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Plastic zone and effective distance under mixed mode fracture -Volumetric approach-

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Abstract: *Majority of the structures contain singularities, and that cause a high stress concentration. Near the root of defect, the singularity of stresses enables to activate strong plastic deformation. In the field of fracture mechanics, the investigation of plastic deformation at a notch tip under mixed mode loading is important to the development of understanding of material fracture and failure.*

In this paper, a study was conducted using U-notched circular specimens subjected to compression load. The analysis consists in studying the region of high stress concentration under elastoplastic behaviors. Two local methods are used: the volumetric approach and Irwin models for different values of angle and notch radius under mixed-mode I+II.

At the notch root, there exists a plastic zone which affects the evolution of the crack at notch tip. The value of this plastic zone will be compared to effective distance generated by relative stress gradient. A new model are proposed to evaluate the plastic zone is proposed using the effective stress and notch stress intensity factor. The effective distance can be determined accepting that this distance is supposed to be larger than the plastic zone diameter.

Keywords: *Effective distance,Irwin method,Mixed mode, Plastic zone, Relative stress gradient, Volumetric approach.*

I. Introduction

Majority of the structures contain singularities, and that cause a high concentration of constraints. Near the root of defect, the singularity of stresses enables to activate strong plastic deformation. In the field of fracture mechanics, the investigation of plastic deformation at a crack tip under mixed mode loading is important to the development of understanding of material fracture and failure.

Plastic deformation at a crack tip in material produces a plastic zone around the crack which keeps radially decaying stresses away from the crack tip. Various theoretical analyses have been reported over the last few decades to gain a comprehensive understanding of the mixed mode fracture, as done by Golos and Wasiluk[1], Erdogan and Sih[2]proposed a criterion based on the maximum tangential stress, Sih[2, 3] presented a criterion based on the energy study, Theocaris et al.[4] used von Mises yield criterion for predicting the radius of plastic zone. But Irwin considered that the presence of a plastic zone at the bottom of crack, fact that the length of the crack behaves as if it was longer than its physical size and the stress distribution is equivalent to a crack elastic length (a+r) [5,6], so its effective length , a_{eff} is :

$a_{eff} = a + r_{eff}$ With $R_p = 2\, r_{eff}$

For simple estimation of the size of plastic zone along 'θ' equal to zero degree, considering a first approximation that plastic zone is circular with diameter Rp, for a perfectly elastic plastic material, according to:

$$R_p(\theta_0) = \lambda \left(\frac{K}{\sigma_V}\right)^2 \qquad \text{i.e., (1)}$$

With: σv stress tangential, K stress intensity factor and λ varies between 0.30 – 0.39 (Irwin: 0.318 and Dugdale[7]: 0.342).

II. Materials and methodology

1. Material

The material studied is a high strength steel named 45CDS6 according to French standard. Mechanical properties are listed in Table 1.

TABLE 1 MECHANICAL PROPERTIES OF 45CDS6 STEEL

E (MPa)	ν	σ_Y(MPa)	σ_U(MPa)	A%	Density (Kg/m^3)	K_{IC}(MPa)
210065	0.28	1463	1662	2.8	7800	97

The microanalysis of the material givesthe following chemical composition:

TABLE 2
CHEMICAL COMPOSITION GIVEN IN ATOMIC PERCENTAGE

%C	%Mn	%Si	%Cr	%Mo
0.45	0.60	1.60	0.60	0.25

2. Specimens

The experiments have carried out by examining various U-notched circular ring specimens (see Fig.1.), with various geometries and boundary conditions (Fig.2.) [8].
With: external radiusRe=20mm, internal radius Ri=10mm, thickness B=7mm, and notch length a=4mm.
Different notch radii are introduced using a wire-cutting electrical discharge machine and using wiresof different diameter. The notch root radius was measured using a profile projector.
Five notchradius values are used: ρ = {0.15, 0.3, 0.5, 1, 2 mm}. With: 0° < α < 33° (mixed mode I+II). [9]

