

Modeling and Parameter Identification of a DC Motor Using Constraint Optimization Technique

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Abstract: This paper presents mathematical modeling and simulation, and experimental study of an electromechanical system (a direct current motor) to identify the mechanical and electrical parameters (mass moment of inertia, damping, armature resistance and motor constant) of the motor and its accessories. Laboratory experiments were performed to obtain the step responses of the motor. The parameters were identified in two stages viz: 1) determination of the time constant and gain of the first order transfer function of the motor's model using MATLAB Parameter Identification Toolbox. 2) MATLAB codes were written to invoke MATLAB Constraint Optimization Toolbox and minimize the root mean squared error between the experimental and simulated step responses using the relationships between the motor parameters and the time constant and gain as constraint equations. The comparisons between the experimental and simulated step responses showed excellent agreement. The mechanical and electrical parameters of the motor were found to be the following: mass moment of inertia = $9.85 \times 10^{-3} \text{ kgm}^2$, armature resistance = 6.29 Ohms, damping = $2.52 \times 10^{-3} \text{ Nms/rad}$, and motor constant = $1.57 \times 10^{-2} \text{ Nm/Amp}$.

Keywords: dc motor model, constraint optimization, parameter identification, time constant, step response.

Nomenclature

DC	Direct current
$v(t)$	The motor's input voltage (Volts).
R	The motor's armature resistance (Ohms).
L	The armature's inductance (Henry)
i	The armature's current (Amp)
k_m	DC motor constant (Nm/Amp.)
v_b	The motor's back electromotive force (emf) (Volts).
τ_m	Motor's supplied torque (Nm)
T	Time constant (seconds)
b	Damping of the DC motor and accessories (Ns/rad).
I	Mass moment of inertia of the flywheel (kgm^2).
J	Combined mass moment of inertia of the flywheel and shaft and armature (kgm^2).
G_1	Motor gain or proportionality constant
ω	Angular speed (rad/s)
PI	Proportional and integral controller
TQ	TecQuipment (Trademark for servo motor manufacturer)

I. Introduction

Electromechanical devices (DC and alternating current motors) are widely used as prime movers for mechanical systems and machines. In some cases where automation and/or control of the mechanical systems is required, a control unit is attached to the electric motors and these type of motors are generally referred to as servo motor. Due to the importance of the servo motors in the automation of systems and processes, research studies on the characterization, mathematical modeling and parameter identification of electromechanical devices have been published [1-6]. Modeling and simulation of physical systems, a realistic compromise, are widely used in engineering for better understanding of the characteristics of systems in order to control and/or improve systems' performance and reduce costs by building and testing a prototype at the preliminary stage instead of the exact machine. The concepts of modeling and parameter identification are more useful during modification of existing systems when little or no information about the existing system is available. This paper is the outcome of the author's efforts during the development of laboratory manual for Systems Dynamic and Control course at the Department of Mechanical Engineering, Jubail University College, Saudi Arabia. The manual for the TQ servo trainer contains no information about the mechanical and electrical parameters or

properties of the system, which are needed to teach students about designing and testing of PI controller for servo motor to simulate speed control and responses.

The mechanical and electrical properties of electromechanical devices are needed to derive mathematical models. Even if these parameters are documented in the user manuals, it may be necessary to determine them if the system has been used for a few years. The reason for this is that these parameters will change due to aging, wear and tear of the system. Different parameter identification techniques and algorithms have been developed and used by different researchers to determine some important mechanical and electrical properties of various models of electromechanical systems. For examples, a multi-objective elitist genetic algorithm has been used to minimize the error between the model and measured responses of a DC motor [7]. Artificial Neural Network has also been used to determine model parameters and control dynamic systems [8]. One of the advantages of neural network is that parameter identification could be done online while the system is working by using adaptive learning method. Furthermore, a parameter estimation method using a block pulse function has been used to determine the parameters of DC servo motor systems [9]. The block pulse approximates a function as a linear combination of sets of orthogonal basis functions. The most common technique for parameter identification is the least square error (LSE) method [10]. LSE involves the minimization of the sum of the squared error between the model and experimental responses.

