

## **Development of a vehicle dynamic model to evaluation of the vertical behavior on the road**

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**Abstract :** *During a century of development of automobile industry, major technological advances were achieved. Detailed studies with relation to the dynamic behavior of vehicles are needed for a better understanding of design and development of auto-vehicles. This work aims to develop a computational model used for dynamic analysis of vehicles, offering to the user a better knowledge about the mathematical methods implemented, through a friendly interface. The Vehicle Dynamic Model (VDM) software provides four types of results. The first one presents the results in a steady state. The second shows the frequency response curves of the vehicle. The third present's animation features of the vehicle running the track profile. The forth present's natural frequencies and vibration modes of the vehicle. The software allowed the definition of the characteristics of tires, springs, dampers and geometric parameters of the vehicle. The dynamic response of the vehicle is then checked. The model was run using data based on literature to demonstrate their capabilities. The program allows the dynamical analysis of a vehicle and offers the possibility to change the vehicle parameters and check the ride response, covering a track profile.*

**Keywords -** *Computational Model, Dynamic analysis, Software, Vehicles*

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### **I. Introduction**

The vehicle dynamics studies the relationship between movements and forces acting under the various conditions applied to a vehicle. The dynamic behavior of a vehicle is determined by the forces imposed by the tires, gravity and aerodynamics. The vehicle and its components are studied to obtain the forces generated in a route and calculate the vehicle response. A motor vehicle is composed of several components. For some elementary analysis, it can be assumed that all components move together. But for the analysis of ride, it is often necessary to treat the wheels separately. In this case, the concentrated mass representing the body of the vehicle is called "sprung mass" and the wheel is called "un-sprung mass".

For more realistic analysis, it is necessary to model each component, or at least the components of a given system (suspension, steering, transmission, brakes, etc.). In the several journeys performed by a vehicle, two components are very important: the suspension and tires. According to Gillespie [1], the main functions of the tires are: To support the vertical load while absorbing the impact of the runway; Dissipate the longitudinal forces of acceleration and braking; dissipate lateral forces when cornering. For the suspension, the principal functions are: To provide vertical flexibility, allowing the wheels to follow the irregularities of the track and isolate the chassis of these irregularities; Keeping the wheels on the right conditions to steer and camber; Reacting to the forces and moments generated by the tires; Resist the roll motion of the body; and keep the tires in contact with the ground, with minimal variations of loading. The suspensions can be generally divided into two categories: rigid axle suspensions (solid axles) and independent suspensions.

Despite the variety of geometries, the suspensions are basically composed of three components: spring, damper and supporting components. The springs absorb the movements from the runway. Damper function is to reduce the amplitudes of the suspension and reduce fluctuations caused by springs. These components are nonlinear and difficult to model. The spring-damper assembly is also known as shock absorbers or vibration isolator.

The supporting components alloy wheels to the body or chassis of the vehicle. Most studies on automotive suspensions are aimed to calculate the geometry of these components, because they determine the movement of the wheels relative to the rest of the vehicle. The suspensions are often calculated as a multibody mechanism and optimized according to parameters such as ride and handling. According to Attia [2], there are several ways to simulate the dynamic behavior of mechanisms. However, it is interesting to use a methodology that can minimize the number of differential equations describing the problem, in order to obtain a more efficient computing solution. According to Kim et al. [3], the parameters of ride and handling are conflicting in the performance of automotive suspensions. To optimize both parameters, numerous solutions have been proposed. These solutions consist not only in different suspension geometry as well as in computerized control (semi-active and active suspension systems).

Several models have been developed to analyze the dynamic behavior of vehicles. Shiiba and Suda [4], used the multibody dynamics of a real vehicle in a simulator. Thus, the characteristics of this simulator have become very close to the real car. The simulator could be used to predict the dynamic performance and comfort in a subjective way. The operator of the simulator can evaluate the behavior of the vehicle without the necessity of creating a prototype. Alexandru [5], study the the differences between a rigid model and a compliant model on the dynamic behaviour of the multilink guiding system used for the rear axle of the commercial vehicles. The main parameters used in the comparative analysis refer to the vertical displacement of the car body, the roll and the pitch oscillations, as well as the specific linear and angular accelerations.