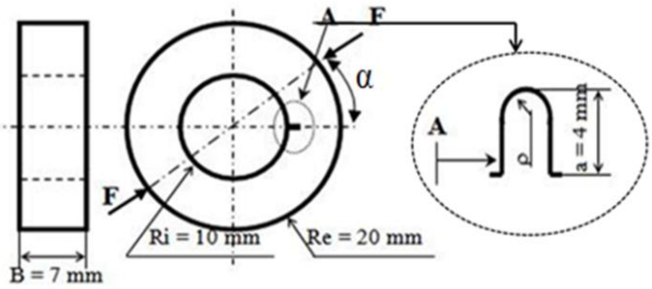


Fig.1. U-notched circular ring specimen

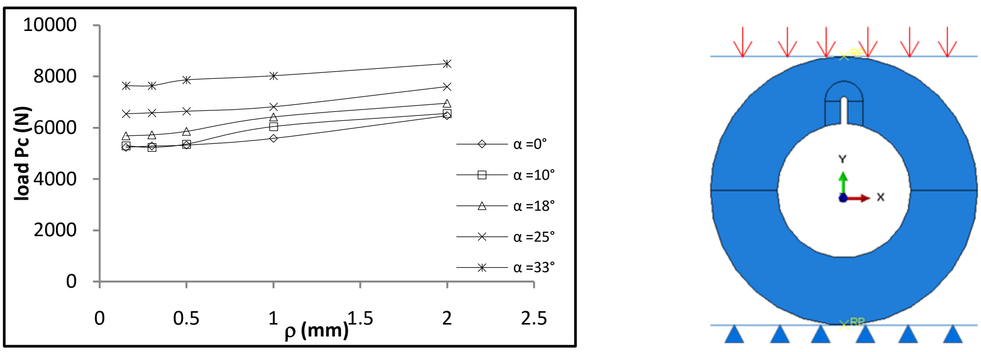


Fig.2. Loading mode of the specimen

The specimens are submitted to compression load in order to determine the critical loads when the fracture occurs. These loads are introducing to the simulation computation to finally evaluate the stress triaxiality evolution (Fig.3.).

I. FINITE ELEMENT ANALYSIS

The nonlinear finite element simulations are performed usingABAQUS 6.10[10].The geometry of the U-notched circular was simplified by considering a plane part worthless thickness such as z=1.

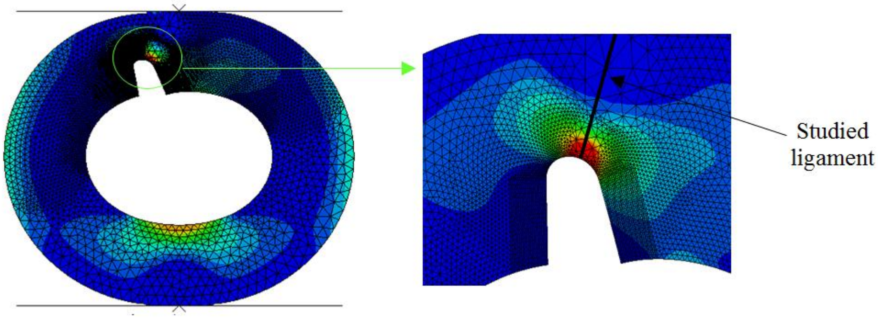


Fig.4. Studied ligament

To study the transitional stages of mode I to the mode II, it's necessary to define new orientations of stress. It exist an angle corresponding to each mode of application of load named θ_0(fig.4)[15]. Method XFEM is very effective to study this kind of problem (applicable on Abaqus/CAE).
The following figure (fig.5) summarizes the values of the angles drawn on (Abaqus/CAE) and raised [10].