It should be noted that the different methods of model parameter identification reported [1, 2, 3, 5, 10] are similar to a curve fitting, which involves changing the model parameters until a good match between the model and experimental responses is achieved. However, it is well-known that curve-fitting technique has multiple solutions. This means that there are infinite sets of values that would yield a good match between the model and experimental responses. Therefore, care needs to be taken to ensure that the obtained set of parameters are realistic, that is the parameters can give a unique solution, which can be related to the actual physical system. A unique solution implies that regardless of the assumed initial parameters while solving the error minimization of the model and experimental responses, the final values of the parameters will always be the same. One of the ways to ensure the uniqueness of the identified parameters is by using constraint optimization method, which checks the relationship between the parameters in addition to the minimization of error between experimental and simulated responses. The constraint optimization technique has been used in the parameter identification of models for the human hand-arm system [11]. The reported studies [1, 2, 3, 5, 10] did not check the uniqueness of the identified parameters. The present study uses constraint optimization technique in addition to the conventional root-mean-square error method in order to ensure that the identified DC motor parameters yield unique solution.

In this study, the mechanical and electrical parameters (mass moment of inertia, damping, armature resistance and motor constant) of an electromechanical device (TQ DC servo motor and accessories) are determined from a linear model of the system and experimental responses of the device. Both the MATLAB System Identification and Constraint Optimization Toolboxes are used for the parameter identification of the model for the electromechanical device in order to ensure the uniqueness of the identified parameters. The parameters of the electromechanical device are needed in order to design and simulate a PI controller for speed control of the DC motor for demonstration to students in Systems Dynamic and Control course.

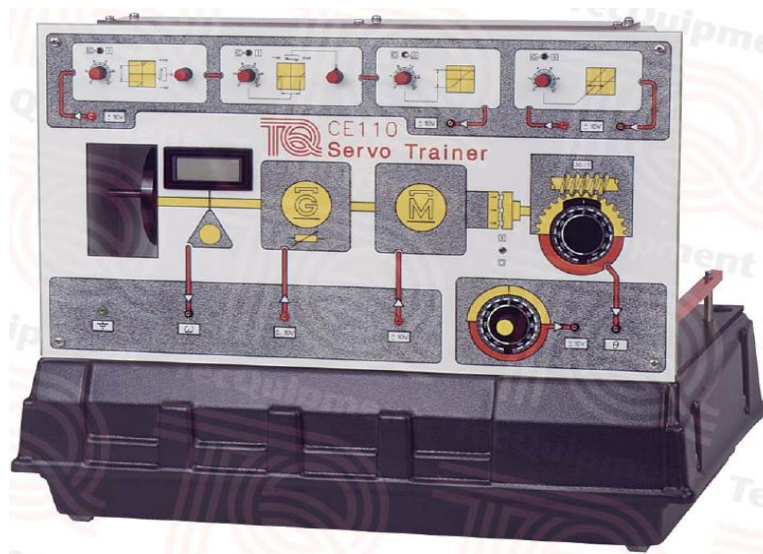
II. Methods

2.1 Experimental Set-up

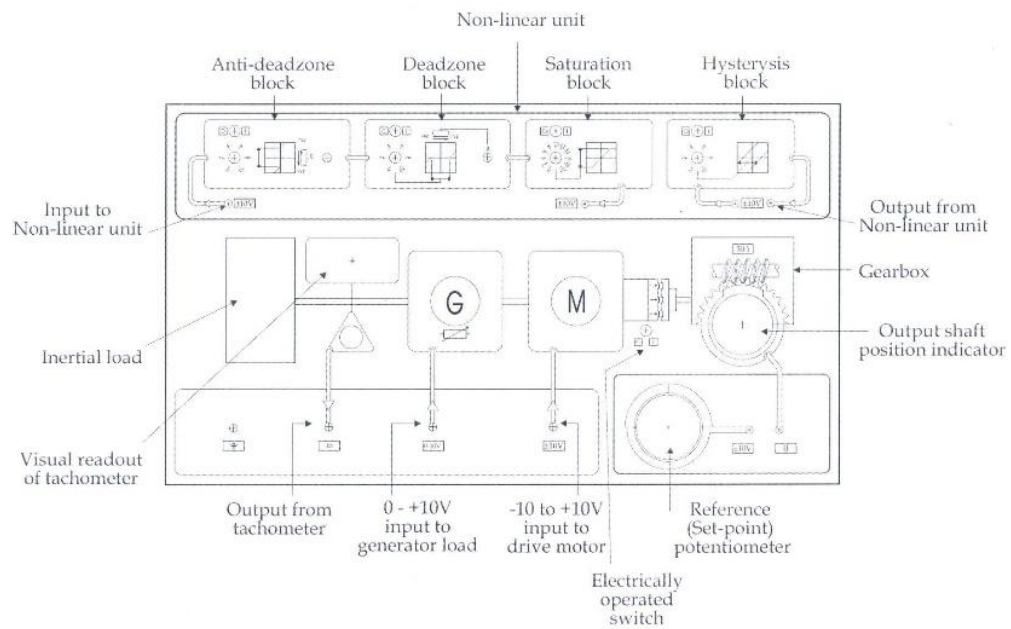
Figure 1 shows the electromechanical device that was used in the present study, the TecEquipment (TQ) CE110 Servo Trainer Apparatus [12, 13]. The CE110 Servo Trainer Apparatus is designed for DC servo motor speed and position control using a typical PID controller (TQ CE 120 Controller). The relevant parts of the apparatus, shown in Fig. 1, for the experiment comprises a DC motor, a DC generator and a flywheel mounted on a common shaft. An analogue input signal to the motor circuit in the range 0 to $\pm 10V$ enables variable shaft speed of rotation in either direction to be achieved. Shaft speeds, up to 1999 revolutions per minute (RPM), are continually sensed optically and indicated on a panel mounted digital meter. An analogue voltage signal proportional to speed, in the range 0 to $\pm 10V$, is available at an adjacent socket. The DC motor may be loaded, statically or dynamically, using the DC generator. The full range of loads may be applied by inputting an analogue voltage in the range 0 to 10V. This electronically varies the load on the generator. The DC generator was not used in the present study.