According to Atsumi et al. [6], several studies have been developed in order to improve the ride parameters, due to growing interest in vehicle comfort. Gobbi and Mastinu [7], analyzed a system with two degrees of freedom that represents the tire-suspension. Using two types of track profiles, three performance parameters were computed: comfort, road holding and working range of the suspension.

Margolis e Shim [8], modeled the dynamics of a four wheels vehicle, emphasizing the control systems related to security. The motion equations were obtained directly from the links charts and simulated using software called “ACSL”, capable of solving differential equations of first order explicit. Thoresson et al. [9], [10], proposed a methodology for the efficient determination of gradient information, when optimizing a vehicle’s suspension characteristics for ride comfort and handling. The non-linear full vehicle model, and simplified models for gradient information has been discussed, and validated.

A method for simulation of the dynamic interaction between vehicle and railway track is proposed by Baeza et al. [11]. The model has been designed to take into account the complexity of wheel–rail contact, rail pad and ballast, with low computational requirements.

Some authors also analyze the influence of the vehicle power train in the dynamic behaviour. El-Gohary, et al [12], developed an eight-state mathematical model describing vehicle power train for a mid-size passenger car. The inverse dynamics model determines the throttle angle required to produce a predetermined vehicle velocity. Fakhrabadi, et al [13], study the effective parameters on the torsional vibration of the vehicle clutch system. The vehicle powertrain system is simulated using a model with eight degrees of freedom. The effects of oscillations in clutch compression force and engine torque due to the variation in system natural frequencies are studied. The results present some helpful techniques to reduce the undesirable vibrations in the vehicle.

This work aims to develop a computational model that enables the evaluation of the dynamic behavior of a vehicle when driving on different types of track, allowing the test accomplish of different geometrical arrangements of body, sets of springs and dampers.

## II. Methods

### II.1-Vehicle Dynamic Model - VDM

The Vehicle Dynamic Model (VDM) program was developed for modeling the dynamics of a two axle’s vehicle. Basically, the VDM uses MATLAB software and provides the user the ability to change a number of vehicle parameters and check the ride response, covering a track profile.

The VDM program offers four types of responses: (a) response in a steady state of the vehicle traversing the track profile (b) frequency response obtained from a unit amplitude sinusoidal input applied simultaneously to each vehicle axle (c) Animation of steady state response, (d) Animation of the vibration modes of the vehicle. The coordinate systems and distances presented in Fig. 1 were adopted in the development of the vehicle model.

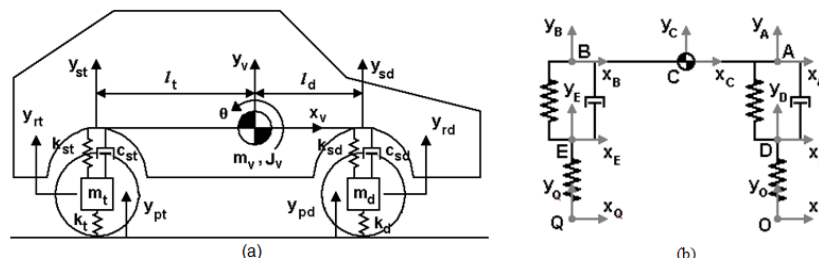


Fig. 1. – (a) Coordinate systems and distances adopted on vehicle model. (b) Local coordinate systems.

