IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 13, Issue 1 Ver. III (Jan. - Feb. 2016), PP 01-11
www.iosrjournals.org

Contact Mechanics Studies for an Elliptical Curvature Deep Groove Ball Bearing Using Continuum Solid Modeling based on FEM Simulation Approach

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Abstract: *The present paper investigates the role of modifying bearing raceway curvatures to elliptical geometry from conventional circular geometry and its subsequent effect on performance characteristics of a deep Groove ball bearing. A continuum solid modeling based FEM approach has been used to analyze the performance characteristics of a Deep Groove ball bearing DGBB 6312 having raceway curvatures with elliptical geometry. The obtained simulation results have been compared with the analytical results for three different elliptical geometries, starting with "Normal Ellipse" type of curvatures for both inner and outer rings, followed by "Inverted Ellipse" type of curvature for outer ring and considering the conventional "Circular" type of curvature for inner rings. To be more precise in present analysis, the values of bearing Axial Play and Contact Angle have been calculated analytically for all three cases with the help of algebraic formulations for different values of Radial Clearance. Further, Finite element methodology has been used to obtain the values of contact stresses for all the three cases. The obtained results and other excerpts from this research provide grounds for a case involving the use of elliptical curvature Deep groove ball bearings based on their suitability, which in turn is subject to the corresponding application.*

Keywords*: Axial Play, Contact angle, Contact stress, DGBB, Normal & Inverted Ellipse, Truncation*

I. Introduction

The precision rolling-element bearing of twentieth century is a product of extracting technology and sophisticated science. New and improved design concepts have extended the range of applications and life of the rolling element bearings. Macro-geometry of bearing plays a vital role in enhancing the bearing performance. In the determination of stresses and deflections, the relative conformities of balls and rollers to their contacting raceways are of vital interest. The present study underlines new raceway geometry, precisely elliptical raceway geometry for the deep groove ball bearings. The purpose of the study is to understand how the newly conceived raceway geometry affects the axial play, contact angle and contact stresses under radial and axial loads. In order to better materialize the excerpts of this research, one needs to understand the macro geometry of deep groove ball bearing. The same can be looked up in comprehensive details in Essential Concepts of Bearing Technology [1]. All the formulations for the above mentioned. Above mentioned parameters play a significant role in controlling bearing life, stiffness and several other parameters [2].

Prior work has been done on this theory by Hans Flander in his patented work, Radial, Radial Angular-Contact, and Axial Angular-Contact Ball Bearing [3]. His work disclosed bearings with "Normal elliptical curvature". Further, Ishiguro et al., in his patent Single row deep grove radial ball bearing [4] disclosed a deep groove ball bearing with step curvatures where in the bearing curvatures were constant for a zone and changed by a fixed discrete amount as one moved from one zone to another azimuthally . Work has also been done by Ling Wang [5], wherein he employs an elliptical curvature for the sockets of a prosthetic hip joint. His paper validates how an ellipsoidal curvature can reduce wear and add to component life. Moving their work along, as bearing parameters depend significantly of contact angle, axial play & contact stresses, values of all these variables has been calculated and verified to understand their behavior based on several cases with changing radial clearance.

In the present study, three different elliptical geometries has been studied, starting with "Normal Ellipse" type of curvatures for both inner and outer rings, followed by "Inverted Ellipse" type of curvature for outer ring and considering the conventional "Circular" type of curvature for inner rings. This is based on the nature of contact between the interacting bodies, which is conformal for the outer raceway and non-conformal for inner raceway [6]. Subsequently, the curvature for inner raceway is kept lower than that of outer raceway. Axial play based on radial clearances has been derived in the following section for both inner and outer raceways. Unlike standard

Deep Groove ball bearings, elliptical curvature ball bearings have different axial play and contact angles for inner and outer raceways, pertaining to difference in curvatures.

The parameters of bearing design for a Deep Groove ball bearing with elliptical curvature have been formulated in the following section. The basics of an elliptical geometry have been mentioned below and will be recurrently used in the calculation that follows

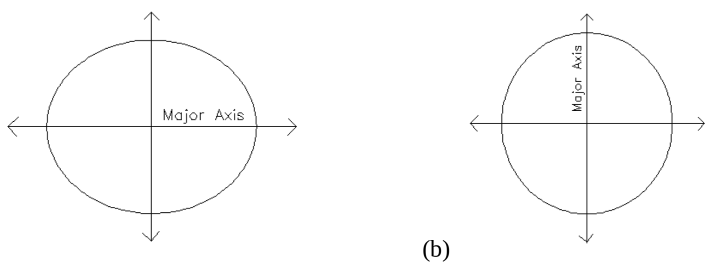
NOMENCLATURE

d	Nominal Ball Diameter
f_i	r_i/d
f_o	r_o/d
r_i	Radius of inner ring raceway
r_o	Radius of outer ring raceway
P_d	Bearing Radial Internal clearance (RIC)
P_e	Bearing axial play
Z	Number of rolling elements
R_N	Radius of Curvature of elliptical geometry
Φ, Φ’	Angle between line segment forming R_N and the vertical
F_a	Axial Load applied
F_r	Radial Load applied
X, Y	Cartesian Coordinates
α, α’	Angle between line segment forming R_N and the horizontal
e	Ellipse eccentricity
a	Major Axis of raceway ellipse
b	Minor Axis of raceway ellipse
o	Center of circle
o’	Center of circle

II. Mathematical Modeling

Contact angle and axial play have been calculated analytically using mathematical modeling of the previously defined cases. We have considered two cases in order to understand the Elliptical geometry.