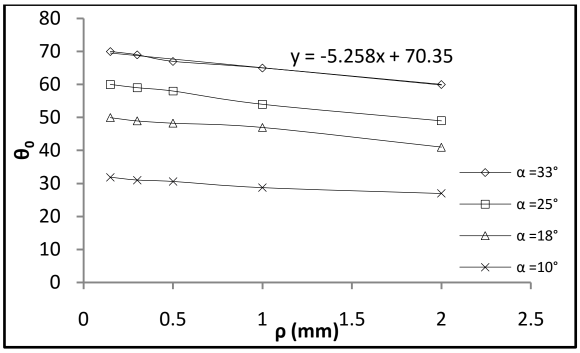


Fig.5. Fracture angle θ_0 vs radii ρ

The numerical values obtained according to θare compared to the experimental values measured by an optical microscope [4]. In mode II (for α=33°), for a radius notch ρ=0:
θ_0= 70,35°

This value is compared to others results, such as: θ_0≈ 70,39° [8, 9] and ≈70,5° [11], and≈70,33° [12]

The evolutions of stress with various radii are presented in the sections at the bottom, in mixed-mode (I+II) fracture crack initiation from notches is governed by the tangential stress.The stress evolutions areplotted versus the notch tip distance for each angle β, for (ρ=1) (Fig.6.). The maximum stress values decrease when the notch radius increases [15].

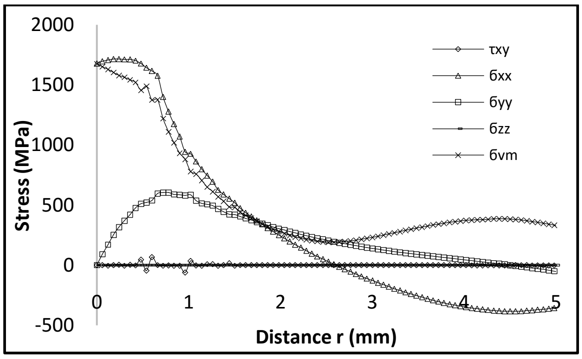


Fig.6.a) Stress distributions vs distance r

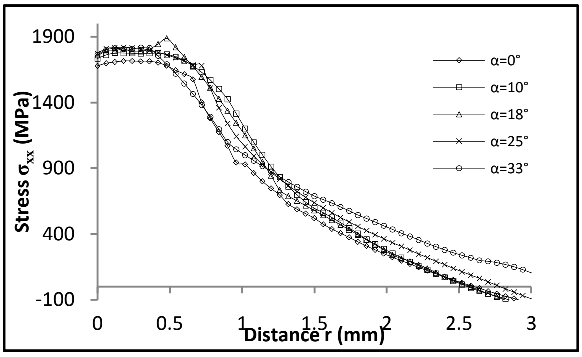


Fig.6.b) Stress σxx distributions in the notch tip ligament for the different angles

II. EFFECTIVE DISTANCE AND VOLUMETRIC METHOD

Stress distributions around the notch defect have been converted into so called notch stress intensity factor using the notch fracture mechanics and particularly the volumetricmethod.

The volumetric method is a local fracture criterion, which supposes that the fracture process requires a certain fracture volume. This volume is assumed as a cylinder with effective distance at its diameter. The elastic-plastic stress distribution along the ligament is plotted in the bi-logarithmic diagram as can be seen in figure 7.

Three distinct zones in the diagram can be distinguished:

- Zone I: the elastic-plastic stress opening stress increases and attains a peak value.
- Zone II: the elastic-plastic stress drops gradually in the elastic regime.
- Zone III: starts at a certain distance which is named the effective distance. It represents linear behaviour in the bi-logarithmic diagram.

