernoulli's equation, we must have clear understanding of the assumptions involved in its derivation, and the corresponding **limitations** of its applications.

- Flow is steady, i.e, at a given point there is no variation in fluid properties withrespect to time
- Fluid is ideal ,i.e. it does not exhibit any frictional effects due to fluid viscosity.
- Flow is incompressible; no variation in fluid density.
- Flow is essentially one-dimensional, i,e. along a streamline. However, the
- Bernoulli's equation can be applied across streamlines if the flow is irrotational.
- Flow is continuous and velocity is uniform over a section.
- Only gravity and pressure forces are present. No energy in the form of heat or

Work is either added to or subtracted from the fluid.

Daniel Bernoulli (1700-1782) was a Swiss mathematician and physicist, born in Basel, Switzerland, to Johan Bernoulli and Dorothea Faulkner. He had an older brother, nicolaus II, who passed away in 1725, and a younger brother, Jhonan II.

Daniel Bernoulli made significant contributions to calculus, probability, medicine, physiology, mechanics, and atomic theory, he wrote on problems of acoustics and fluid flow and earned a medical degree in 1721. Daniel was a professor of experimental philosophy, anatomy, and botany at the universities of Groningen in the Netherlands and Basel in Switzerland. He was called to tech botany and physiology at the most ambitious Enlightenment scientific institution in the Baltic states, the St. Petersburg Academy of Science. Later in his academic career he obtained the chair of physics, which he kept for 30 years, St. Petersburg Academy offered mathematics, physics, anatomy, chemistry, and botany courses, its buildings included an observatory, a physics cabinet, a museum, a botanical garden, an anatomy theater, and an instrument-making workshop.

His most important publication, hydrodynamica, discussed many topics, but most importantly, it advanced the kinetic molecular theory of gases and fluids in which Bernoulli used the new concepts of atomic structure and atomic behavior. He explained gas pressure in terms of atoms flying into the walls of the containing vessel, laying the groundwork for the kinetic theory. His ideas contradicted the theory accepted by many of his contemporaries, including Newton's explanation of pressure published in Principia Methematica. Newton thought that particles at rest could cause pressure because they repelled each other.

Because of the brilliance of Newton's numerous discoveries, it was assumed by most scientists of the inaccurate. Although Bernoulli disagreed with Newton's theory. Bernoulli supported the physics of Issac Newton, as dis his female contemporary, Emillie du Chateslet, who translated newton's work into French in the late 1740s. In section X of his book, Bernoulli also offered his explanation of pressure measured with a new instrument named by Boyle, the barometer.

In Hydrodynamica, Bernoulli took on the task of solving difficult mechanical problems mentioned in Newton's Principia Mathematica. In the 10th chapter, Bernoulli imagines that gases, which he called "elastic fluid," were composed of particles in constant motion and describes the behavior of the particles trapped in a cylinder. As he depressed the movable piston, he calculated the increase in pressure, deducing Boyle's law. Then he described how a rise in temperature increases pressure, as well as the speed of the atomic particles of the gas, a relationship that would later become a scientific law.

He attributed the change in atosmospheric pressure to gases being heated in the cavities of the Earth's crust that would rush out and dries, increasing barometric pressure. The pressure dropped when the internal heat of the Earth decreased the air contracted. Bernoulli not only used his mathematical expertise to contribute to physical science. He attempted to statistically predict the difference in the number of deaths from smallpox that would occur in the population if people were properly inoculated against the horrible disease. [10]

But in modern physical science textbooks, Daniel Bernoulli is best recognized for Bernoulli's principle, or the Bernoulli effects, which describes the inverse relationship between the speed of air and pressure. In 1738, Bernoulli stated his famous principle in Section XII of Hydrodynamica, where described the relationship between the speed of a fluid and pressure. He deduced this relationship by observing water flowing through tubes of various diameters. Bernoulli proposed that the total energy in a flowing fluid system is a constant along the flow path. Therefore if the speed of flow increases, the pressure must decrease to keep the energy of flow lat that constant. Today we apply this relationship to the flow of air over a surface, as well as. Because of Daniel Bernoulli, we are able to build aircraft, fly helicopters, water the lawn, and even pitch a curve ball. Daniel Bernoulli, physicist, mathematician, natural scientist, and professor, died in his native Base. Switzerland, on March 17, 1782 and fittingly, was buried in the Peterskirche, meaning St. Peter's Church, in Vienna. It is believed that the location of St. Peter's Church has supported a place of worship since the second half of the fourth century. [10]

2.5 Applications Of Bernoulli's Theorem

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices: [2]

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

2.5.1. Venturimeter

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts:

i. A short converging part
ii. Throat and
iii. Diverging part.

It is based on the Principle of Bernoulli's equation.

Expression for Rate of Flow ThroughVenturimeter

We consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water),

Let , d_1 = diameter of inlet or at one section
P_1 = pressure at the section
V_1 = Velocity of fluid at the section
a_1 = area of the section $=\frac{\pi}{4}{d_1}^2$

and d_2, p_2, v_2, a_2 are corresponding values at another section

Applying Bernoulli's equation at sections, we get

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p_2}{w} + \frac{v_2^2}{2g}$$

Or $$\frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g}$$

But $\frac{P_1 - P_2}{w}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h

Or $\frac{P_1 - P_2}{w} = h$

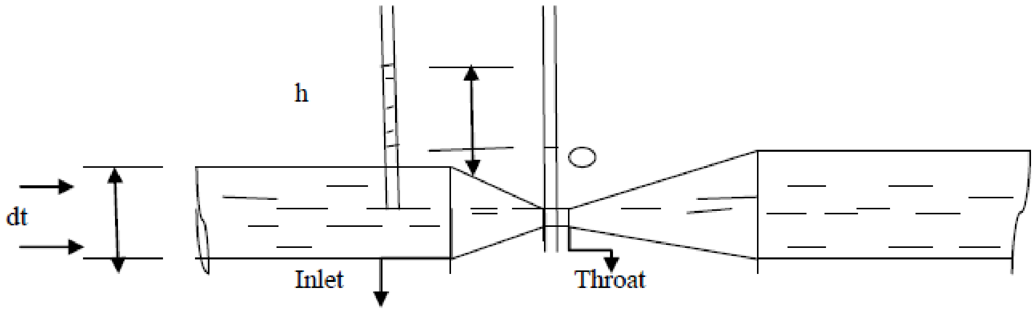


Figure: 2.2 Venturimeter

Substituting this value of $\frac{P_1 - P_2}{w}$ in the above equation, we get

$$h = \frac{V_2^2 - V_1^2}{2g} \quad [2.12]$$

now applying continuity equation at Sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or } v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation [2.12]

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)\left(\frac{a_2 v_2}{a_1}\right)}{2g} = \left[1 - \frac{a_2^2}{a_1^2}\right]\frac{v_2^2}{2g}$$

$$= \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or $v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$

$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$\therefore$ = Discharge, $Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad [2.13]$$

Equation [2.13] gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore Q_{act} = c_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad [2.14]$$

where c_d = co-efficient of venture meter and its value is its less than 1.

Value of 'h' given by differential U-tube manometer

Case I . Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = sp gravity of the heavier liquid

S_o = sp gravity of the liquid flowing through pipe

X = difference of the heavier liquid column in U-tube

Then $$h = x\left[\frac{s_h}{s_o} - 1\right] \quad [2.15]$$

Case II. If the differential manometer contains a liquid which is lighter than the liquid than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{s_h}{s_o}\right] \qquad [2.16]$$

Where, S_l=sp gr. Of lighter liquid in U-tube
S_o = sp. gr. of fluid flowing through pipe
X = difference of the lighter liquid columns in U-tube.