In the present analysis, we would be referring the elliptical curvature where in the major axis is coincident with X axis as “Normal Ellipse” and the case in which the major axis is coincident with the Y axis of the Cartesian plane as “Inverted Ellipse”. The figure below highlights the basic geometrical parameters of an elliptical geometry which will be used recurrently in the consequent sections.



(a) (b)

Figure 1: (a) Normal & (b) Inverted Ellipse.

DGBB 6312 has been used as a reference Deep group ball bearing for present analysis.Fig.2 gives all the relevant boundary dimensions (mm). Radius of curvature determines the ball raceway curvature ratio in case of elliptical curvature.

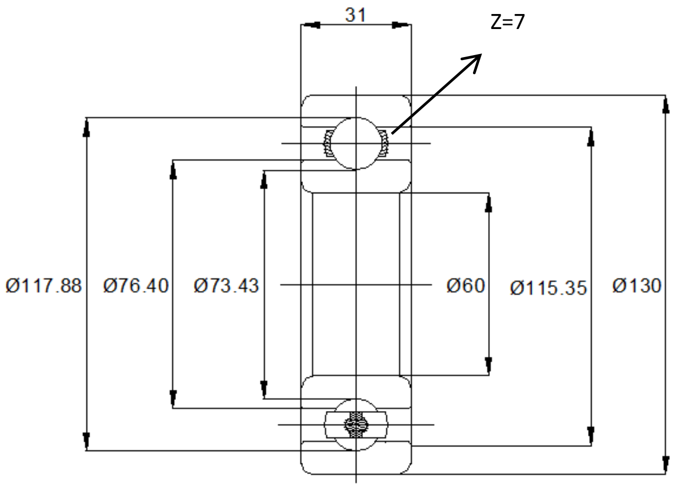


Figure 2: Boundary dimensions and number of rolling elements DGBB 6312

1. Radius of curvature

Radius of curvature is parameter of interest, as this parameter in essential in calculation of axial play. Complete elliptical & ellipsoidal geometry was studied in comprehensive details by referring to an appendix on ellipse geometry [7] Fig. 3 illustrates the tangent line to the ellipse at a point P. The line perpendicular to the point of tangency is also drawn and is labeled PQ. The distance from P to Q, the point of intersection with the y axis, defines the radius of curvature, R_N.

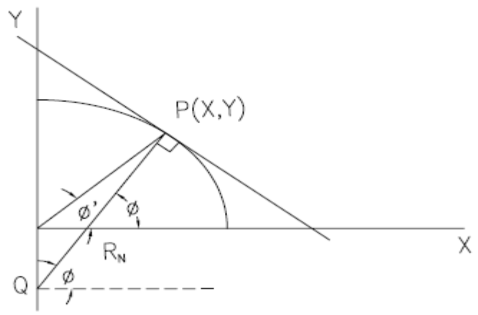


Figure 3: Radius of Curvature for Elliptical geometry

And the radius of curvature, R_N, is obtained from Equation (1)

$$R_N = \frac{a}{(1-(e\sin\varphi)^2)^{\frac{1}{2}}} \tag{1}$$

The Cartesian coordinates for R_N are obtained by

$$X = R_N \cos\varphi \tag{2}$$

$$Y = R_N (1 - e^2) \sin\varphi \tag{3}$$

1.1. Normal Ellipse

As evident from the Fig.4 above, the dimensions (mm) of the ellipse generated for major and minor ellipse is as follows:

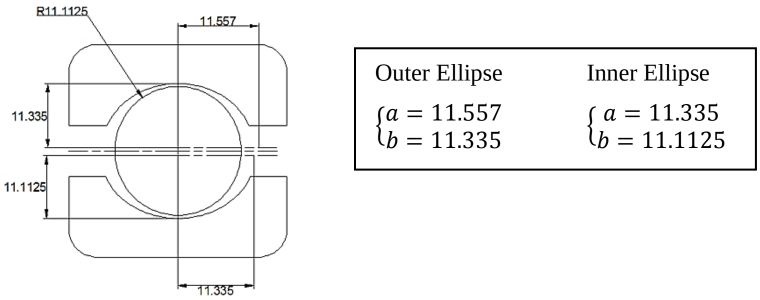


Figure 4: Defining Normal Ellipse parameters.

1.2. Inverted Ellipse

We have analyzed Inverted Ellipse stress and axial play using two different macro geometries. Dimensional (mm) inputs for both the cases have been explained below in Fig.5.