The CE 110 servo trainer has two additional removable inertia discs which may be added to the flywheel permanently fixed on the common shaft. An electrically operated clutch, enabled by a toggle switch, is used to study angular position responses by engaging an output shaft via a 30:1 reduction gearbox. The apparatus may also be used for the practical introduction into the design, operation and application of controllers with linear and non-linear parameters and inputs.

Figure 2 shows the TQ CE120 external analogue controller for the CE110 Servo Trainer. The CE 120 Controller board consists of signal generator, proportional, integral, derivative controllers and their



(a)



(b)

Figure 1: (a) TQ CE110 Servo Trainer; (b) schematic diagram of the servo trainer showing all components.

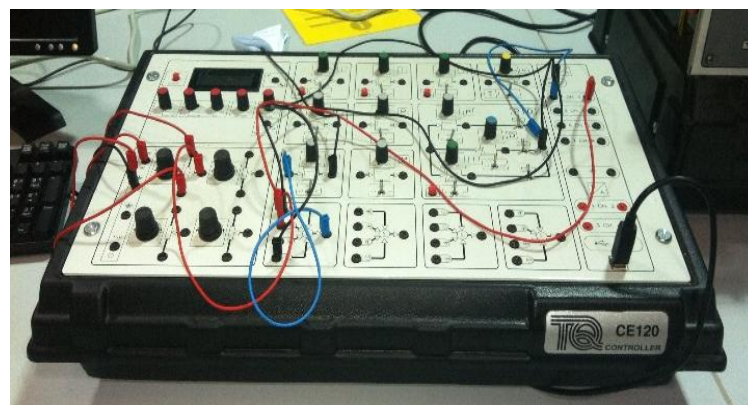


Figure 2: TQ CE 120 analogue controller.

combinations, and other accessories necessary to build either an open loop or closed loop control actions for the CE 110 servo trainer. Since all of the power supplies for the built-in devices and systems are included, only low power (0 to $\pm 10V$) connections between the CE110 and the external controller are required. Finally, Fig. 3 shows the PC in which the TQ CE2000 software is installed for input and output data acquisitions, analysis, display and storage. The CE 2000 software has a dongle without which the software will not work.

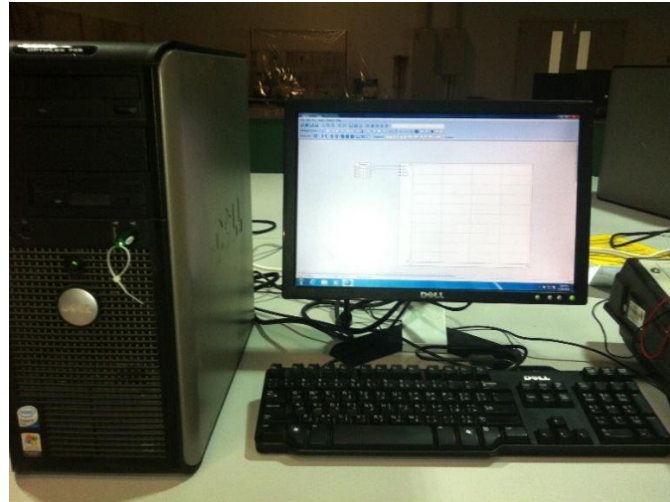


Figure 3: PC with CE 2000 software to measure and display responses of the DC servo motor.

