

Chemical Reaction on Heat and Mass Transfer Flow through an Infinite Inclined Plate with Soret and Dufour Effects in Porous Medium

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Abstract : The numerical studies are performed to examine the mass transfer flow with thermal diffusion and diffusion thermo effect past an infinite, inclined vertical plate in a porous medium in the presence of chemical reaction. First of all, the governing equations are transformed to a system of dimensionless coupled partial equations. Explicit finite difference method has been used to solve these dimensionless equations for momentum, concentration and energy equations. During the course of discussion, it is found that various parameters related to the problem influence the calculated result. Finally, the profiles of velocity, concentration and temperature are analyzed and illustrated with graphs.

Keywords: Chemical Reaction, Dufour effect, Finite Difference Method, Inclined infinite plate, Soret effect.

I. Introduction

Effect of heat and mass transfer on fluid has become important through the last few years due to a number of industrial processes and engineering applications which experiences not only temperature difference but also concentration difference. In many mass transfer processes, heat transfer considerations arise owing to chemical reaction and are often due to the nature of the process. In processes such as drying, evaporation at the surface water body, energy transfer in a water cooling tower and the flow in a desert cooler heat and mass transfer occur simultaneously. In many of these processes, the determination of the total energy transfer, natural convection processes involving the convection along with inclined plate has received less attention than the cases of vertical and horizontal plates. But natural convection processes involving the combined mechanism are encountered in many natural processes, in various industrial applications involving solutions and mixtures in an infinitely large inclined surface in absence of an externally induced flow and in different chemical processing systems.

Sparrow and Cess [1] studied the free convection fluid flow past a semi infinite vertical plate in two dimensions where buoyancy and magnetic forces acted simultaneously. The study of natural convection flow of inclined plate is presented by the authors Ganesan and Palani [2, 3] and Sparrow and Husar [4]. Dufour and Soret effects on steady MHD free convection and mass transfer fluid flow through a porous medium in a rotating system have been investigated by N. Islam and M. Alam [5].

Motivated by all these studies, we have had our research on the result of chemical reaction on unsteady boundary layer flow with Soret and Dufour effect in a porous medium past through an inclined infinite plate. To solve the problem numerically, governing partial differential equation has been transformed into non-similar coupled partial differential equation by usual transformation. These equations has been solved numerically by FORTRAN-95 and effects of parameters such as Dufour number, Soret number and chemical reaction parameter on velocity, temperature and concentration profile have been illustrated by graphs.

II. Mathematical Formulation

An unsteady, Two dimensional, laminar boundary layer flow of an incompressible, viscous, radiating Boussinesq fluid near impermeable inclined vertical plate stretching with velocity U , temperature distribution T_w and concentration distribution C_w is considered, in the presence of thermal diffusion (Soret) and diffusion-

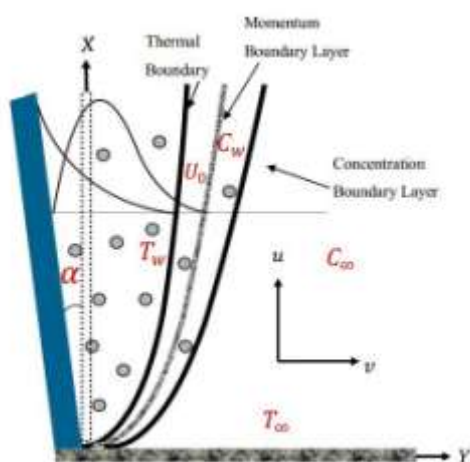


Fig.1. Physical configuration

thermo (Dufour) effects. The positive X coordinate is measured along the plate in the direction of fluid motion and the positive Y coordinate is measured normal to the plate. Now, under the usual Boussinesq's and boundary layer approximations, the governing equations for the flow field under consideration are

Continuity equation

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation in X -direction

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g\beta_C(C - C_\infty)\cos\alpha + g\beta_T(T - T_\infty)\cos\alpha + \nu \frac{\partial^2 u}{\partial y^2} - \frac{g}{k'}u \tag{2}$$