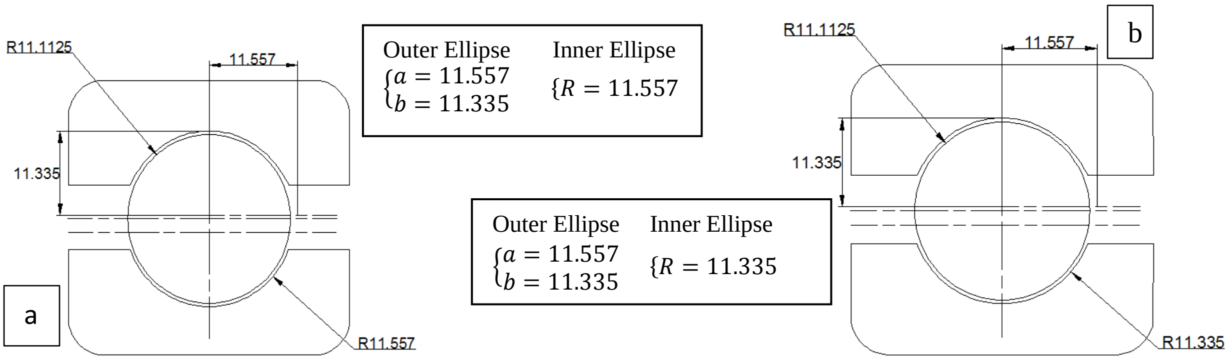


Figure 5: (a) Case 1 (b) Case 2 of defining Inverted Ellipse parameters.

These parameters have also been chosen based on the same logic so as to conform to the ISO guidelines [8]. For both types of macro geometries, we have calculated the axial play and contact stresses for varying values of Radial clearances. Different values of radial clearances have been taken with a synchronized step of 10 micrometers. However, while calculating the stress values for Inverted Ellipse, we observed that truncation occurs at values of radial clearance higher than a certain maximum value. Hence we have calculated the stress value for a comprehensive value lower than the maximum possible radial clearance value without truncation, in order to observe the values and define their significance.

Table 1: Inputs of radial internal clearance for calculations of axial play

Different Values of Radial Clearance used as Input			
Normal Ellipse		**Inverted Ellipse**	
RIC for Axial play (µm)	**RIC for Contact Stress (µm)**	**RIC for Axial play (µm)**	**RIC for Contact Stress (µm)**
20	20	20	
30	30	30	
40	40	40	4
50	50	50	
60	60	60	

For stress analysis, we have applied radial as well as axial load independently to observe stress values in both the cases, so as to understand the role that changing curvature plays in the determination of stresses. Fig.6 shows the load application.

Figure 6: Applied loads

We have taken a value of F_r =15000N and F_a = 1500N and calculated corresponding stress values through various iterations of static FEM.

2. Contact angle calculation

Axial Play in a Ball Bearing or end play is the maximum relative axial movement of the inner ring with respect to the outer ring. The end play is directly related to the radial play of the ball bearing for a conventional bearing curvature.

In the discussion that follows, we have been able to calculate the end play for both Normal and Inverted elliptical geometries for raceway curvature using the equations which were derived using complex mathematical formulations. The same has been shown below in Fig. 7

2.1. Macro geometry "Normal Ellipse" type of Curvature

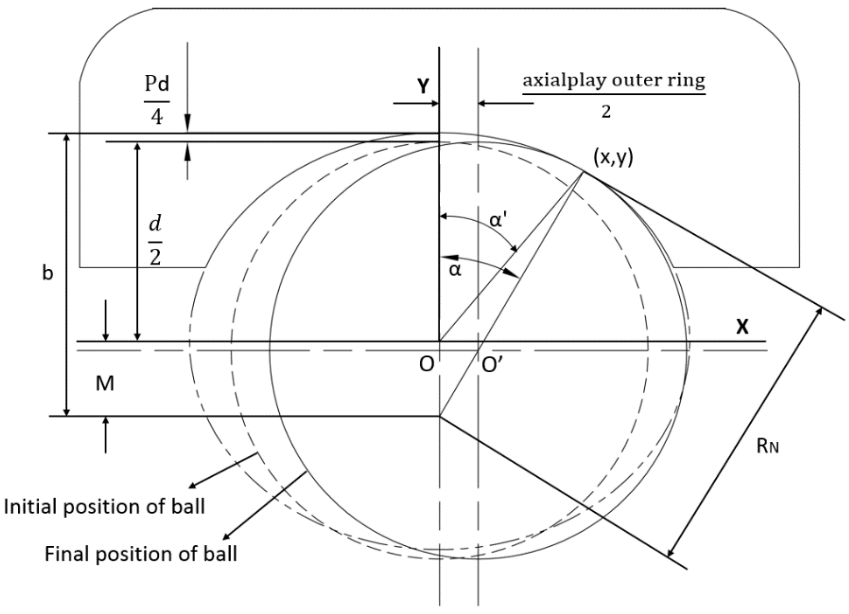


Figure 7: Internal geometry for derivation of axial play for "Normal Ellipse".

In Fig.7, the outer raceway with elliptical curvature has a radial clearance with respect to the rolling element. This will induce axial play in the system. Unlike the conventional circular curvature, the radius of curvature for an elliptical curvature varies azimuthally. Hence, in order to calculate axial play, a different approach has been used.