2.2 Experimental Setup to Measure Responses of the DC Motor to Step Input

Figure 4 shows the schematic diagram for the experimental set-up to measure the open-loop responses of the DC motor to step inputs. The equipment used include CE110 Servo Trainer, CE120 Controller (only signal generation unit). CE 2000 Software installed on PC and accessories (connecting wires/cables). Prior to

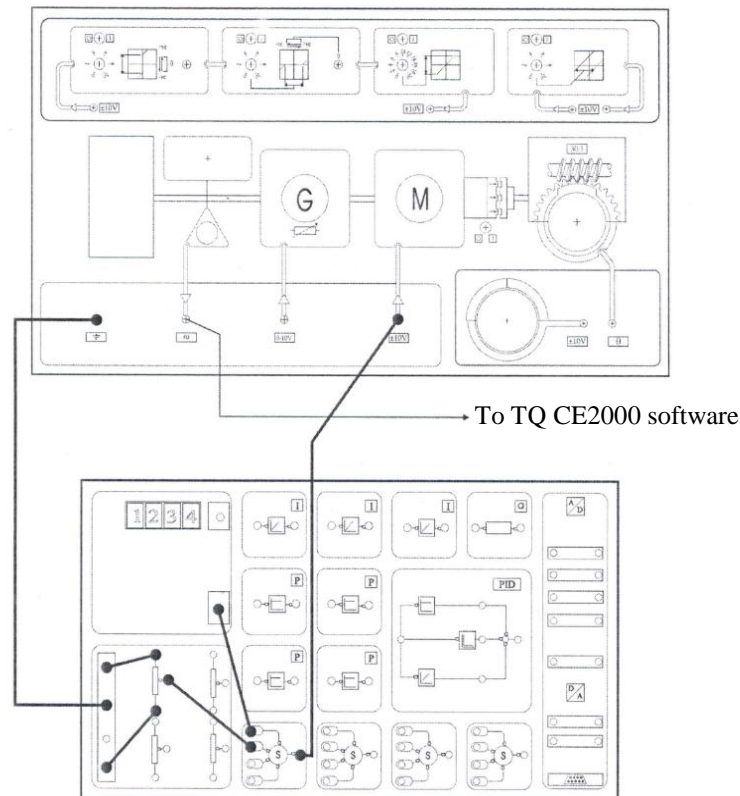


Figure 4: Experimental set-up for step response [13].

the experiment, the CE110's clutch for the gear system was disengaged and the additional two removable inertia discs were also removed. Step input to the DC motor was achieved by using square wave signal of 0.5 Hz from the signal generator on the CE120 Controller. The transitions in the square wave signal provide step changes in the input, and the output of the speed sensor will be a series of step responses, as shown in Fig. 5 from the CE 2000 Software monitor.

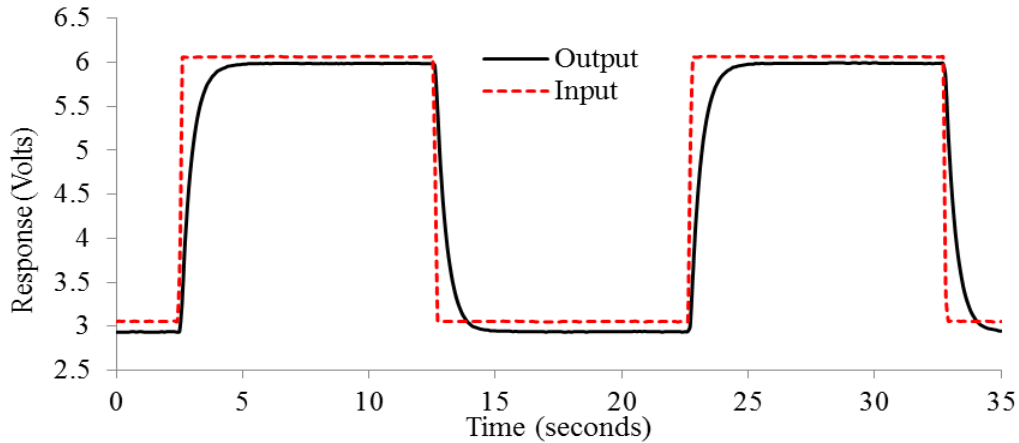


Figure 5: Experimental responses to step input.