Case III. Inclined venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = x\left[\frac{s_h}{s_o} - 1\right] \qquad [2.17]$$

Case IV. Similarly for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = x\left[1 - \frac{S_l}{s_o}\right] \qquad [2.18]$$

2.5.2. Orifice meter

It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It consists of a flat circular plate which has a circular sharp edge hole called orifice, which is concentric with the pipe, through it may from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1) , which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2) ,which is at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

Let P_1 = pressure at section (1)
V_1 = velocity at section (1)
a_1 = area of pipe at section (1) , and
P_2 , V_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at both sections. We get,

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

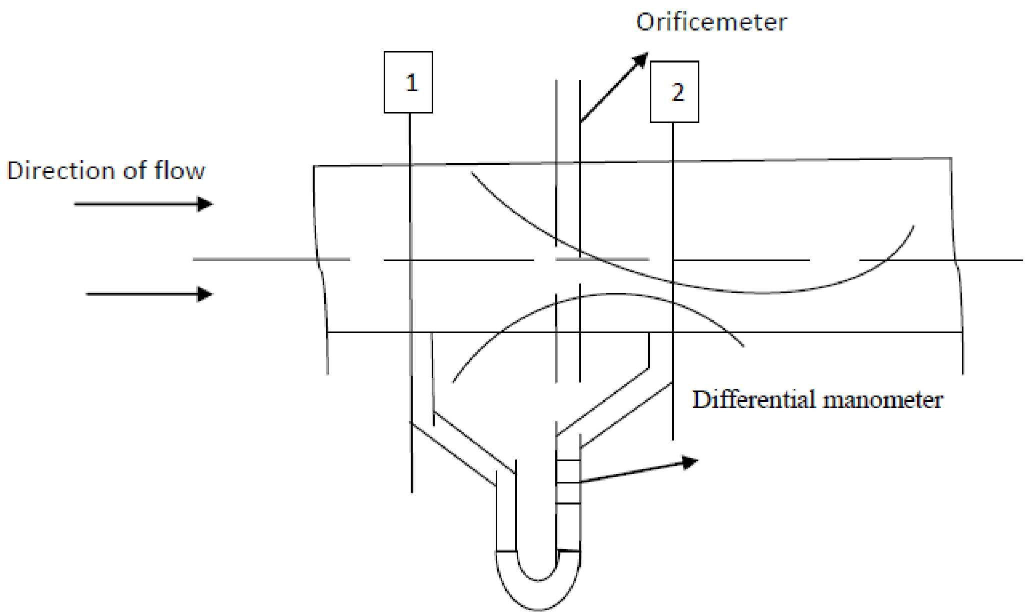


Figure: 2.3 Orificemeter

$$\left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = \frac{V_2^2 - V_1^2}{2g}$$

$\left(\frac{p_1}{w}+z_1\right)$ - $\left(\frac{p_2}{w}+z_2\right)$ = h = Differential head = $\frac{V_2^2-V_1^2}{2g}$

$v_2 = \sqrt{2gh+V_1^2}$ [2.19]

∴ The discharge Q = $v_2 \times a_2 = v_2 \times a_0 = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 c_0^2}}$ [2.20]

The above equation is simplified by using

$$c_d = c_c \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 c_0^2}}$$

$$\therefore c_c = c_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 c_0^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Substituting the value of c_c in equation (iv) we get

$$Q = \frac{c_d a_o \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{c_d a_{o} a_1 \sqrt{2gh}}{\sqrt{a_1^2-a_0^2}}$$

Where C_d = Co-efficient of discharge for orifice meter. The Co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

a_0 = croosection area at '0' any section

a_1= croosection area at '1' any section

2.5.3 Pitot-tube.

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in figure 2.2.[2]

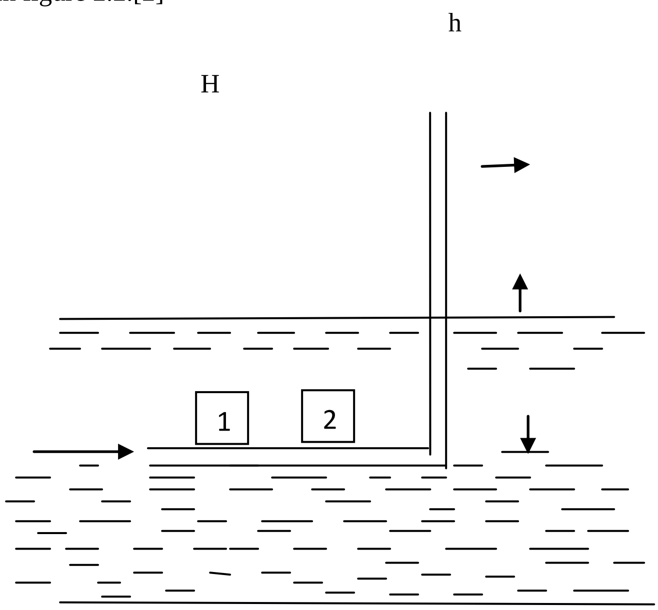


Fig: 2.4 Pitot-tube

The lower end, which is bent through 90^0 is directed in the upstream direction as shown in fig. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at the same level in such a way that point (2) is just at the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)
V_1 = velocity of flow at (1)
P_2 = pressure at point (2)
V_2 = velocity at point (2), which is zero
H = depth of tube in the liquid
h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equations at point (1) and (2), we get

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$

$\frac{p_1}{w}$ = pressure head at (1) = H

$\frac{p_2}{w}$ = pressure head at (2) = (h + H)

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h+H)$$

$$\therefore \quad h = \frac{v_1^2}{2g}$$

This is theoretical velocity $v_1 = \sqrt{2gh}$

Actual velocity is given by, $(v_1)_{act} = C_d\sqrt{2gh}$

III. Bernoulli's Apparatus

3.1 Technical Specification:

Common data table for all runs

Table no: - 02

Piezometer No., i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Distance from inlet Section, Xi $\times 10^{-2}$ m	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
Area of duct.(Ai) $\times 10^{-3} m^2$	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Dimensions of the uniform duct at the inlet and outlet sections = 6cm×5cm
Dimensions of the duct at the throat section = 6cm×5cm
Length of the duct, L =98.5cm
Width of the duct, B =5cm
Distance between the two piezometers = 5cm
Number of piezometers = 20

We are considering the length of duct is 100 cm ,but in actual present set up as in figure [3.1] lenth of the duct is 98.5 cm. The error in length may cause in variation of some readings.

3.2 Bottlenecks In Bernoulli's Apparatus:

- System was not in running condition since a decade.
- Piezometers tube were broken.
- Inlet and outlet fluid flow pipes were destroyed
- Apparatus was corroded due to ageing.
- Water supply line was disturbed.
- The discharge measurement bucket of volume 9350 cc was not in good condition .

Figure: 3.1 Bottlenecks of Bernoulli's apparatus

3.3 Debottlenecking Of Components:

- First apparatus was cleaned and get painted.
- New piezometer tubes were installed in apparatus.
- New inlet and outlet pipes were fitted.
- Discharge bucket was repaired and painted.
- Water supply lines get corrected for proper fluid supply.