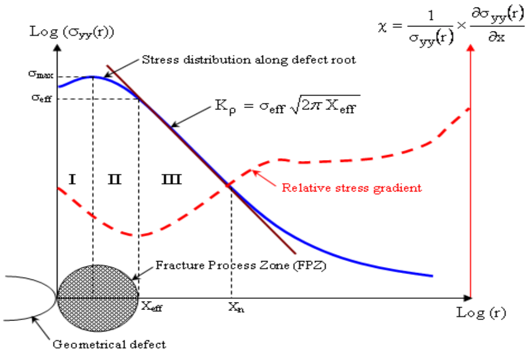


Fig.7. Schematic elastic-plastic stress distribution along notch ligament and stress intensity concept

The notch stress intensity factor is defined as the function of effective distance and effective stress:

$K\rho = \sigma_{eff}\sqrt{2\,\pi\, X_{eff}}$ i.e., (2)

By definition, the effective distance is the diameter of the process volume assuming it has a cylindrical shape. To determine this effective distance [4] studied the evolution of the function of the gradient relative of stress to the bottom of notch. This function represents a minimum corresponding to the effective distance X_{eff}:

$\chi(r) = \frac{1}{\sigma_{xx}(r)}\frac{d\sigma_{xx}(r)}{dr}$ i.e., (3)

Where: $\chi(r)$ and $\sigma_{xx}(r)$, are relative stress gradient and maximum principal stresses or crack opening stress, respectively.

Average volume of the stress distribution over the effective distance. However stresses are multiplied by a weight function in order to take into account the influence of stress gradient due to geometry and loading mode. The effective stress is defined as:

$\sigma_{eff} = \frac{1}{X_{eff}}\int_0^{X_{eff}} \sigma_{xx}(r)\big(1 - r\,\chi(r)\big)dr$ i.e., (4)

III. VON-MISES STRESS AND PLASTIC RADIUS

The criterion of plasticity Rp is defined by the point of intersection of the stress Von Mises and the yield stress of material.

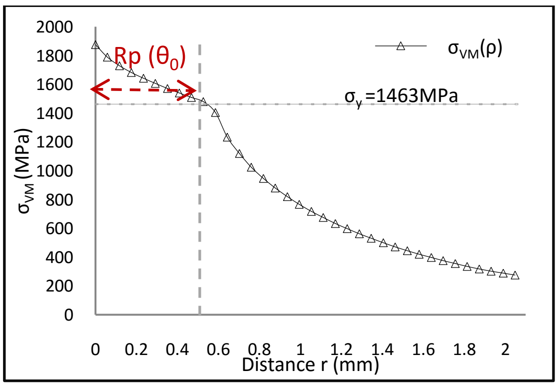
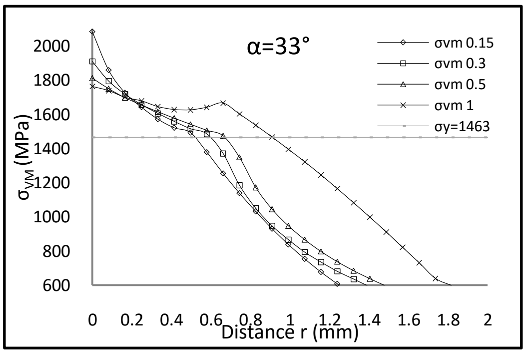


Fig.7. Determination of the plastic radius by stress Von Mises

This model of behavior of criterion of plasticity considers that the threshold from which the plastic flow develops, the constraint equivalent to the zone of development of rupture is in intersection with the yield stress (with σ_y=1463MPa).

The notch generates a concentration to the front of defect, which involves a fall of the resistance of the U-notched circular ring specimens. There is then risk of rupture.The values of R_P then present an agreement with the distance where the material joined the mode of plasticity.

The examination of (Fig.8.) shows that the extent of the plastic zone at the bottom of notch varies according to the values of bifurcation angle.



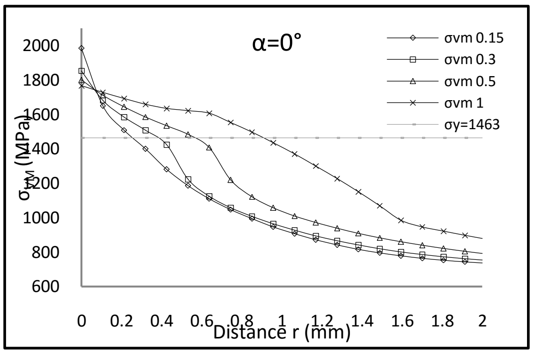


Fig.8. a) and b) Distribution of Von Mises Stress vs distance r for different angles of notch in mode II and I

FIGURE 8 SHOWS THE DISTRIBUTION OF VON MISES STRESS FOR DIFFERENT ANGLES OF NOTCH: 0°, 10°, 18°, 25° AND 33°. THE CHANGE IN PARAMETERS OF NOTCH, RADIUS AND ANGLE, CHANGES THE MORPHOLOGY OF PLASTIC ZONE NEAR THE BOTTOM OF NOTCH.