III. Mathematical Modeling

The mathematical model of the DC servo motor is derived from the schematic diagram shown in Fig. 6. The Load Generator in Fig. 6(a) was not used in the study hence the load control voltage v_l is zero, however the mass of the rotating parts of the generator shaft is considered in the total mass moment of inertia J of the system. Fig. 6(b) shows a simplified schematic diagram of the DC motor and its accessories. For the mechanical components of the DC motor, Newton's second law of motion can be applied to obtain the mathematical model equation of motion. Newton's second law can be applied as:

$$\tau_m = b\omega + J \frac{d\omega}{dt} \tag{1}$$

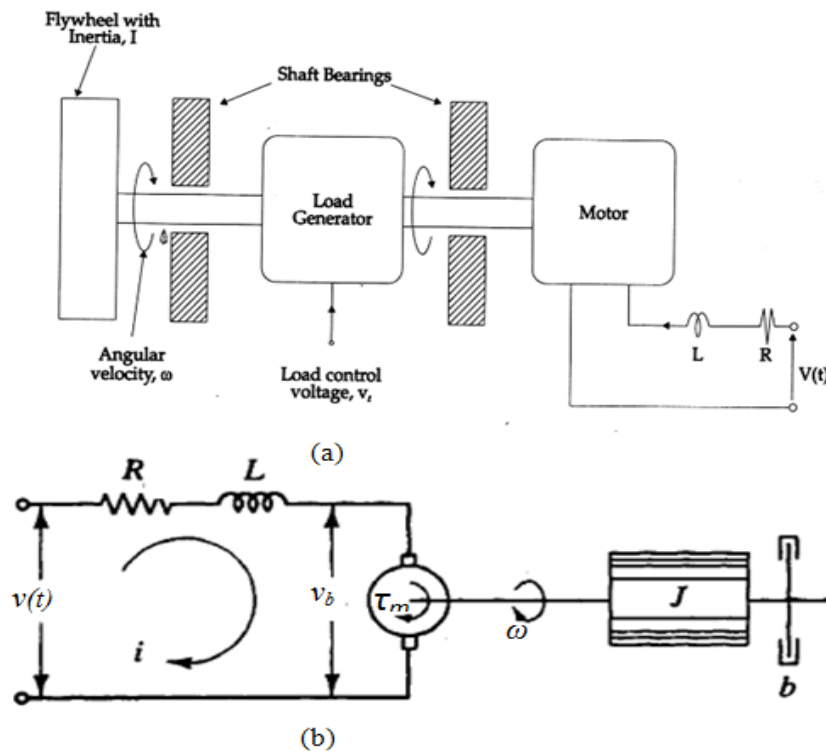


Figure 6: (a) Schematic diagram of DC motor and accessories [13]; (b) Simplified schematic diagram of DC motor and accessories.

where τ_m represents the DC motor's supplied torque (Nm), b is the effective viscous damping of the bearings in Ns/rad, J is the effective mass moment of inertia of the flywheel, and the shaft and armature of the DC motor and generator in kgm^2 , $d\omega/dt$ is the angular acceleration of the shaft of the DC motor in m/s^2 , and ω is the angular speed of the DC motor in rad/s. The mathematical model of the electrical circuit of the DC motor can be obtained by applying Kirchoff's Voltage Law (KVL). The electrical circuit equation is:

$$v(t) = Ri + L\frac{di}{dt} + v_b \tag{2}$$

where $v(t)$ is the input voltage to the DC motor in volts, R and i are the resistance and the current of the armature, respectively, L is the inductance of the armature windings, and v_b is the back emf. According to Faraday's laws of electromagnetic induction, the torques generated by the DC motor is proportional to the armature current, and the motor's back emf is proportional to the angular speed, where the proportionality constant is the motor constant k_m . Faraday's laws can be expressed as:

$$\tau_m = k_m i \tag{3}$$

$$v_b = k_m \omega \tag{4}$$

Taking Laplace transforms and combining Equations (1) through (4), the transfer function relating the output speed $\omega(s)$ to the input voltage $V(s)$ can be expressed, in frequency domain, as:

$$\omega(s) = \frac{k_m V(s)}{(sJ + b)(sL + R) + k_m^2} \tag{5}$$

The transfer function can be simplified by using the fact that the inductance L of the armature circuit is usually small compared with the inertia of the flywheel. This will give the following first order transfer function:

$$\frac{\omega(s)}{V(s)} = \frac{G_1}{Ts + 1} \tag{6}$$

where T , the time constant, is given as:

$$T = \frac{JR}{bR + k_m^2} \tag{7}$$

and G_1 , the gain of the DC motor, is given as:

$$G_1 = \frac{k_m}{bR + k_m^2} \tag{8}$$

3.1 Parameter Identification Techniques

The mechanical parameters (the effective mass moment of inertia J and damping b) and the electrical parameters (armature resistance R and servo motor constant k_m) are identified in two stages. The first stage involves the use of the System Identification Toolbox in MATLAB to determine the time constant T (Eq. (7)) and the motor gain G_1 (Eq. (8)) using the transfer function of the mathematical model expressed in Eq. (6). It should be noted that the time constant T of the electromechanical device and the motor gain G_1 are related to the mechanical and electrical parameters of the servo motor that are to be determined, as shown in Eqs. (7) and (8), respectively.

The second stage of the analysis of the experimental response of the servo motor was the writing of codes for Constraint Optimization Toolbox in MATLAB to determine the mechanical and electrical parameters (J , b , R and k_m related by Eqs. (7) and (8)) by minimizing the error between the experimental and model responses using the root-mean-square method. These two stages are to ensure the uniqueness of the parameter identification procedures.

3.1.1 Determination of time constant and gain of the DC motor

The time constant T and the proportionality constant or gain G_1 of the DC motor and accessories were first estimated from the experimental step response graph shown in Fig. 5 using the established concept presented in Fig. 7. The time constant is the time, in seconds, that the system response takes to reach 63.2 % of the steady-state response and the proportionality constant or gain can be determine from the steady-state value of the response, such that:

$$U G_1 = \text{steady-state value of response} \tag{9}$$

Figure 8 shows the experimental output response to step input after the DC offset was removed from Fig. 5 to ensure that the response starts from zero at time equals to zero to conform to Fig. 7. The amplitude of the step input U is obtained from Fig. 8 as 3.125 Volts, the steady state output response was also obtained from Fig. 8 as 3.045 Volts. From Eq. (9), G_1 was calculated to be 0.974. Similarly, the time constant T was calculated to be 0.5 seconds. These values of T and G_1 were used as the starting values (initial guess) in MATLAB System Identification Toolbox, which is discussed in the next subsection.

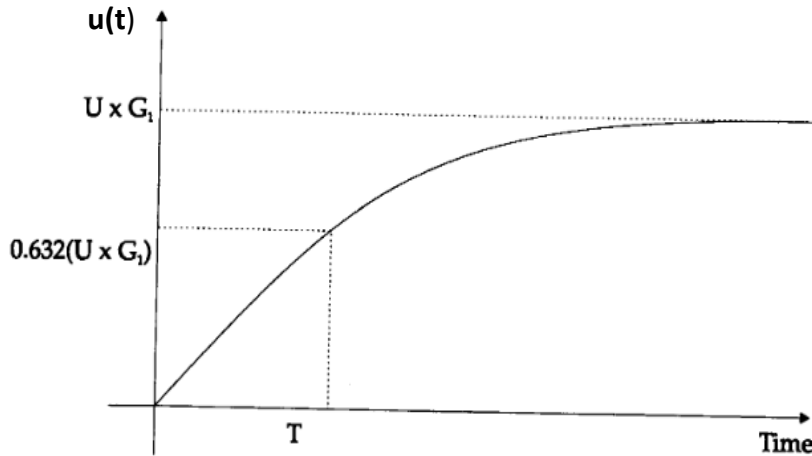


Figure 7: Standard step response graph [13].