Figure: 3.2 Experimental set up of Bernoulli's apparatus at BIT Sindri Hydraulic Machines laboratory

3.4 Experimental Set Up:

The set-up consists of a horizontal uniform duct having constant width and depth Fig: 3.2 . The duct is made of Galvanized Iron sheets, which are joined together to form duct of required shape. A number of piezometers are fitted on the duct at equal intervals for measuring the pressure heads at different gauge points. The duct is connected to two tanks, one at the upstream end (inlet tank) and the other at the downstream end (outlet tank). The inlet tank is fitted with a piezometer for indicating the water level in the tank. The outlet tank is provided with an outlet valve for controlling the outflow. The set-up is placed on a hydraulic bench. Water is supplied to the inlet tank by a supply pipeline provided with an inlet valve (supply valve) and connected to a constant overhead water tank.

Figure: 3.3 Experimental set up in presence of lab instructor

3.5 Experimental Procedure:

1. I opened the inlet valve gradually to fill the inlet tank. The water level starts rising in various piezometers. I removed the air bubbles in the piezometer,
2. I opened the exit valve and adjusted the inflow and the outflow so that the water level in the piezometers was constant, i.e.so that the flow be steady.
3. I measured the levels of water in various piezometers with respect to an arbitrary selected suitable horizontal plane as datum i.e., MS platform.
4. I measured the discharge, from the bucket of 9350cc.
5. I repeated the above steps for six runs by regulating the supply valve.
6. I calculated the pressure head, velocity head and datum head for each runs.

Figure: 3.4 Running condition of Bernoulli's apparatus at BIT Sindri Hydraulic machines laboratory

IV. Observations And Calculations

4.1 Computation Of Total Head:

Run No. 1

Discharge calculations:-

For head = 32.5 × 10^{-2} m

The bucket used for discharge calculation is = 9350 cc

For first run the time taken to fill the bucket is = 19 seconds

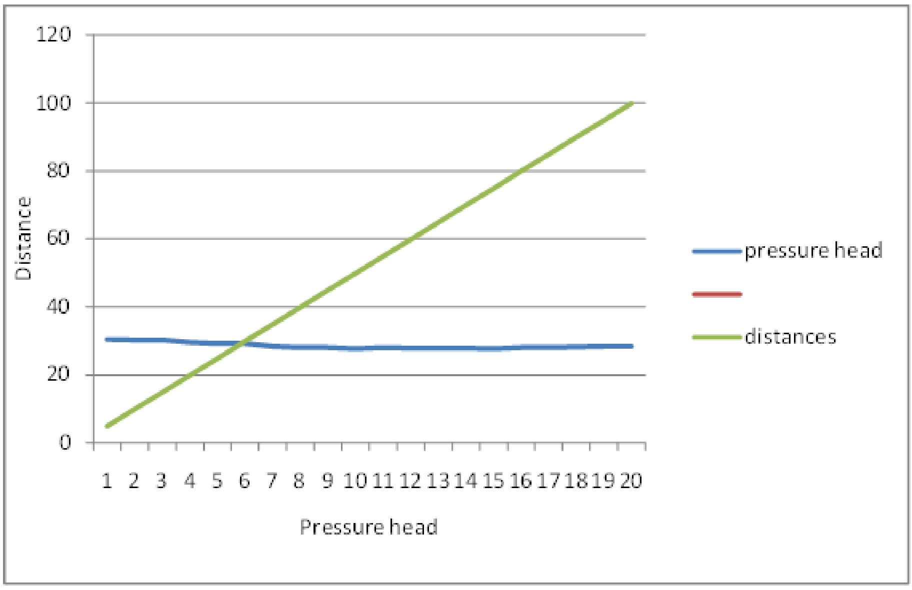
Discharge (Q) = 9350 ÷ 19 × $10^{-6} m^3$/s = 4.9210 × $10^{-4} m^3$/s

Mean velocity $=\frac{Q_i}{A_i}$ = Discharge ÷ crossecsional area
$= 4.9210 \times 10^{-4} m^3$/s ÷ (6 × 5 × 10^{-4}) m^2
= 0.16403 m/s
Velocity head = $(Meanvelocity)^2$ ÷ (2 × g)
= (0.16403 $m/s)^2$ ÷ (2 × 9.81m/s^2)
= 1.3713 × 10^{-3} m
Pressure head = p/ρg = 30.4 × 10^{-2} m of water ÷ (1000 ×9.81 N)
= 30.4 × 10^{-6} m

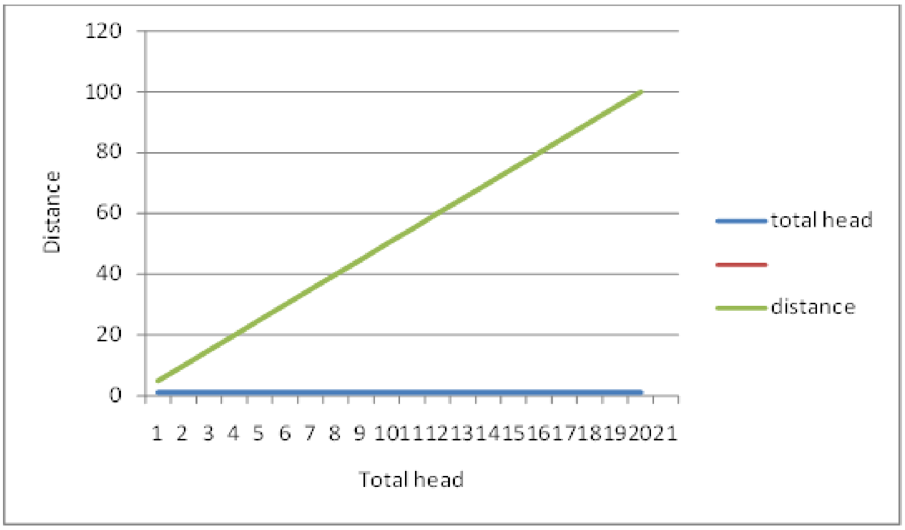
Table no: - 03

Piezometer No., i	Mean velocity, Vi = Q/Ai (m/s)	Velocity head, $Vi^2/2g \times 10^{-3}$ m	Pressure head $(p/\rho g) \times 10^{-6}$ m	Datum head	Total head, H = $p/\rho g + \frac{Vi^2}{2g} + y$
1	0.164	1.3713	30.4	103	1.031670
2	0.164	1.3713	30.2	103	1.031674
3	0.164	1.3713	30.3	103	1.031674
4	0.164	1.3713	29.6	103	1.031667
5	0.164	1.3713	29.3	103	1.031664
6	0.164	1.3713	29.2	103	1.031663
7	0.164	1.3713	28.5	103	1.031656
8	0.164	1.3713	28.2	103	1.031653
9	0.164	1.3713	28.1	103	1.031652
10	0.164	1.3713	27.8	103	1.031649
11	0.164	1.3713	28.1	103	1.031652
12	0.164	1.3713	28.0	103	1.031651
13	0.164	1.3713	27.9	103	1.031650
14	0.164	1.3713	28.0	103	1.031651
15	0.164	1.3713	27.8	103	1.031649
16	0.164	1.3713	28.2	103	1.031653
17	0.164	1.3713	28.2	103	1.031653
18	0.164	1.3713	28.3	103	1.031654
19	0.164	1.3713	28.4	103	1.031655
20	0.164	1.3713	28.5	103	1.031656

For first run



Graph no: - 01 Pressure head vs. Distance



Graph no: - 02 Total head vs. Distance

4.2 Computation Of Total Head:

Run No. 02

Discharge calculations:-

For head = 34.3 × 10^{-2} m

The bucket used for discharge calculation is = 9350 cc

For first run the time taken to fill the bucket is = 14.35 seconds

Discharge (Q) = $\frac{Q_i}{A_i}$ = 9350 ÷ 14.35 × $10^6 m^3$/s = 6.5156 × $10^{-4} m^3$/s