Energy equation (neglecting viscous dissipation and Joule heating term)

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + D_m \frac{\partial^2 C}{\partial y^2} \tag{3}$$

Concentration equation

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D_m \left(\frac{\partial^2 C}{\partial y^2} \right) + D_T \left(\frac{\partial^2 T}{\partial y^2} \right) - K_c(C - C_\infty) \tag{4}$$

With corresponding boundary conditions

$$u = U(X), T = T_w, C = C_w \text{ at } y = 0$$

$$(5) u = 0, T = T_\infty, C = C_\infty \text{ at } y = \infty \tag{6}$$

where v is the y components of velocity vector, ν is the kinematic coefficient viscosity, ρ is the density of the fluid, C_p is the specific heat at the constant pressure, K_c is the rate of chemical reaction and D_m is the coefficient of mass diffusivity, K is the thermal diffusion ratio.

To obtain the governing equations and the boundary condition in dimension less form, the following non-dimensional quantities are introduced as;

$$Y = \frac{yU_\infty}{\nu}, U = \frac{u}{U_\infty}, V = \frac{v}{U_\infty}, \tau = \frac{tU_\infty^2}{\nu}, \bar{T} = \frac{T - T_\infty}{T_w - T_\infty}, \bar{C} = \frac{C - C_\infty}{C_w - C_\infty}$$

Substituting the above relations in equations (1)-(4) and corresponding boundary conditions (5) and (6) are;

Momentum equation

$$\frac{\partial U}{\partial \tau} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + G_m \bar{T} \cos\alpha + G_m \bar{C} \cos\alpha - KU \tag{7}$$

Energy equation

$$\frac{\partial \bar{T}}{\partial \tau} + V \frac{\partial \bar{T}}{\partial Y} = \frac{1}{P_r} \frac{\partial^2 \bar{T}}{\partial Y^2} + D_u \frac{\partial^2 \bar{C}}{\partial Y^2} \tag{8}$$

Concentration equation

$$\frac{\partial \bar{C}}{\partial \tau} + V \frac{\partial \bar{C}}{\partial Y} = \frac{1}{S_c} \frac{\partial^2 \bar{C}}{\partial Y^2} + S_o \frac{\partial^2 \bar{T}}{\partial Y^2} - \gamma \bar{C} \tag{9}$$

Boundary conditions are

$$U = 1, \bar{T} = 1, \bar{C} = 1 \text{ at } y = 0 \tag{10}$$

$$U = 0, \bar{T} = 0, \bar{C} = 0 \text{ at } y = \infty \tag{11}$$

where τ represents the dimensionless time, Y is the dimensionless Cartesian coordinate, U and V are the dimensionless velocity component in X and Y direction, \bar{T} is the dimensionless temperature, \bar{C} is the dimensionless concentration, $G_r = \frac{g\beta(T-T_\infty)\nu}{U_\infty^3}$ (Grashof number), $G_m = \frac{g\beta^*(C-C_\infty)\nu}{U_\infty^3}$ (modified Grashof number), $P_r = \frac{\rho c_p \nu}{\kappa}$ (Prandtl number), $D_u = \frac{Dk_t(C_w - C_\infty)}{\nu c_s c_p (T_w - T_\infty)}$ (Dufour number), $S_c = \frac{\nu}{D}$ (Schmidt number), $S_o = \frac{Dk_t(T_w - T_\infty)}{\nu T_m (C - C_\infty)}$ (Soret Number) and $\gamma = \frac{K_c \nu (C - C_\infty)}{U_\infty^2}$ (Chemical reaction parameter).