From Fig.7,

$$M = b - \frac{d}{2} - \frac{P_d}{4} \tag{4}$$

Also,

$$M = Y - \frac{d}{2}\sin(90 - \alpha) \tag{5}$$

From Equation (3),

$$b - \frac{d}{2} - \frac{P_d}{4} = R_N\,(1 - e^2)\sin(\,90 - \alpha) - \frac{d}{2}\sin(\,90 - \alpha) \tag{6}$$

From Equation (1),

$$b - \frac{d}{2} - \frac{P_d}{4} = \left[\frac{a}{(1-(e\cos\alpha)^2)^{\frac{1}{2}}}(1 - e^2)\cos(\alpha)\right] - \frac{d}{2}\cos\alpha \tag{7}$$

As the above equations (4-7) involve higher level mathematical complexities, MATLAB was used to calculate the values of α corresponding to different values of P_d. The same is tabulated in subsequent section.

2.2. Macro geometry "Inverted Ellipse" type of Curvature

Similar approach, as used above has been followed to calculate axial play for Inverted Ellipse type of curvature. Fig. 8 defines the macro geometry for Inverted Ellipse type of curvature.

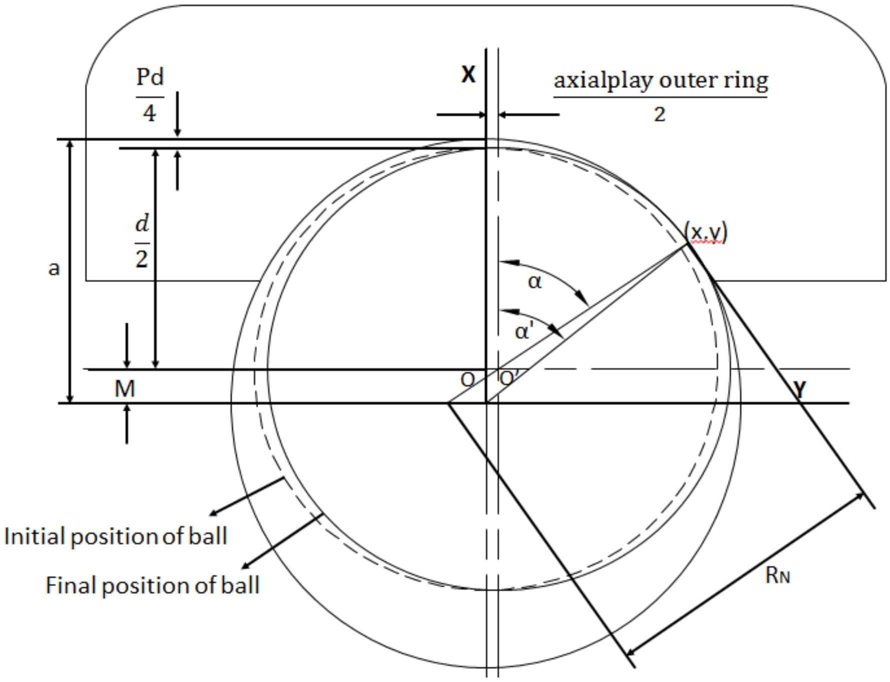


Figure 8: Internal geometry for derivation of axial play for "Inverted Ellipse".

From Fig.8,

$$M = a - \frac{d}{2} - \frac{P_d}{4} \tag{8}$$

Also,

$$M = X - \frac{d}{2}\cos\alpha \tag{9}$$

From Equation (2),

$$a - \frac{d}{2} - \frac{P_d}{4} = R_N \cos\alpha - \frac{d}{2}\cos\alpha \tag{10}$$

From Equation (1),

$$a - \frac{d}{2} - \frac{P_d}{4} = \left[\frac{a \cos\alpha}{(1-(e\sin\alpha)^2)^{\frac{1}{2}}}\right] - \frac{d}{2}\sin\alpha \tag{11}$$

As the above equations (8-11) involve higher level mathematical complexities, MATLAB was used to calculate the values of φ corresponding to different values of P_d. The same is tabulated in subsequent section. In the subsequent section, methodology to calculate axial play for elliptical curvature has been discussed. Same has been described below in detail:

3. Axial Play Calculations

Based on the contact angles obtained above, values of axial play were obtained by using the following methodology.

3.1 Axial Play for Normal Ellipse

Using Fig.7,

$$P_e = X - \frac{d}{2}\sin\alpha \tag{12}$$

From Equation (2),

$$P_e = \left[\frac{a \sin\alpha}{(1-(e\sin\alpha)^2)^{\frac{1}{2}}}\right] - \frac{d}{2}\sin\alpha \tag{13}$$

3.2 Axial Play for Inverted Ellipse

From Equation (2),

$$P_e = Y - \frac{d}{2}\sin\alpha \tag{14}$$

From Equation (3),

$$P_e = \left[\frac{a \sin\alpha\,(1-e^2)}{(1-(e\sin\alpha)^2)^{\frac{1}{2}}}\right] - \frac{d}{2}\sin\alpha \tag{15}$$

Values for contact angle and axial play were calculated for all cases using As the above equations (1-15) and have been tabulated in Table 2-4 in the subsequent section.

III. Numerical Analysis

1. FEM Model and Methodology for Axial Play

Standard and Explicit model for Dynamic Analysis in ABAQUS is created. Fig.9a shows the 2D model used for the calculation of axial play.

A two dimensional Dynamic Axis Symmetric FE Model is constructed to reduce the intricacy of the modeling steps and bring down the iteration time and resources to a minimum. The ball of the bearing is considered to be rigid with six degrees of freedom restricted. The inner and outer rings of the bearing are given different displacements (µm) incrementally synchronized with the step size in the direction shown in the Fig.9 a.