$l_d$ : horizontal distance from Center of Gravity (CG) to the front axle;  $l_t$ : horizontal distance from CG to the rear axle;  $y_{pd}, y_{pt}$ : vertical displacement of the track for the points O and Q;  $y_{rd}, y_{rt}$ : vertical displacement of the wheels for the points D and E;  $y_{sd}, y_{st}$ : vertical displacement at the suspension top for the points A and B;  $y_v$ : vertical displacement of the vehicle relative to point C; Tetha ( $\theta$ ): vehicle rotation (pitch) around the point C.

The model was solved using the variables that define the vehicle. Relating the displacements  $y_{sd}$  and  $y_{st}$  with displacement  $y_v$  and rotation  $\theta$ , equation (1) and (2):

$$y_{sd} = y_v + l_d \cdot \tan \theta \approx y_v + l_d \theta \tag{1}$$

$$y_{st} = y_v - l_t \cdot \tan \theta \approx y_v - l_t \theta \tag{2}$$

Isolating  $\theta$  in Equation (1) and (2):

$$\theta = \frac{y_{sd} - y_{st}}{l_d + l_t} \tag{3}$$

and taking  $y_v$  in the resulting equation (4):

$$y_v = \frac{l_d y_{st} + l_t y_{sd}}{l_d + l_t} \tag{4}$$

The force balance in the vehicle Center of Gravity (CG) is given by (5):

$$\sum F_y = m_v \ddot{y}_v$$

$$m_v \ddot{y}_v = -k_u (y_u - y_n) - c_u (\dot{y}_u - \dot{y}_n) - k_{sd} (y_{sd} - y_{rd}) - c_{sd} (\dot{y}_{sd} - \dot{y}_{rd}) \tag{5}$$

and the momentum balance (6):

$$\sum M_{CG} = J_v \ddot{\theta}$$

$$J_v \ddot{\theta} = l_t k_u (y_u - y_n) + l_t c_u (\dot{y}_u - \dot{y}_n) - l_d k_{sd} (y_{sd} - y_{rd}) - l_d c_{sd} (\dot{y}_{sd} - \dot{y}_{rd}) \tag{6}$$

To the front wheels, the forces balance is given by (7):

$$\sum F_y = m_d \ddot{y}_{rd}$$

$$m_d \ddot{y}_{rd} = -k_d (y_{rd} - y_{pd}) + k_{sd} (y_{sd} - y_{rd}) + c_{sd} (\dot{y}_{sd} - \dot{y}_{rd}) \tag{7}$$

and to the rear wheels, the forces balance is (8):

$$\sum F_y = m_t \ddot{y}_{rt}$$

$$m_t \ddot{y}_{rt} = -k_t (y_{rt} - y_{pt}) + k_{st} (y_{st} - y_{rt}) + c_{st} (\dot{y}_{st} - \dot{y}_{rt}) \tag{8}$$

The excitations applied to the model are due to displacement  $y_{pd}$  and  $y_{pt}$ , resulting from the runway. Due the model is two dimensional, it was considered that the vehicle travels in a straight line. Therefore, the front and rear wheels travel the same path and  $y_{pt}$  is related to  $y_{pd}$  with a delay  $\Delta t$  (9):

$$y_{pt}(t) = y_{pd}(t - \Delta t) \tag{9}$$

The delay can be calculated by dividing the wheelbase by vehicle speed. The force transmitted by the tires and suspension can be calculated from Fig. 2 (a), for the front axle (10) and for the rear axle (11).

$$F_{T,rd} = k_d (y_{rd} - y_{pd}) \tag{10}$$

$$F_{T,rt} = k_t (y_{rt} - y_{pt}) \tag{11}$$

and from Fig. 2 (b), for the front axle (12) and for the rear axle (13).

$$F_{T,sd} = k_{sd} (y_{sd} - y_{rd}) + c_{sd} (\dot{y}_{sd} - \dot{y}_{rd}) \tag{12}$$

$$F_{T,st} = k_{st} (y_{st} - y_{rt}) + c_{st} (\dot{y}_{st} - \dot{y}_{rt}) \tag{13}$$