III. Results and discussion

1. $\mathbf{R_p}$ and $\mathbf{R_p^{\rho}(\theta_o)}$ analytical

The parameters introduced into the elastoplastic zone $R_P(\theta_o)$defined by Irwin i.e., (1)are valid only in case of a crack (ρ=0 and ψ=0). After results analysis and for a perfectly elastoplastic materialaccording to a rupture under notch effect,$R_p^{\rho}(\theta_o)$can be proposed and written in the following form:

$$R_p^{\rho}(\theta_o) = \frac{1}{2\pi}\left(\frac{K_\rho}{\sigma_{eff}}\right)^2 \quad \text{i.e., (5)}$$

For critical loads, the expression i.e., (5) will be written:

$$R_p^{\rho c}(\theta_o) = \frac{1}{2\pi}\left(\frac{K_\rho^c}{\sigma_{eff}^c}\right)^2 \quad \text{i.e., (6)}$$

With: K_ρ^c critical stress intensity factor, σ_{eff}^c critical effective stress.

By the analytical method i.e., (2) and i.e., (6), we calculate the values of$R_p^{\rho c}(\theta_o)$, and its well be compared with the values of Rp by Von Mises yieldcriterion. The table 3 summarizes the various values of Rpand $R_p^{\rho}(\theta_o)$.

TABLE 3
COMPARISON OF PLASTIC ZONE BY VON MISES CRITERION AND ANALYTICAL METHOD

Mode	ρ	σ_{eff}^c	K_ρ^c	R_p (mm)	$R_p^{\rho c}(\theta_o)$ (mm)	$\frac{R_p}{R_p^{\rho c}(\theta_o)}$		Mode	ρ	σ_{eff}^c	K_ρ^c	R_p (mm)	$R_p^{\rho c}(\theta_o)$ (mm)	$\frac{R_p}{R_p^{\rho c}(\theta_o)}$
	0,15	1554,65	98,89	0,585	0,64	0,91			0,15	1511,65	95,5	0,564	0,65	0,87
	0,3	1610,24	98,89	0,6004	0,60	1,00			0,3	1626,04	95,47	0,513	0,56	0,92
α=0°	0,5	1441,12	92,83	0,57	0,66	0,86		α=10°	0,5	1478,49	95,62	0,532	0,68	0,78
	1	1401,4	94,27	0,5401	0,73	0,74			1	1678,92	108,85	0,729	0,69	1,06
	2	1250	100,06	0,84	1,03	0,81			2	1386,78	106,78	0,794	0,97	0,82
	0,15	1601,16	96,25	0,495	0,60	0,83			0,15	1566,02	89,41	0,465	0,55	0,85

	0,3	1590,35	101,15	0,643	0,67	0,96			0,3	1698,25	92,86	0,723	0,50	1,44
α=18°	0,5	1359,98	95,01	0,699	0,81	0,86		α =25°	0,5	1649,12	109,16	0,697	0,74	0,94
	1	1676,41	117,22	0,778	0,81	0,96			1	1680,76	121,49	0,95	0,89	1,07
	2	1637,86	132,24	1,04	1,09	0,96			2	1686,42	143,66	1,222	1,23	0,99
	0,15	1590,16	71,17	0,312	0,34	0,91								
	0,3	1384,86	78,93	0,436	0,56	0,78								
α=33°	0,5	1386,39	91,56	0,595	0,75	0,79								
	1	1691,2	114,85	0,825	0,80	1,04								
	2	1742,77	156,41	1,433	1,39	1,03								