The basic properties of steel are given to the inner and outer rings. Surface to surface interaction properties are given between the inner ring and ball and outer ring and ball. All contacts are assumed to be frictionless. To restrict the motion of the inner and outer rings only is the X- direction, boundary conditions are provided accordingly. The inner and outer rings are meshed using the meshing type as 4 node bilinear plane stress quadrilateral elements with reduced integration and hourglass control, abbreviated as CPS4R.The ball is meshed with the meshing type as 2-node 2-D linear rigid link (used only for plane stress or plane strain), abbreviated as

R2D2. Two partitions are made for smoother contact of the ball with the inner and outer rings. A meshed sectional view has been shown in Fig.9b.

Advanced methodology is used to conduct the Finite Element Analysis for the calculation of the axial play of Ball Bearing with different elliptical track curvatures with different radial clearances. Standard and Explicit model for FE Analysis in ABAQUS is created. Fig.9 shows the model used for the calculation of Contact Stress.

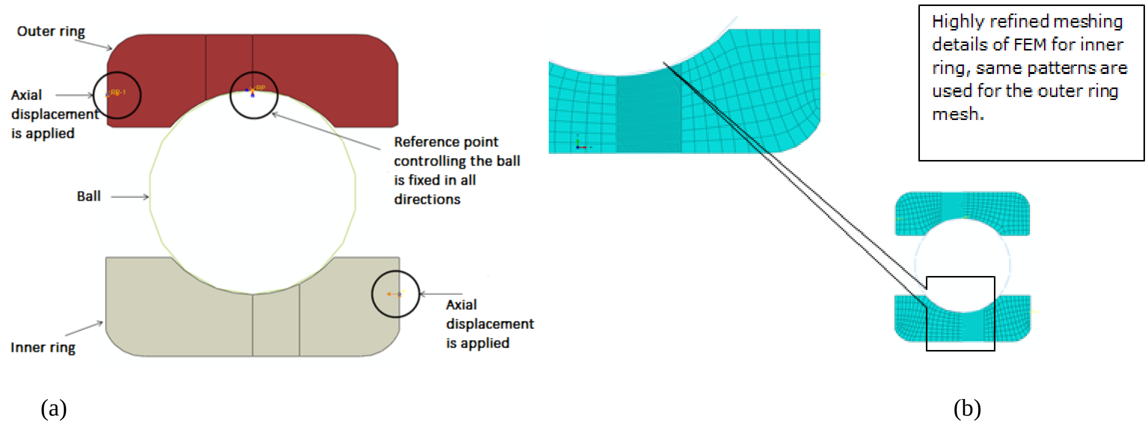


Figure 9: Two dimensional dynamic FE Model highlighting (a) Axial displacement (b) Refined mesh for axial play calculations

In order to calculate the value of end play for multiple cases, incremental displacement was given to the ball in axial direction and the corresponding graphical interpretations of stress was obtained. As illustrated in Fig.10, we have taken a sample case to explain the methodology in a comprehensive manner. In Fig.10, X-axis represents the axial displacement (mm) provided to the ball and Y-axis denotes the reaction force obtained on the raceway member. As is evident from the illustration, the reaction force remains zero for a range of values of axial displacement, after which it starts to increase, pertaining to the contact and generation of contact stresses.

2. FEM methodology for contact Stress

Theoretically, in an ideal situation, the contact area of two spheres would be a point, and it would be a line for two parallel cylinders. Consequently the pressure required to cause immediate yielding between the two curved surfaces would be infinite. However, a small contact area gets generated through elastic deformation in reality, thereby limiting the stresses considerably. These contact stresses are called Hertz contact stresses, which was first studies by Hertz in 1881 and the corresponding stress theory is called Hertzian stress theory. For a detailed study on the Hertzian contact stress theory, one can look up Tutorial on Hertz Contact Stress by Xiaoyin Zhu [9]. The Hertz contact stress theory is used generally to obtain contact stresses at the contact of two radii of different curvatures. In the present analysis, we have used the same to calculate the contact stresses analytically. The sum and difference of curvatures in a ball bearing are needed in order to obtain the normal loads on the ball. One can refer T.A Harris in order to go into details of and understand the formulations and calculations of the above mentioned [1].Also, to go in more detail, one can refer work of Sjoväll on load distribution within ball and roller bearings [10], H For an elliptical curvature, the calculation of stresses requires high level integrations as the contact area, which is formed as a result of Hertzian contact stress theory, has a span which encompasses varying values of curvature (As R_N varies with the azimuthal angle). In order to understand the behavior of stresses in the changed geometry, modeling and simulation was used and the obtained result were found to be in accordance with the expected behavior, which will be dealt in detail in the subsequent sections.

An axis symmetric FE model is constructed for static analysis to bring down the iteration time and resources to a minimum. All the three, ball, inner ring and outer ring, are considered to be deformable. A radial load of 15000 N and axial load of 1500N is applied to find out the contact pressure on the inner and outer races of the bearing.