Mean velocity = Discharge ÷ Crossecsional area

= 6.5156 × $10^{-4} m^3$/s ÷ (6 × 5 × 10^{-4}) m^2

= 0.21718 m/s

Velocity head =$(Meanvelocity)^2$ ÷ (2 × g)

= $(0.21718\ m/s)^2$ ÷ (2 × 9.81m/s^2)

= 2.4042 × 10^{-3} m

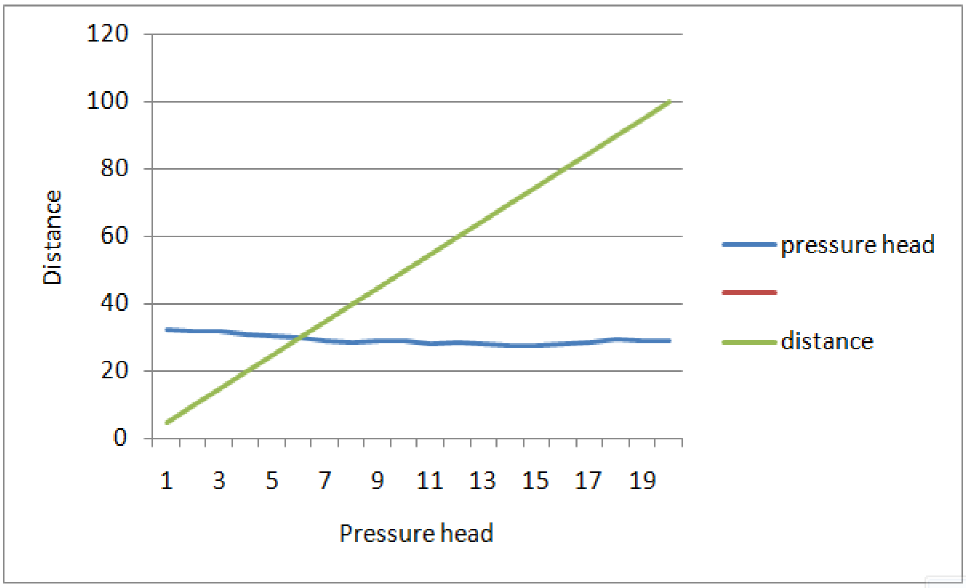
Pressure head = p/ρg = 32.5 × 10^{-2} m of water ÷ (1000 ×9.81N)

= 32.5 × 10^{-6} m

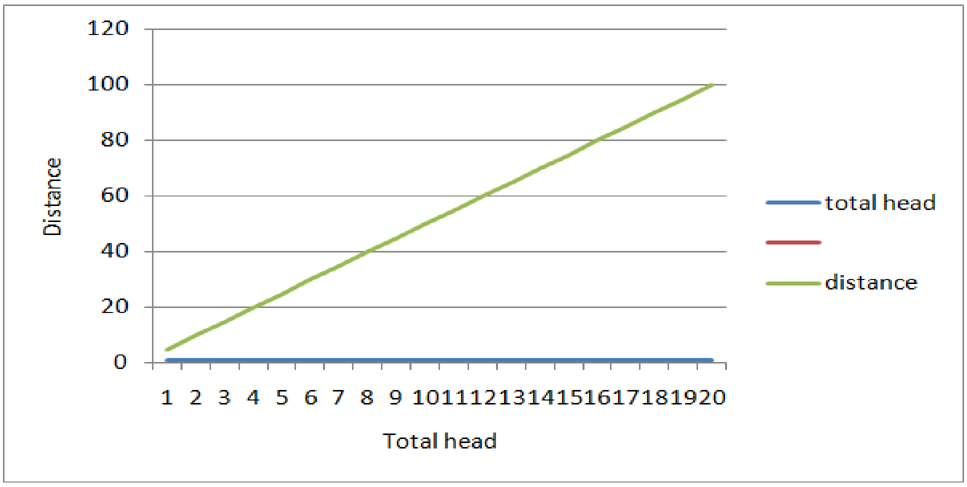
Table no: - 04

Piezometer No., i	Mean velocity, $V_i = Q/A_i$ (m/s)	Velocity head, $V_i^2/2g \times 10^{-3}$ m	Pressure head $(p/\rho g)\times10^{-6}$ m	Datum head $y\times 10^{-1}$ m
1	0.21718	2.4042	32.5	103
2	0.21718	2.4042	32.0	103
3	0.21718	2.4042	31.8	103
4	0.21718	2.4042	30.8	103
5	0.21718	2.4042	30.3	103
6	0.21718	2.4042	30.1	103
7	0.21718	2.4042	29.3	103
8	0.21718	2.4042	28.8	103
9	0.21718	2.4042	28.9	103
10	0.21718	2.4042	29.0	103
11	0.21718	2.4042	28.7	103
12	0.21718	2.4042	28.6	103
13	0.21718	2.4042	28.0	103
14	0.21718	2.4042	27.7	103
15	0.21718	2.4042	27.6	103
16	0.21718	2.4042	28.0	103
17	0.21718	2.4042	28.8	103
18	0.21718	2.4042	29.4	103
19	0.21718	2.4042	29.3	103
20	0.21718	2.4042	29.1	103

For second run



Graph no: - 03 Pressure head vs. Distance



Graph no: - 04 Total head vs. Distance

4.3 Computation Of Total Head:

Run No. 03

Discharge calculations:-

For head = 45 × 10^{-2} m

The bucket used for discharge calculation is = 9350 cc

For first run the time taken to fill the bucket is = 15.6 seconds

Discharge (Q) $=\frac{Q_i}{A_i}$= 9350 ÷ 15.6 × $10^6 m^3$/s = 5.9935 × $10^{-4} m^3$/s

Mean velocity = Discharge ÷ Crossecsional area

= 5.9935 × $10^{-4} m^3$/s ÷ (6 × 5 × 10^{-4}) m^2

= 0.19978 m/s

Velocity head =$(Meanvelocity)^2$ ÷ (2 × g)

= (0.19978 $m/s)^2$ ÷ (2 × 9.81m/s^2)

= 2.034 × 10^{-3} m

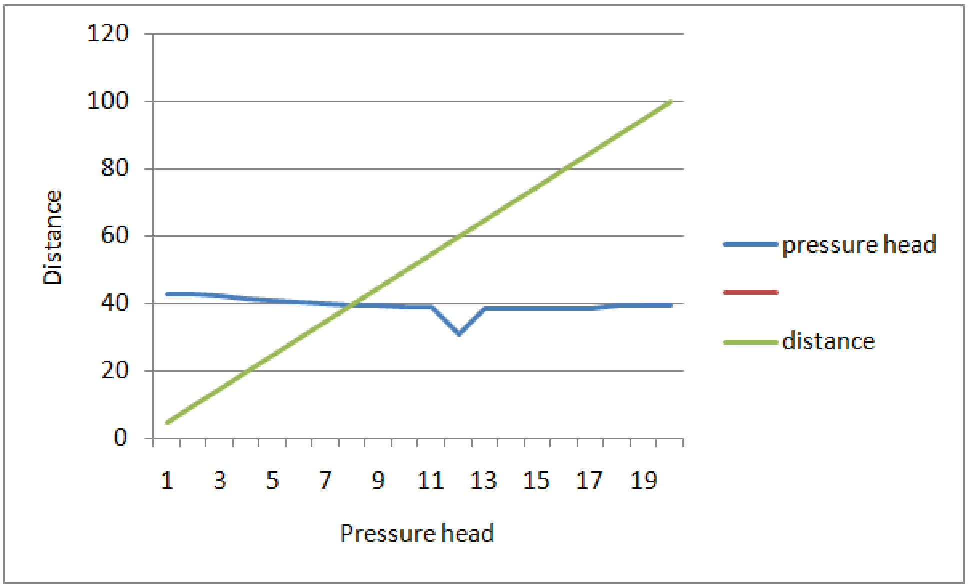
Pressure head = p/ρg = 42.9 × 10^{-2} m of water ÷ (1000 ×9.81N)