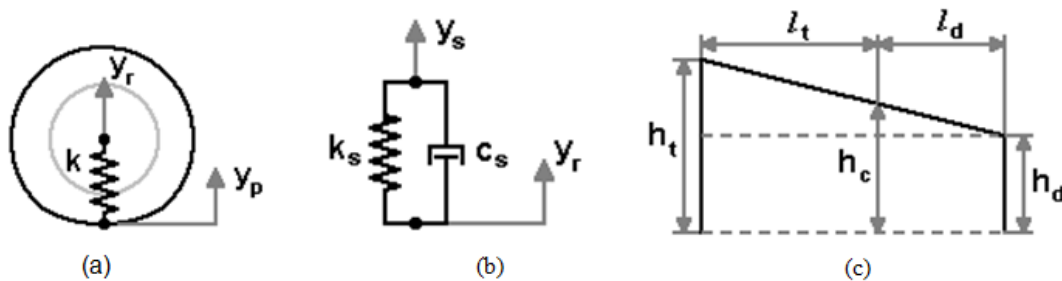


Fig. 2. – (a) Forces transmitted by the tires. (b) Force transmitted through the suspension. (c) Height of suspension-tire assemblies.

$h_d$ : height of the tire-suspension assembly of the front axle;  $h_t$ : height of the tire-suspension assembly of the rear axle;  $h_c$ : height of the reference point vertically aligned with the CG;  $r_d$ : front tire radius (without

deformations);  $r_r$ : rear tire radius (without deformations);  $s_f$ : free height of the front suspension;  $s_r$ : free height of the rear suspension.

To calculate the origins of local coordinates systems (14), it is considered that the vehicle axles may have different tires radius or suspensions with different heights. Fig. 2 (c) shows the height of the tire and suspension assemblies for front and rear axles.

$$h_d = r_d + s_d$$

$$h_r = r_r + s_r \tag{14}$$

where  $r_d$  is the front tire radius (without deformations),  $r_r$  is the rear tire radius (without deformations),  $s_d$  is the free height of the front suspension and  $s_r$  is the free height of the rear suspension.

### III. Results

#### iii.1- Vdm Program Interface

Six user interfaces were created for the VDM program, two designed for input data and four designed for viewing and interpreting the results. The first VDM interface "Vehicle" (Fig. 3) contains the vehicle parameters showing a figure that indicates the degree of freedom model and the variables to be provided.

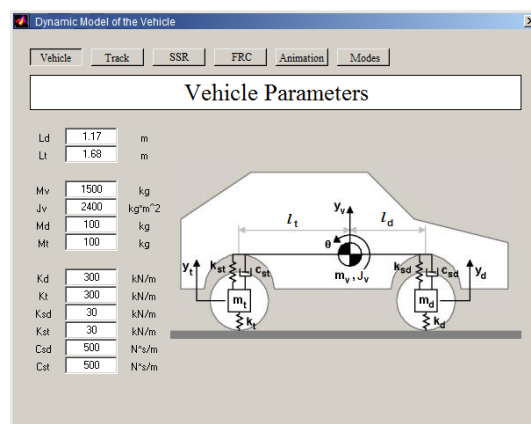


Fig. 3. – Vehicle parameters

$L_d$ : horizontal distance from the vehicle CG to front axle;  $L_r$ : horizontal distance from the vehicle CG to rear axle;  $M_v$ : vehicle mass;  $J_v$ : vehicle pitch inertia;  $M_d$ : front axle non-suspended mass;  $M_r$ : rear axle non-suspended mass;  $K_d$ : front tire stiffness;  $K_r$ : rear tire stiffness;  $K_{sd}$ : front suspension stiffness;  $K_{sr}$ : rear suspension stiffness;  $C_{sd}$ : damping coefficient of the front suspension;  $C_{sr}$ : damping coefficient of rear suspension.