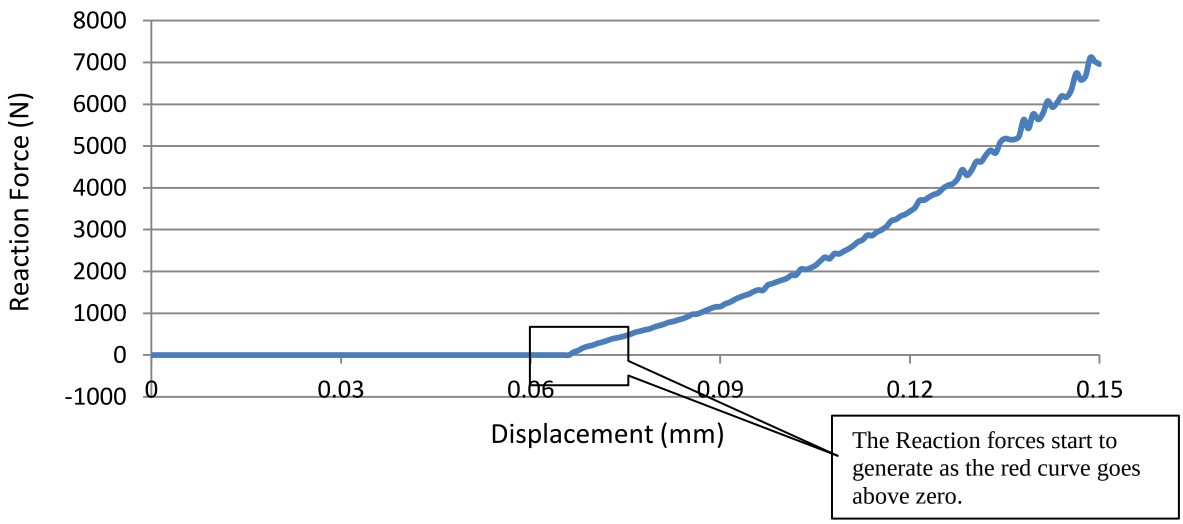
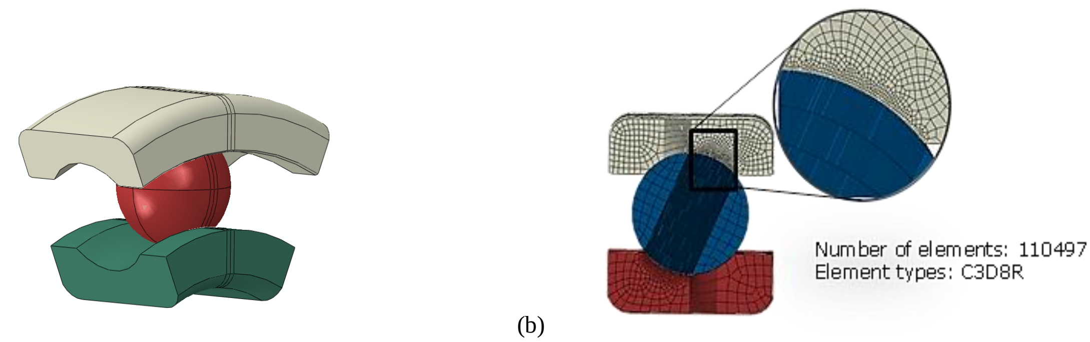


Figure 10: A case illustrating methodology for calculation of axial play

Standard and Explicit model for FE Analysis in ABAQUS is created. Fig.11 a shows the model used for the calculation of Contact Stress.



(a) (b)

Figure 11: Standard axis symmetric 3D model with refined meshing is developed in ABAQUS 6.12

The basic properties of steel are given to the inner and outer rings. Surface to surface interaction properties are given between the inner ring and ball and outer ring and ball. All contacts are assumed at a Penalty with a coefficient of friction of 0.08. To restrict the motion of the inner and outer rings only is the Y- direction, boundary conditions are provided accordingly. All the three part instances, namely, ball, inner ring and outer ring, are meshed using the meshing type as 8-node linear brick with reduced integration and hourglass control, abbreviated as C3D8R.

In Fig.11b, refined mesh has been shown for a particular case of axial loading Fig.11b highlights the refined mesh that was generated in order to obtain accurate results and continuous flow of stress through the boundary. The stress patterns have been discussed in detail in the subsequent sections.

The above methodologies are used for both the cases of the elliptical track curvatures and the results of the same are captured in Table 5-6.

IV Results and Discussion

Results were obtained based on the preceding methodologies and will be discussed in detail in this sections. We have summarized the outputs three sections, pertaining to results of contact angle, axial play and contact stresses respectively.

1. Contact Angle Results

The analytical values of contact angles which have been tabulated in Table 2 as below:

Table 2: Values of the contact obtained by applying axial load.

Contact Angle					
	Normal Ellipse		Inverted Ellipse		
RIC (μm)	Outer (Degrees)	Inner(Degrees)	Outer(Degrees)	Inner(Degrees)	
				Case 1	Case 2
20	7.03	8.635	23.2	12.17	8.60
30	8.63	10.63	26.02	14.92	10.54
40	9.98	12.34	28.21	17.24	12.18
50	11.19	13.87	30.03	19.30	13.62
60	12.28	15.28	31.59	21.16	14.93

As evident from the Table 2, unlike conventional circular curvature DGBB, the values of contact angles in case of elliptical curvature DGBB is different for inner and outer rings, owing to different kinds of geometry parameters. Also, in case of Normal Ellipse type of geometry for inner and outer rings, angle of inner ring is 22.8% higher than that of outer ring. Also, as we increase radial internal clearance, as expected, angle in both the cases increases by approximately 22% with a step increase of 10 micrometers.