= 42.9 × 10^{-6} m

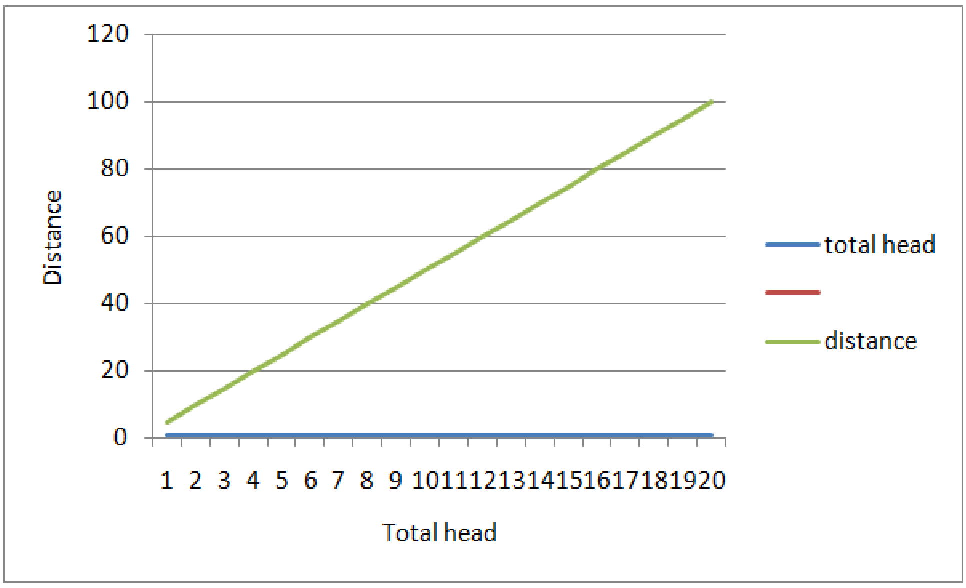
Table no: - 05

Piezometer No., i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Mean velocity, Vi = Q/Ai (m/s)	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978
Velocity head, $Vi^2/2g \times 10^{-3}$ m	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425	2.03425
Pressure head $(p/\rho g) \times 10^{-6}$ m	42.9	42.8	42.6	41.7	41.0	40.7	40.1	39.4	39.3	39.2	39.2	30.9	38.8	38.6	38.8	38.6	38.5	39.6	39.3	39.5
Datum head $y \times 10^{-2}$ m	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103	103

For third run



Graph no: - 05 Pressure head vs. Distance



Graph no: - 06 Total head vs. Distance

4.4 COMPUTATION OF TOTAL HEAD:

Run No. 04

Discharge calculations:-

For head = 46 × 10^{-2} m

The bucket used for discharge calculation is = 9350 cc

For first run the time taken to fill the bucket is = 17.22 seconds

Discharge (Q) = $\frac{Q_i}{A_i}$= 9350 ÷ 17.22 × $10^6 m^3$/s = 5.429 × $10^{-4} m^3$/s

Mean velocity = Discharge ÷ Crossecsional area

= 5.429 × $10^{-4} m^3$/s ÷ (6 × 5 × 10^{-4}) m^2

= 0.18096 m/s

Velocity head =$(Meanvelocity)^2$ ÷ (2 × g)

= $(0.18096\ m/s)^2$ ÷ (2 × 9.81m/s^2)

= 1.669 × 10^{-3} m

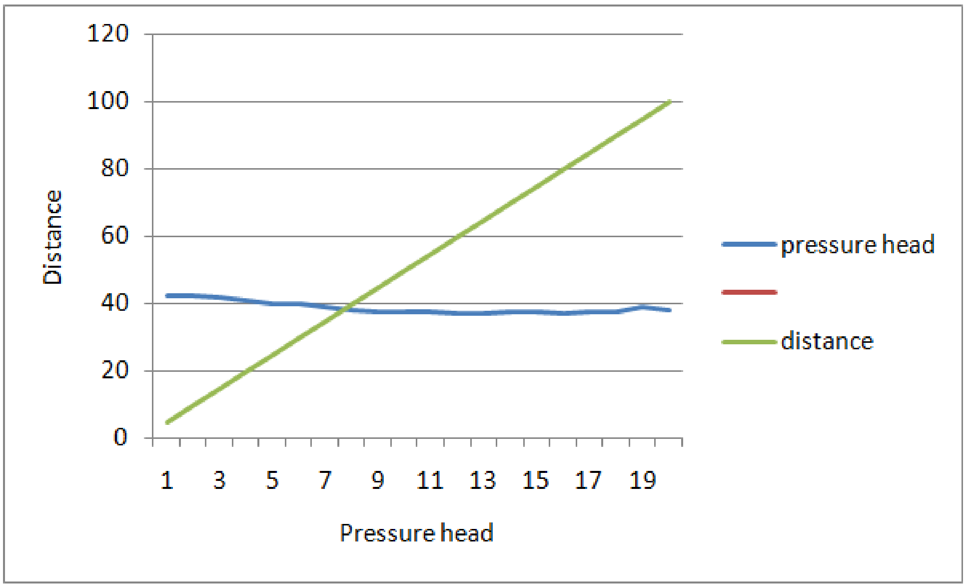
Pressure head = p/ρg = 42.3 × 10^{-2} m of water ÷ (1000 ×9.81N)