Table 3 shows the evolution of the plastic zone by the two methods (analytical and by the constraint of Von Mises) for various specimens. It is shown clearly that the distribution varies with a weak variation, but the values are very close, and the results are in agreement.

$$\frac{R_p}{R_p^{\rho c}(\theta_0)} \approx 1 \qquad \rightarrow \qquad Rp \approx R_p^{\rho c}(\theta_o)$$

2. R_p and effective distance

The values of effective distance X_{eff} determined according to the procedure described by the i.e., (3). To interpret the various criteria for Rpand their convergence towards the determination of the effective distance X_{eff} to the bottom of the notch, the table 4 summarizes all those values.

TABLE 4

COMPARISON OF PLASTIC RADIUS R_p WITH EFFECTIVE DISTANCE X_{eff}

Mode	ρ	X_{eff}	R_p(mm)	X_{eff}/R_p		Mode	ρ	X_{eff}	R_p(mm)	X_{eff}/R_p
	0,15	0,585	0,585	1,0			0,15	0,635	0,564	1,1
	0,3	0,6004	0,6004	1,0			0,3	0,548	0,513	1,1
α=0°	0,5	0,6603	0,57	1,2		α =10°	0,5	0,665	0,532	1,3
	1	0,7202	0,5401	1,3			1	0,669	0,729	0,9
	2	1,0199	0,84	1,2			2	0,943	0,794	1,2
	0,15	0,585	0,495	1,2			0,15	0,635	0,465	1,4
	0,3	0,6004	0,643	0,9			0,3	0,548	0,723	0,8
α =18°	0,5	0,6603	0,699	0,9		α =25°	0,5	0,665	0,697	1,0
	1	0,7202	0,778	0,9			1	0,669	0,95	0,7
	2	1,0199	1,04	1,0			2	0,943	1,222	0,8
	0,15	0,312	0,312	1,0						
	0,3	0,517	0,436	1,2						
α =33°	0,5	0,694	0,595	1,2						
	1	0,734	0,825	0,9						
	2	1,283	1,433	0,9						

The examination of the evolution of the various values shows that the size of the plastic zoneincreases with the radius of notch (table 4)[14].The analyses above (Fig.9.), assume that the zone of fracture process is larger than the plastic zone (for α=10° and ρ=2mm).

In both cases, the values of X_{eff} are considered to be larger than the plastic zone; the fracture process zone is covering the plastic zone.

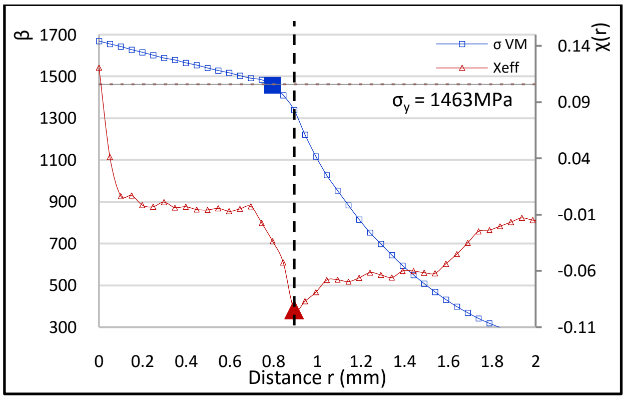


Fig.9. Comparison between effective distance X_{eff} and plastic zone R_p

IV. Conclusion

In order to evaluate the elastoplastic zone, the modeling of a problem is carried out by using two methods to describe it; by using Von Mises's stress andby analytical method; good agreement between the two forms of the plastic zone were be achieved. Furthermore, the result of analysis shows that the effective distance X_{eff}may be determined by taking into account the following principles:

- The effective distance X_{eff}can be determined accepting that this distance is supposed to be larger than the plastic zone diameter.
- The effective distance is the limit of the most highly stressed zone.

Moreover, the calculation of the size of the plastic zone depends mainly on the type of specimen, the radius and angle of notch too.

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