The initial values used for the vehicle parameters were taken by Margolis and Shim [7]. The second VDM interface "track" (Fig. 04), presents the parameters that define the track profile. The parameters that can be modified by the user define the parameters of the track profile.

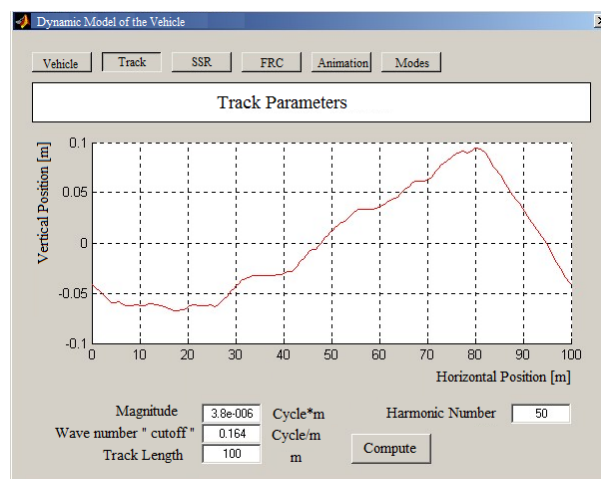


Fig. 4. – VDM interface "track". Parameters that define the track profile.

The third VDM interface "SSR" (Steady State Response) (Fig. 5) presents the results in a steady state, while the vehicle travels the road profile at a defined speed. The value of vehicle speed could be changed in order to check the response variation. The result can be displayed in units of displacement, velocity and acceleration (angular and linear). The program also allows the user to select which results are displayed. Fig. 5 displayed  $Y_v$  (vertical displacement of the vehicle's CG),  $\theta$  angle (Pitch),  $Y_d$  (vertical displacement of the front suspension),  $Y_t$  (vertical displacement of the rear suspension).

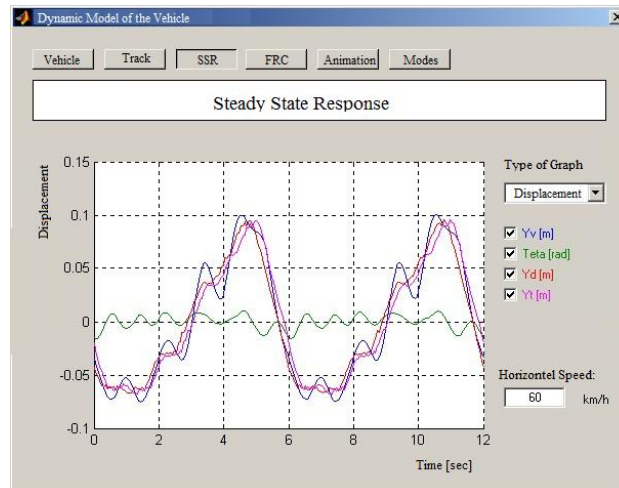


Fig. 5. – VDM interface "SSR". Steady State Response.

The fourth VDM interface "FRC (Frequency Response Curves) (Fig. 6) shows the frequency response of the vehicle subjected to a sine wave input of unit amplitude in each axis. The amplitudes and phases as a function of frequency are presented. The frequency scale can be viewed in a linear or logarithmic scale and amplitude in decibels or linear scale. The result can be displayed in units of displacement, velocity and acceleration. The program also allows the user to select which results are displayed.

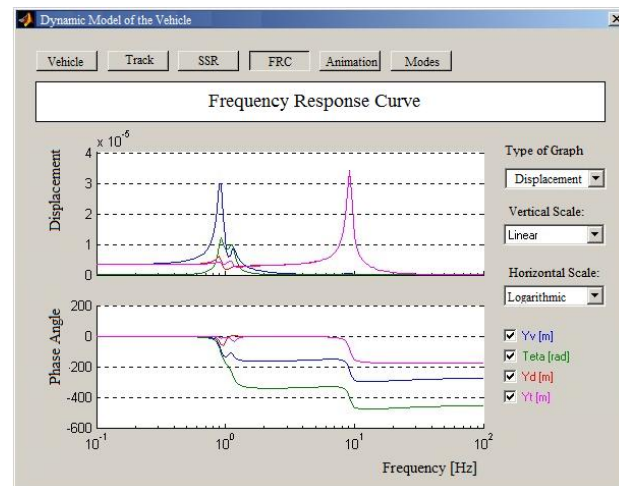


Fig. 6. – VDM interface "FRC". Frequency Response Curve.