= 42.3 × 10^{-6} m

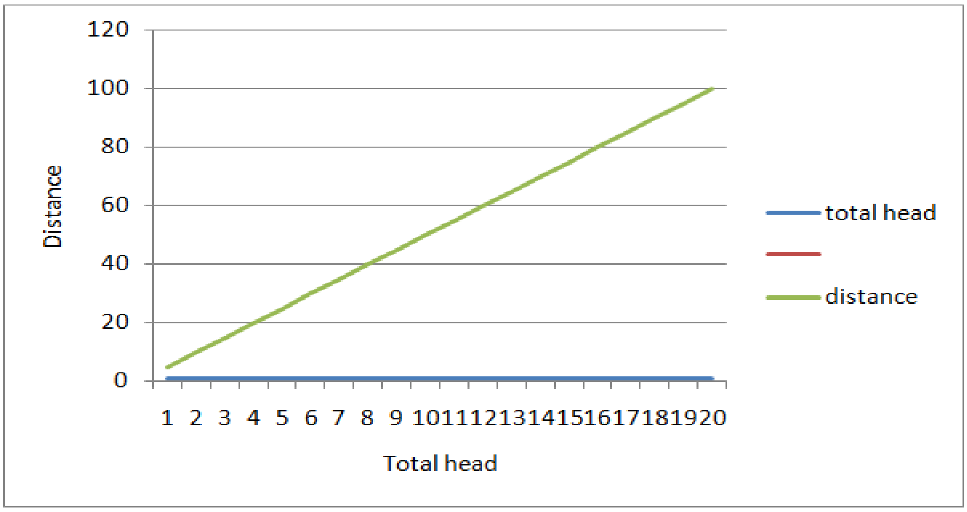
Table no: - 06

Piezometer No., i	Mean velocity, Vi = Q/Ai (m/s)	Velocity head, $Vi^2/2g \times 10^{-3}$ m	Pressure head $(p/\rho g) \times 10^{-6}$ m	Datum head $y \times 10^{-2}$ m	Total head, $H = p/\rho g + \frac{Vi2}{2g} + y$ in meters
1	0.18096	1.669	42.3	103	1.032092
2	0.18096	1.669	42.1	103	1.032090
3	0.18096	1.669	41.9	103	1.032088
4	0.18096	1.669	40.7	103	1.032076
5	0.18096	1.669	40.0	103	1.032069
6	0.18096	1.669	39.7	103	1.032066
7	0.18096	1.669	39.0	103	1.032059
8	0.18096	1.669	38.0	103	1.032049
9	0.18096	1.669	37,8	103	1.032047
10	0.18096	1.669	37.6	103	1.032045
11	0.18096	1.669	37.4	103	1.032043
12	0.18096	1.669	37.2	103	1.032041
13	0.18096	1.669	37.3	103	1.032042
14	0.18096	1.669	37.4	103	1.032043
15	0.18096	1.669	37.4	103	1.032043
16	0.18096	1.669	37.1	103	1.032040
17	0.18096	1.669	37.4	103	1.032043
18	0.18096	1.669	37.7	103	1.032046
19	0.18096	1.669	38.8	103	1.032057
20	0.18096	1.669	38.2	103	1.032051

For forth run



Graph no: - 07 Pressure head vs. Distance



Graph no: - 08 Total head vs. Distance

4.5 Computation Of Total Head:

Run No. 5

Discharge calculations:-

For head = 48 × 10^{-2} m

The bucket used for discharge calculation is = 9350 cc

For first run the time taken to fill the bucket is = 18.7 seconds

Discharge (Q) = $\frac{Q_i}{A_i}$ = 9350 ÷ 18.7 × $10^6 m^3$/s = 5.000 × $10^{-4} m^3$/s

Mean velocity = Discharge ÷ Crossecsional area

= 5.000 × $10^{-4} m^3$/s ÷ (6 × 5 × 10^{-4}) m^2

= 0.16666 m/s

Velocity head = $(Meanvelocity)^2$ ÷ (2 × g)

= $(0.16666\ m/s)^2$ ÷ (2 × 9.81m/s^2)

= 1.4157 × 10^{-3} m

Pressure head = p/ρg = 46.00 × 10^{-2} m of water ÷ (1000 ×9.81N)

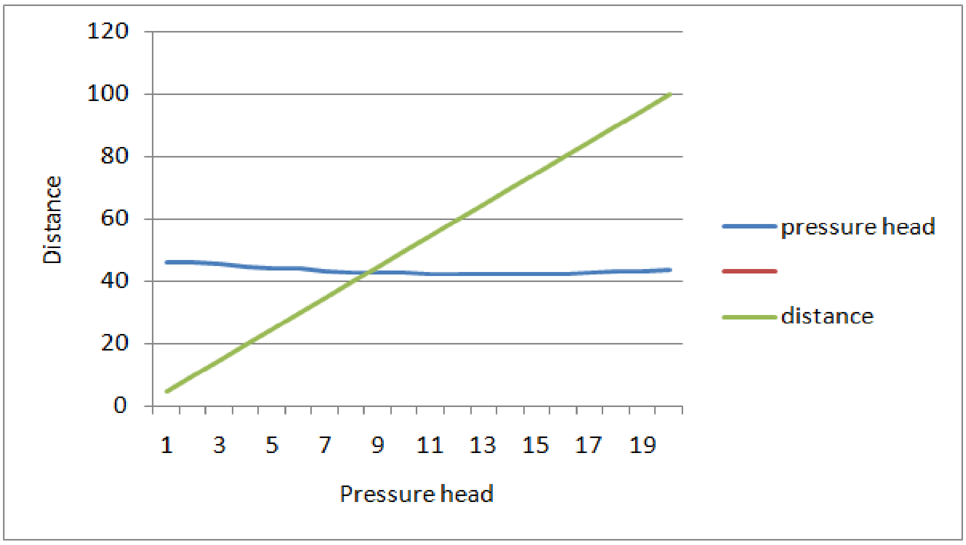
= 42.00 × 10^{-6} m

Table no: - 07

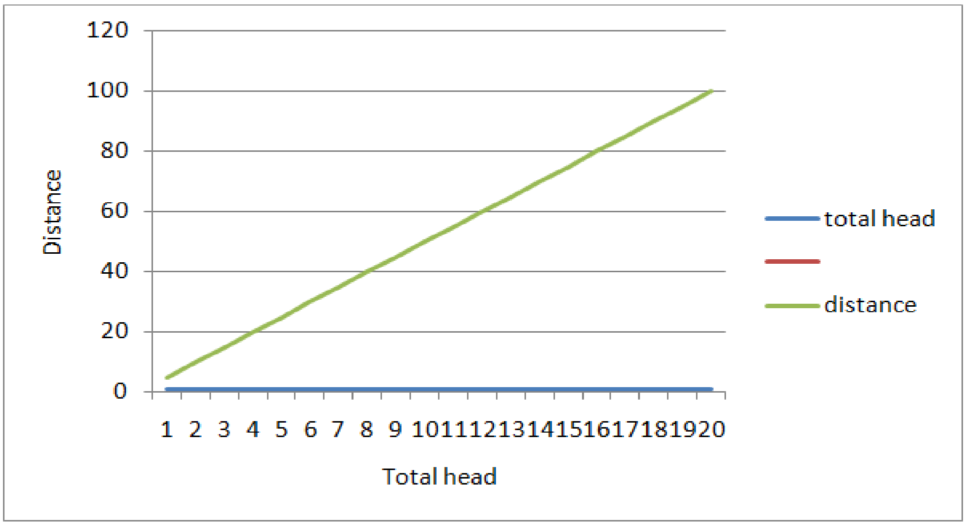
Piezometer No., i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Mean velocity, Vi = Q/Ai (m/s)	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666	0.16666
Velocity head, $Vi^2/2g \times 10^{-3}$ m	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157	1.4157
Pressure head $(p/\rho g) \times 10^{-6}$ m	46.00	45.9	45.7	44.8	44.2	44.0	43.3	42.9	42.8	42.6	42.4	42.3	42.4	42.3	42.4	42.2	42.8	43.1	43.0	43.4

Total head, $H = p/\rho g + \frac{Vi2}{2g} + y$ in meters	Datum head $y \times 10^{-2}$ m
1.031875	103
1.031874	103
1.031872	103
1.031863	103
1.031857	103
1.031855	103
1.031848	103
1.031844	103
1.031843	103
1.031841	103
1.031839	103
1.031838	103
1.031839	103
1.031838	103
1.031839	103
1.031837	103
1.031843	103
1.031846	103
1.031845	103
1.031849	103

For fifth run



Graph no: - 09 Pressure head vs. Distance



Graph no: - 10 Total head vs. Distance

4.6 Computation Of Total Head:

Run No. 06

Discharge calculations:-

For head = 54 × 10^{-2} m

The bucket used for discharge calculation is = 9350 cc

For first run the time taken to fill the bucket is = 17.7 seconds

Discharge (Q) $= \frac{Q_i}{A_i}$ = $9350 \div 17.7 \times 10^6 m^3/s$ = $5.2824 \times 10^{-4} m^3/s$