2. Axial Play Results

Table below shows the value of theoretical as well the FEA results for inner and outer as well as the total axial play for the ball bearing with Normal elliptical curvature.
We have calculated the total axial play as:

$$Total\ Endplay = 2(Axial\ play\ inner\ ring + Axial\ Play\ outer\ ring) \qquad (16)$$

It would be worthwhile to keenly observe the changing pattern of axial play in Normal and Inverted Macro geometries. Equation16 has been used to arrive at total endplay in the adjoining Table 3.

Table 3: Axial play results for all cases of Normal Ellipse.

Normal Ellipse						
	Theoretical			FEM		
RIC (μm)	Axial play Outer(μm)	Axial play Inner(μm)	Total(μm)	Axial play Outer(μm)	Axial play Inner(μm)	Total (μm)
20	81	67	296	82	67	298
30	100	81	362	100	82	364
40	115	94	418	115	95	420
50	128	104	464	129	105	468
60	140	114	508	141	114	510

As evident from the values obtained in present analysis, theoretical methodology for the calculation of axial play is comprehensively verified with the help of axial analysis. The difference between theoretical and simulation results is below 5 % (w.r.t theoretical). Further, Fig.12 (a) Displacement vs. reaction force for Normal where Fig.12 (b) Displacement vs. reaction force for Inverted Ellipse Fig. 12 (c) Displacement vs. reaction force for Inverted Ellipse case 2 of various graphs has been obtained from FE Simulation have been provided below.

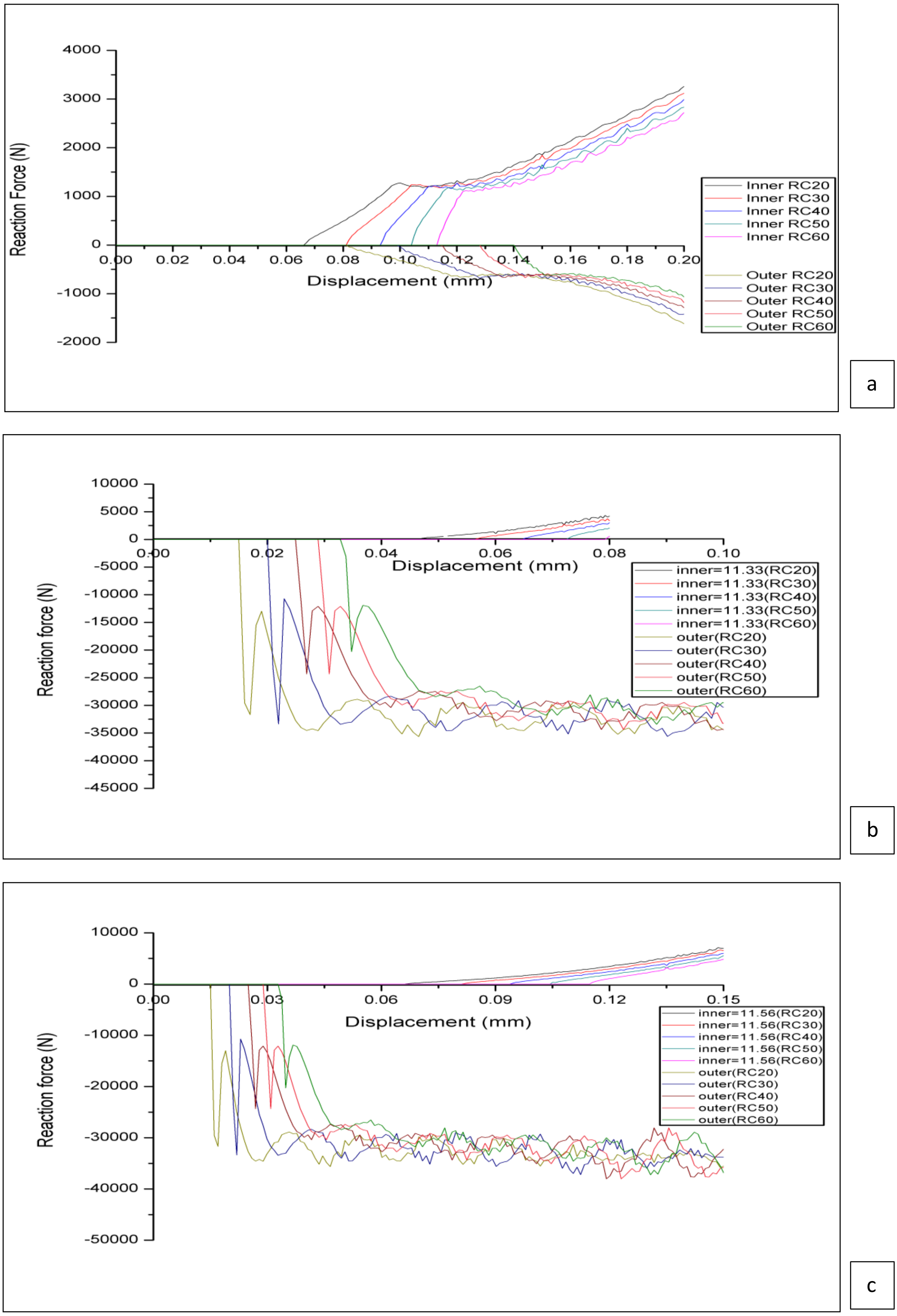
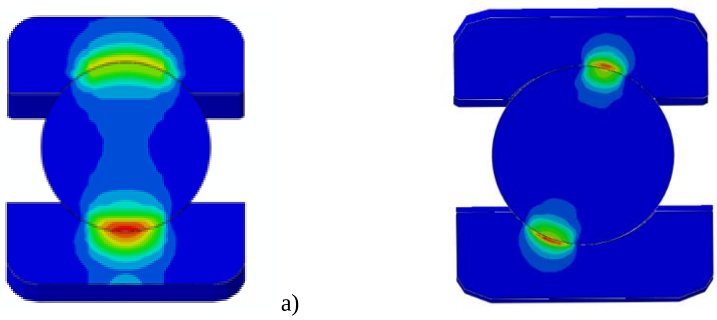