III. Numerical Solution

To solve the non-dimensional system by implicit finite difference method, a set of finite difference equations is required. For this reason the area within the boundary layer is divided by some perpendicular lines of Y axis as shown in Fig.2. It is assumed that the maximum length of boundary layer is $Y_{Max} = 25$. i.e. Y varies from 0 to 25 and the number of grid spacing in Y directions is $n (=100)$ Hence the constant mesh size along Y -axis becomes $\Delta Y = 0.25$ ($0 \leq Y \leq 25$) with a smaller time step $\Delta t = 0.005$.

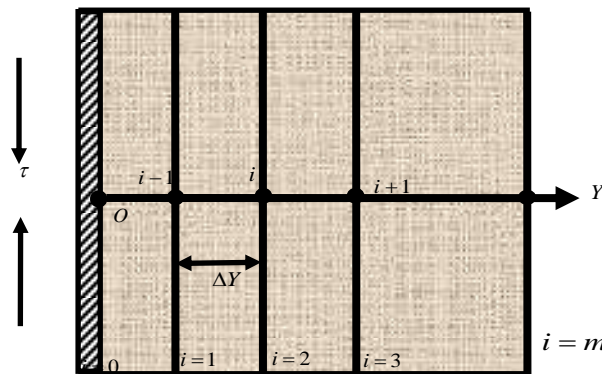


Fig.2. Implicit Finite Difference system grid

Let U' , \bar{T}' and \bar{C}' denotes the values of U , \bar{T} and \bar{C} at the end of the time-step respectively. Using implicit finite difference, the following appropriate set of finite difference equation is obtained as;

$$\frac{U'_i - U_i}{\Delta \tau} + V_i \left(\frac{U_{i+1} - U_i}{\Delta Y} \right) = G_m \bar{C}_i \cos \alpha + G_r \bar{T}_i \cos \alpha + \frac{U_{i+1} - 2U_i + U_{i-1}}{(\Delta Y)^2} \quad (12)$$

$$\frac{\bar{T}'_i - \bar{T}_i}{\Delta \tau} + V_i \frac{\bar{T}_{i+1} - \bar{T}_i}{\Delta Y} = \frac{1}{P_r} \frac{\bar{T}_{i+1} - 2\bar{T}_i + \bar{T}_{i-1}}{(\Delta Y)^2} + D_u \frac{\bar{C}_{i+1} - 2\bar{C}_i + \bar{C}_{i-1}}{(\Delta Y)^2} \quad (13)$$

$$\frac{\bar{C}'_i - \bar{C}_i}{\Delta \tau} + V_i \frac{\bar{C}_{i+1} - \bar{C}_i}{\Delta Y} = \frac{1}{S_c} \frac{\bar{C}_{i+1} - 2\bar{C}_i + \bar{C}_{i-1}}{(\Delta Y)^2} + S_o \frac{\bar{T}_{i+1} - 2\bar{T}_i + \bar{T}_{i-1}}{(\Delta Y)^2} - \gamma \bar{C}_i \quad (14)$$

with the finite difference boundary conditions

$$U_L = 1, \bar{T}_L = 1, \bar{C}_L = 1 \text{ at } L = 0$$

$$U_L = 0, \bar{T}_L = 0, \bar{C}_L = 0 \text{ at } L = \infty$$

Where subscripts i denotes Y and the superscript n represents a value of time, $\tau = n\Delta \tau$. Where $n = 0, 1, 2, 3, \dots$

The primary velocity U , temperature \bar{T} and concentration \bar{C} distributions at all interior nodal points may be computed by successive applications of the above finite difference equations.

IV. Results And Discussions

In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in Figs:3-10. For different values of inclination and permeability parameter velocity profile has been analyzed in Fig.3 and in Fig. 4 respectively. Temperature profile has been illustrated for different values of Dufour number, Prandtl number, Grashof number in Fig. 5.