The fifth VDM interface "Animation" presents an animation feature of the vehicle running the track profile. The scale of the vehicle speed and displacement can be changed. For greater control over the results display, two configuration options for the animation were implemented: the frames number per second and maximum time for animation.

The sixth and final VDM interface "Modes" presents the natural frequencies and vibration modes of the vehicle. The program allows changing the displacement scale. There is an option to animate the vibration mode, multiplying the mode by a sine wave of unitary amplitude in function of time.

### A. Ride Vehicular Response

The program was run with values based on literature to demonstrate their capabilities (Table 1). The vehicle parameters were adapted from Margolis and Shim [7], and the track parameters were taken from Gillespie [1].

TABLE I - Vehicle and track parameters values.

Vehicle Parameters	Value
$l_d$	1.17 m
$l_t$	1.68 m
$m_v$	1500 kg
$J_v$	2400 kg x m <sup>2</sup>
$m_d$	40 kg
$m_t$	40 kg
$k_d$	150 kN/m
$k_t$	150 kN/m
$k_{sd}$	15 kN/m
$k_{st}$	15 kN/m
$c_{sd}$	500 N x s/m
$c_{st}$	500 N x s/m
Track Parameters	Value
Magnitude parameter	3.8x10 <sup>-6</sup> cycles x m
cutoff wave number	0.164 cycles/m
Profile length	100 m
Number of harmonic terms	50

The VDM program presents as numerical results, the natural frequencies, the modal damping and the static displacements of the vehicle (Table 2).

TABLE II - The natural frequencies, the modal damping and the static displacements of the vehicle.

Natural Frequencies	Value [Hz]
1 <sup>o</sup> mode	0.91 ± 0,01
2 <sup>o</sup> mode	1.14 ± 0,01
3 <sup>o</sup> mode	9.14 ± 0,01
4 <sup>o</sup> mode	9.14 ± 0,01
Modal Damping	Value [%]
1 <sup>o</sup> mode	4.33 ± 0,01
2 <sup>o</sup> mode	5.40 ± 0,01
3 <sup>o</sup> mode	4.45 ± 0,01
4 <sup>o</sup> mode	4.50 ± 0,01
Static Displacements	Value [mm]
Front spring	289 ± 1
Rear spring	201 ± 1
Front tire	32 ± 1
Rear tire	23 ± 1

### IV. Conclusion

The Vehicle Dynamic Model program developed for modeling vehicle dynamics showed useful to enables the evaluation of the dynamic behavior of a vehicle when driving on different types of track and to analyze different geometrical arrangements of springs and dampers that adapt to these different types of track.

It can be seen in Fig. 5 that the displacements in the wheels have high frequency components and these components can be neglected in the displacement of the vehicle due to suspension action that filters the high frequency components. Comparing Table 2 to Fig. 6, one can observe that the peaks of the graph correspond to the values of natural frequencies obtained.

The program allows the dynamical analysis of a two axle's vehicle and offers the possibility to change a number of vehicle parameters and check the ride response, covering a track profile. Four interfaces for viewing and interpreting the results were designed. The first one presents the results in a steady state. The second shows the frequency response curves of the vehicle. The third present's animation features of the vehicle running the track profile. The forth present's natural frequencies and modes of vibration of the vehicle. As numerical results the program providing the natural frequencies, the modal damping and the static displacements of the vehicle.

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