Figure 12: Displacement vs. Reaction force for axial play Calculations using FEM

Axial play values are higher for Normal elliptical geometry when compared to the corresponding end play values for Inverted elliptical curvature. Also, the axial play values increase by approximately 23% as we increase the radial internal clearance by a step of 10 micrometers.

3. Contact stress Results

Based on the inputs given in table1 and input loads, we were able to obtain the contact stress for all the cases by using FE simulation using ABAQUS 6.12.Following table lists all the values of contact stresses obtained. It would be worthwhile to keenly observe the changes in stress values pattern as well its variation with radial clearance to understand the connotation of present analysis. Flow of contact stress under both radial and axial load for elliptical curvatures is shown in Fig.13 below.

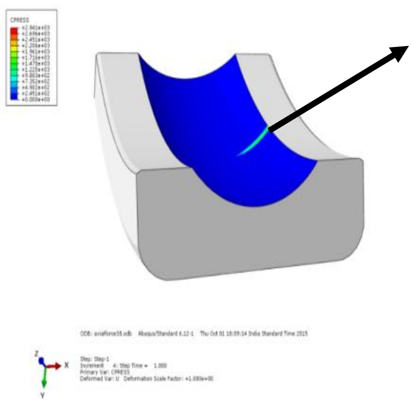


a) b)

Figure 13: Stress flow for (a) Radial (b) Axial stress loads

Table 4 contains values of contact stresses for all the permutations for Normal Ellipse with both the loads applied independently.

For Inverted Ellipse, for a radial clearance of 4 micrometers, we were able to obtain contact stresses for axial loadings. Beyond 0.005 mm radial Clearance, truncation was observed in the system as shown below with the help of Fig.14 highlighting the same.



Stress pattern highlighting that the contact ellipse has gotten truncated due to insufficient shoulder radius in case of RIC above 0.005 mm.

Figure 14: Truncation as observed in cases where RIC is higher than 0.005 mm.

Values of contact stresses for all the cases of Inverted elliptical geometry have been tabulated in Table 5. One can observe that an Inverted Ellipse kind of geometry has lower values of contact stresses when compared to Normal elliptical geometry.

Table 4: Values of contact stresses obtained through FEM for all cases of Normal Ellipse

Normal Ellipse				
	Contact Stresses obtained by FEA			
RIC (µm)	Axial Load		Radial Load	
	Outer (Mpa)	Inner (Mpa)	Outer (Mpa)	Inner (Mpa)
20	1865	2044	2978	3302
30	1836	2011		
40	1810	1981		
50	1786	1951		
60	1762	1925		

.

Also, between case 1 and case 2, inner rings in case 1 has higher contact stresses when compared to case 2, as expected.

Table 5: Values of contact stresses obtained through FEM for all cases of Inverted Ellipse

	Inverted Ellipse			
		RIC (µm)	Inner (MPa)	Outer (MPa)
Axial Load (1500N)	Case 1	04	2620	2293
Radial Load (15000N)		0	2941	1650
Axial Load (1500N)	Case 2	04	1822	1204
Radial Load (15000N)		0	2639	1650

V. Conclusions

Values of contact angle, axial play and contact stresses for all the considered permutations of elliptical geometries for DGBB6312 has been calculated. In applications where requirement of axial play is more, owing to shaft misalignments, or where truncation maybe a major issue, one may go for Normal elliptical type of geometry. Inverted elliptical geometry furnishes higher contact angles with very little axial play, provides relatively lower values of contact stresses which can be useful for applications required to handle high combined loading with very little shaft misalignments. As compared to Normal bearing geometry, Normal and Inverted kind of elliptical geometries can be used based on the requirement of the application. Based on above results, one may go for calculation of other important parameters of Deep groove ball bearing design and analyze the significance of this geometry for bearing raceways.

Acknowledgements

The authors are truly thankful to Mr Maruti Khaire, General Manager Technology function, R & D, NEI Ltd for permission of this research work. Authors are grateful to Mr Deepak Borse, Engineer Product design for improvement of manuscript and also thankful to Advance product technology team, R & D, NEI Ltd for their valuable and significant suggestions which substantially improved the manuscript.

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