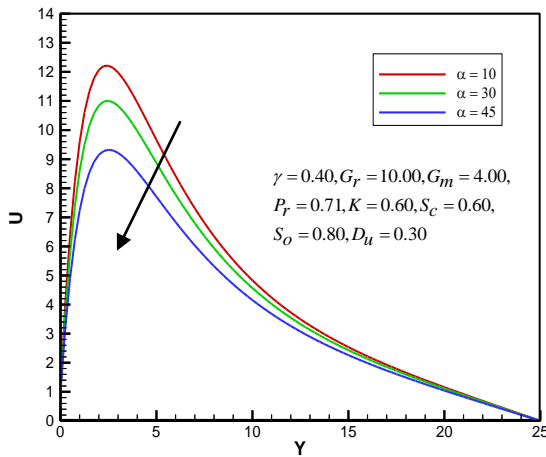


Fig: 3 Velocity profile for various values of inclination

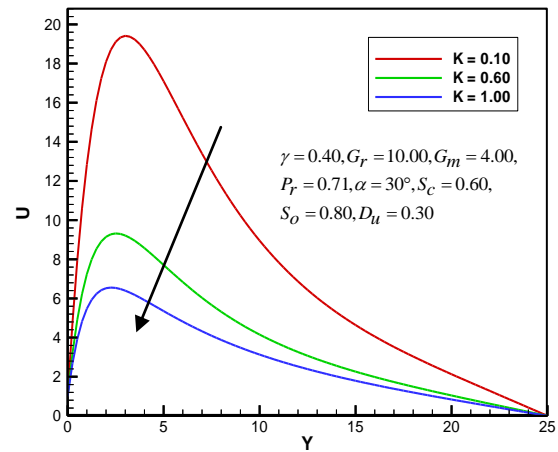


Fig: 4 Velocity profile for various values of K

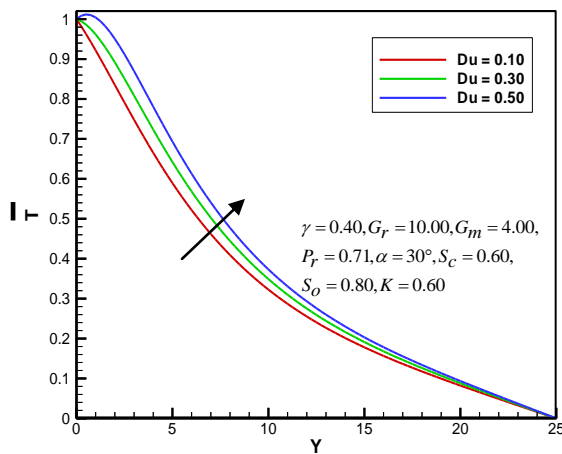


Fig: 5 Temperature profile for various values of D_u

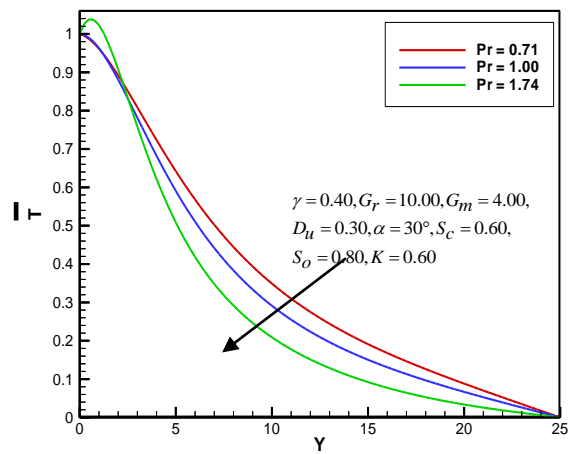


Fig: 6 Temperature profile for various values of P_r

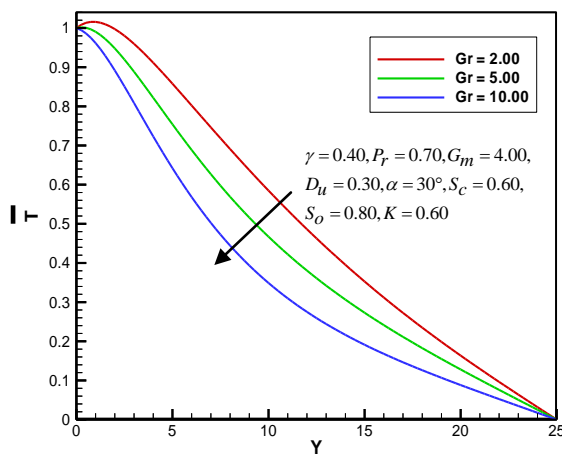


Fig: 7 Temperature profile for various values of G_r

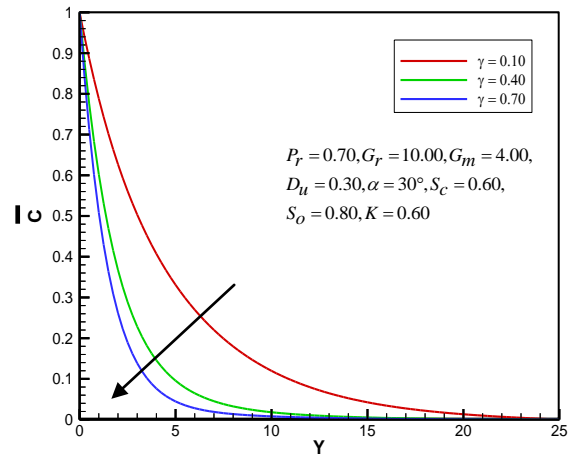


Fig: 8 Concentration profile for various values of γ

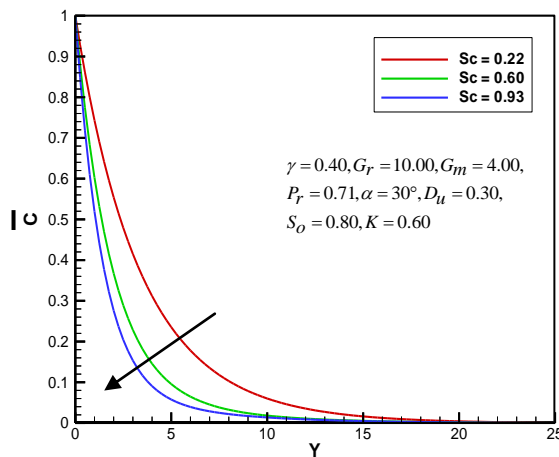


Fig: 9 Concentration profile for various values of S_c

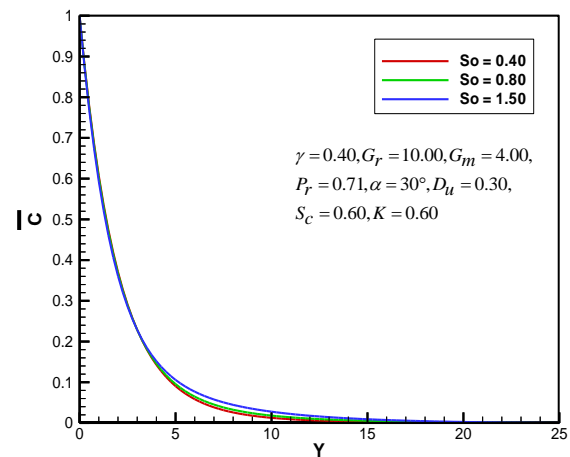


Fig: 10 Concentration profile for various values of S_o

V. Conclusions

In this research, the implicit finite difference solution of unsteady viscous, incompressible boundary layer flow of a fluid has been investigated. The physical properties are illustrated graphically for different values of corresponding parameters. Among them some important findings of this investigation are mentioned here;

1. Velocity profile shows decreasing effect with the increase of inclination (α) and permeability parameter (K).
2. Temperature profile shows decreasing effect with the increase of Prandtl number (P_r) and Grashof number (G_r) but increasing effect for Dufour number (D_u).
3. Concentration profile shows decreasing effect with the increase of chemical reaction parameter (γ) and Schmidt number (S_c) but effect is so minor to study in case of Soret number (S_o).

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