Mean velocity = Discharge ÷ Crossecsional area

$= 5.2824 \times 10^{-4} m^3/s \div (6 \times 5 \times 10^{-4})\ m^2$

$= 0.17608$ m/s

Velocity head = $(\text{Mean velocity})^2 \div (2 \times g)$

$= (0.17608\ m/s)^2 \div (2 \times 9.81 m/s^2)$

$= 1.5802 \times 10^{-3}$ m

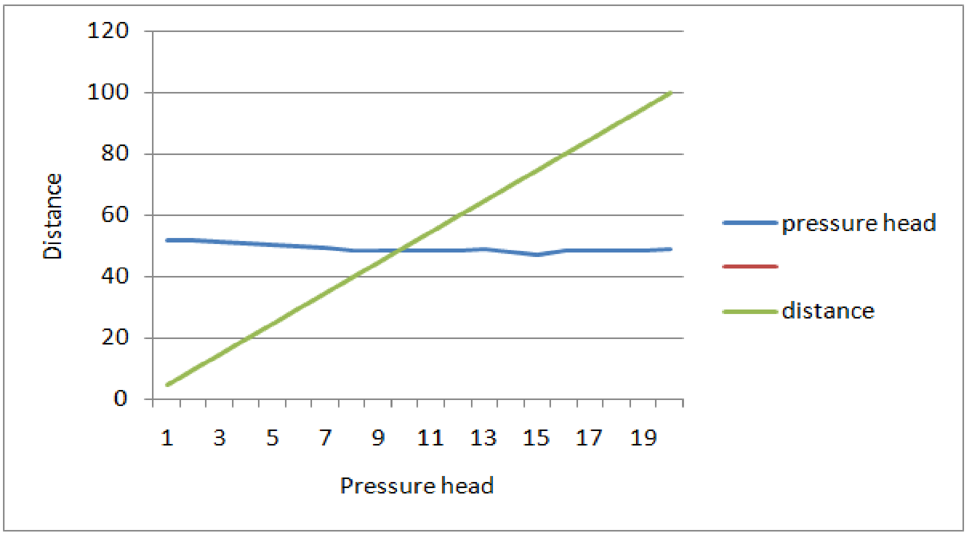
Pressure head = $p/\rho g$ = 51.80×10^{-2} m of water ÷ (1000 ×9.81N)

$= 51.80 \times 10^{-6}$ m

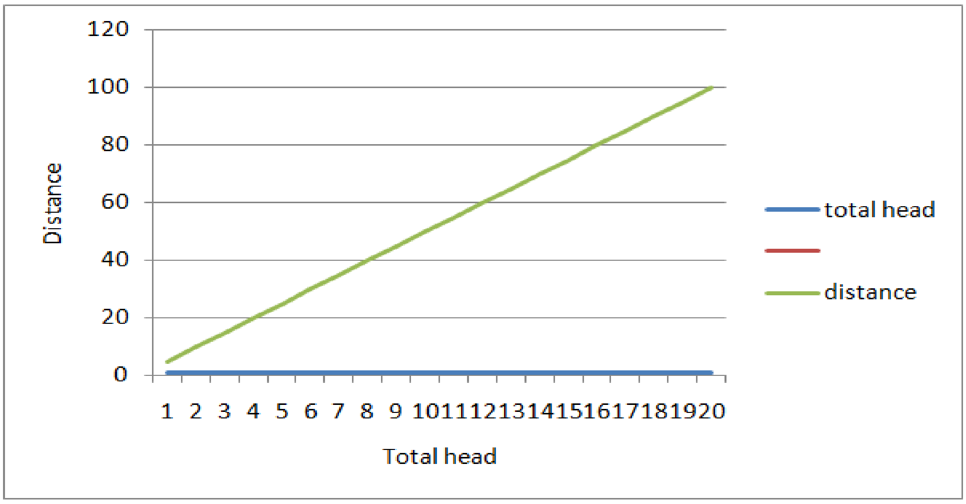
Table no: - 08

Piezometer No., i	Mean velocity, $V_i = Q/A_i$ (m/s)	Velocity head, $V_i^2/2g \times 10^{-3}$ m	Pressure head $(p/\rho g) \times 10^{-6}$ m	Datum head $y \times 10^{-2}$ m	Total head, $H = \frac{p}{\rho g} + \frac{V_i^2}{2g} + y$ in meters
1	0.17608	1.5802	51.8	103	1.032098
2	0.17608	1.5802	51.7	103	1.032097
3	0.17608	1.5802	51.5	103	1.032095
4	0.17608	1.5802	50.7	103	1.032087
5	0.17608	1.5802	50.2	103	1.032082
6	0.17608	1.5802	49.8	103	1.032078
7	0.17608	1.5802	49.3	103	1.032073
8	0.17608	1.5802	48.6	103	1.032066
9	0.17608	1.5802	48.5	103	1.032065
10	0.17608	1.5802	48.6	103	1.032066
11	0.17608	1.5802	48.4	103	1.032064
12	0.17608	1.5802	48.4	103	1.032064
13	0.17608	1.5802	48.9	103	1.032069
14	0.17608	1.5802	48.0	103	1.032060
15	0.17608	1.5802	47.2	103	1.032052
16	0.17608	1.5802	48.6	103	1.032066
17	0.17608	1.5802	48.6	103	1.032066
18	0.17608	1.5802	48.7	103	1.032067
19	0.17608	1.5802	48.8	103	1.032068
20	0.17608	1.5802	49.0	103	1.032070

For sixth run



Graph no: - 11 Pressure head vs. Distance



Graph no: - 12 Total head vs. Distance

V. Results And Discussions

Total head, H = $p/\rho g + \frac{Vi2}{2g} + y$ in meters	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Run 01	1.031670	1.031674	1.031674	1.031667	1.031664	1.031663	1.031656	1.031653	1.031652	1.031649	1.031652	1.031651	1.031650	1.031651	1.031649	1.031653	1.031653	1.031654	1.031655	1.031656
Run 02	1.032729	1.032724	1.032722	1.032712	1.032707	1.032705	1.032697	1.032692	1.032693	1.032694	1.032684	1.032690	1.032684	1.032681	1.032686	1.032684	1.032692	1.032698	1.032697	1.032695
Run 03	1.032463	1.032462	1.032460	1.032451	1.032444	1.032441	1.032435	1.032428	1.032427	1.032426	1.032426	1.032343	1.032422	1.032420	1.032422	1.032420	1.032419	1.032430	1.032427	1.032429
Run 04	1.032092	1.032090	1.032088	1.032076	1.032069	1.032066	1.032059	1.032049	1.032047	1.032045	1.032043	1.032041	1.032042	1.032043	1.032043	1.032040	1.032043	1.032046	1.032057	1.032051
Run 05	1.031875	1.031874	1.031872	1.031863	1.031857	1.031855	1.031848	1.031844	1.031843	1.031841	1.031839	1.031838	1.031839	1.031838	1.031839	1.031837	1.031843	1.031846	1.031845	1.031849
Run 06	1.032098	1.032097	1.032095	1.032087	1.032082	1.032078	1.032073	1.032066	1.032065	1.032066	1.032064	1.032064	1.032069	1.032060	1.032052	1.032066	1.032066	1.032067	1.032068	1.032070

Table no: - 09

5.1 Conclusion from the graphs :

Finally Graphs show that the main conclusion of Bernoulli's Theorem as the total head remains constant. In all the above graphs it is verified that the total head always remains constant with Bernoulli's apparatus used by me.

For all the runs the Bernoulli's Theorem is verified and the total head remains same up to five decimal places, it is checked with [Table No 07]. It verifies the Bernoulli's principle that the total head remains constant throughout the duct length.

As for Example - [From Table No: - 02]
For [1-2] piezometer the difference is = 0.000004 m
For [2-3] piezometer the difference is =0.000000 m
For [3-4] piezometer the difference is =0.000010 m
For [4-5] piezometer the difference is =0.000005 m
For [5-6] piezometer the difference is =0.000002 m
For [6-7] piezometer the difference is =0.000008 m
For [7-8] piezometer the difference is =0.000005 m
And so on……..

Since all real fluid has finite viscosity, i.e. in all actual fluid flows, some energy will be lost in overcoming friction. This is referred to as head loss, i.e. if the fluid were rise in a vertical pipe it will rise to a lower height than predicted by Bernoulli's equation. The head loss will cause the pressure to decrease in the flow direction. If the head loss is denoted by H_1 then Bernoulli's equation can be modified to:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + H_1$$

Figure [5.1] shows the variation of total, static and velocity pressure for steady, incompressible fluid flow through a pipe of uniform cross-section without viscous effects and with viscous effects.

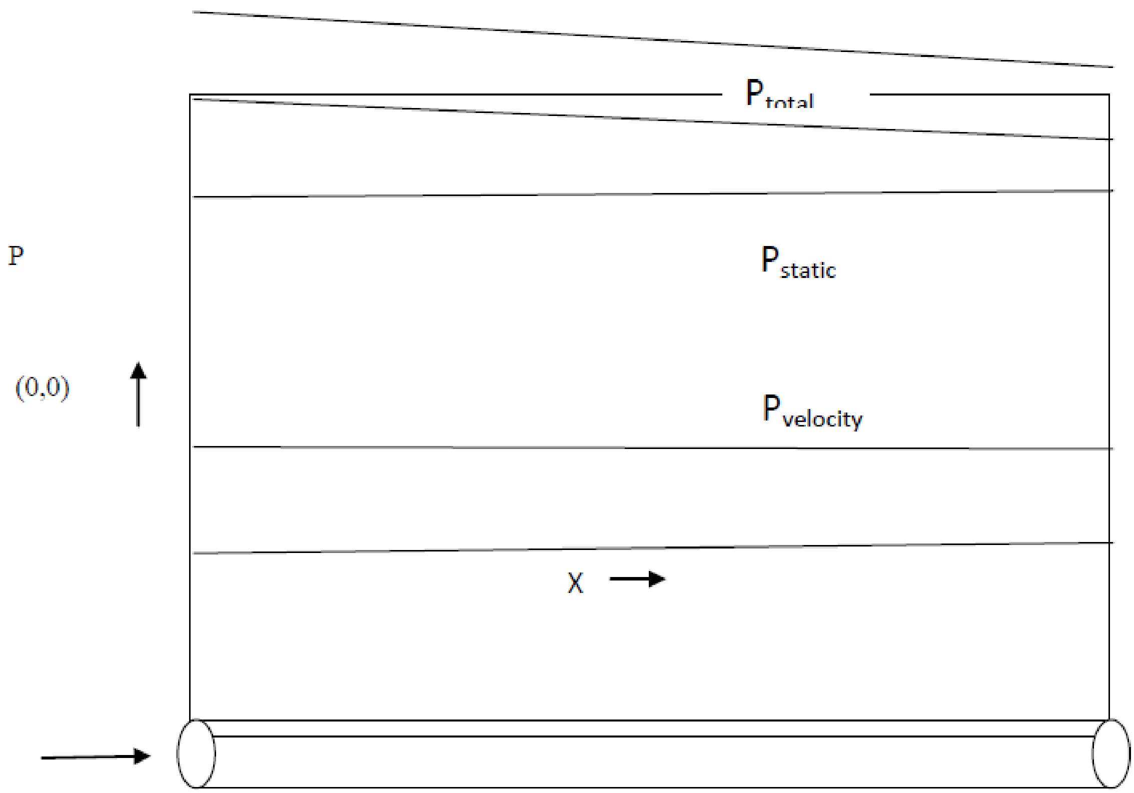


Figure [5.1] variation of total, static and velocity pressure for steady Incompressible fluid flow

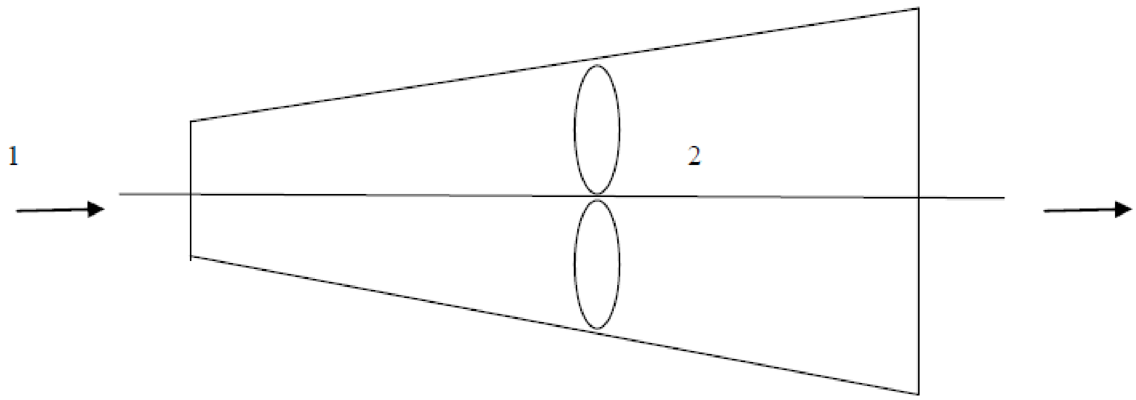


Figure [5.2] Fluid flow behavior through fan

Since the total pressure reduces in the direction of flow, sometimes it becomes necessary to use a pump or a fan to maintain the fluid flows as shown in Fig. [5.2]

Energy is added to the fluid when fan of pump is used in the fluid flow conduit then the modified Bernoulli's equation is written as:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + H_p = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + H_1$$

Where $H_p$is the gain in head due to fan of pump and H_1 is the loss in head due to friction. When fan of pump is used, the power required (W) of drive the fan/pump is given by:

$$W = \left(\frac{m}{\eta_{fan}}\right)\left(\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g\ (z_2 - z_1 + \frac{gH_1}{\rho}\right)$$

Where m is the mass flow rate of the fluid and η_{fan} is the energy efficiency of the fan/pump. Some of the terms in the above equation can be negligibly small, for example, for sir flow the potential energy term g $(z_1 - z_2)$ is quite small compared to the other terms. For liquids, the kinetic energy term $V_2^2 - V_1^2$ is relatively small. If there is no fan of pump then W is zero.

Pressure loss during fluid flow: [11]

The loss in pressure during fluid flow is due to:

a) Fluid friction and turbulence
b) Change in fluid flow cross sectional area, and
c) Abrupt change in the fluid flow direction

Normally pressure drop due to fluid friction is called as major loss of frictional pressure drop Δp_m. The total pressure drop is the summation of frictional pressure drop and major loss. In most of the situations, the temperature of the fluid does not change appreciably along the flow direction due to pressure drop. This is due to the fact that the temperature tends to rise due to energy dissipation by fluid friction and turbulence, at the same time temperature tends to drop due to pressure drop. These two opposing effects more or less cancel each other and hence the temperature remains almost constant (assuming no heat transfer to or from the surroundings).

VI. Scope For Future Work

SCOPE FOR FUTURE WORK

- Apparatus can be used for Homogeneous solutions.
- In place of simple water heterogeneous solution may also be used for verifying
- Bernoulli's Theorem with this apparatus.
- Effect of alkalinity can be evaluated with this apparatus.
- Effect of basicity can be evaluated with this apparatus.
- The same apparatus may be used for other types of fluids (Thyxotropic, Dilatants
- etc.) to check the validity of Bernoulli's theorem.
- This apparatus can be used to check the effect of pH of Fluid on Bernoulli's
- Theorem.
- By changing constant area of duct by variable area of duct the Bernoulli's Principle can also be verified easily.

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