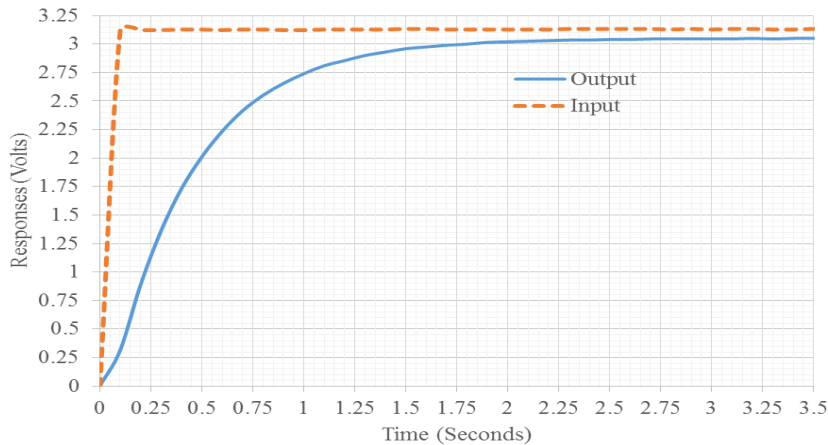


Figure 8: Experimental step response of the DC servo motor.

3.1.2 Determination of time constant and gain of DC motor using MATLAB system identification toolbox

In order to ensure accurate determination of the time constant T and the gain G_1 of the DC motor and accessories, MATLAB System Identification Toolbox was also used. The System Identification Toolbox in MATLAB is used for building accurate, simplified models of complex systems from time-series data. It provides tools for creating mathematical models of dynamic systems based on measured output and input data, and an expression for transfer function. The toolbox features a flexible graphical user interface (GUI) that aids in the organization of data and models. Figure 9 shows the main graphical user interface of the system identification toolbox in MATLAB. The toolbox has the option of specifying the starting values and the range within which the variable parameters could be changed during the error minimization computations. The values of T and G_1 obtained in section 3.1.1 above were used as the starting values (initial guess values) for the MATLAB System Identification Toolbox as shown in Fig. 9.

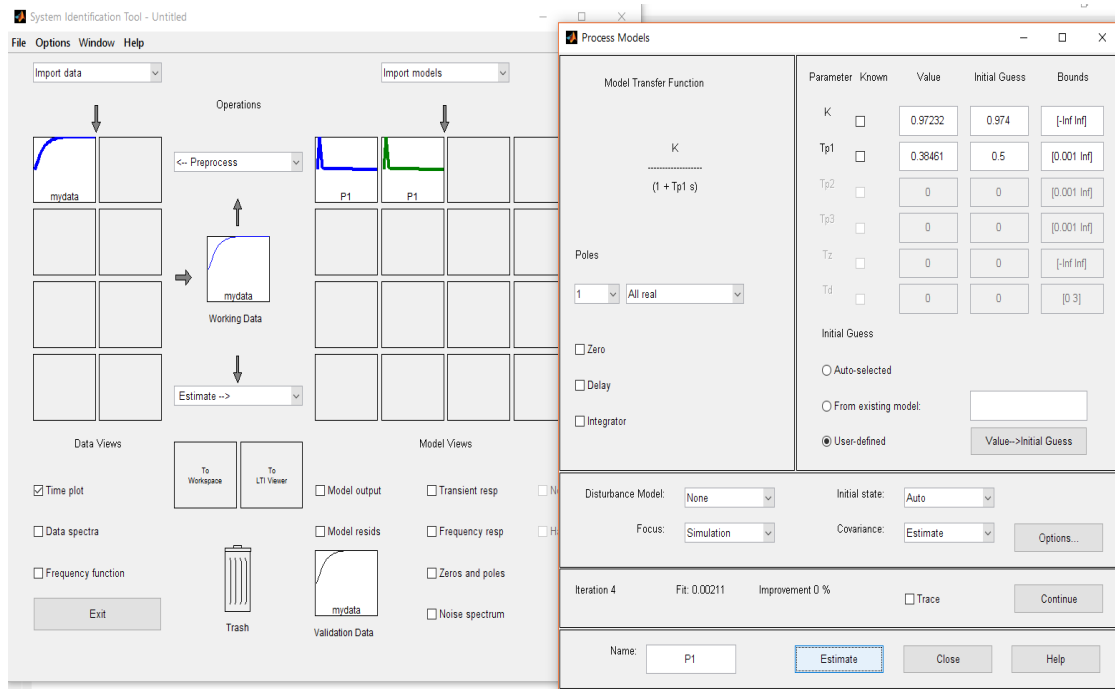


Figure 9: MATLAB System Identification Toolbox GUI

The experimental input and output data, shown in Fig. 8, corresponding to the first step response were imported into the MATLAB System Identification Toolbox using the GUI shown in Fig. 9. First order transfer function, similar to Eq. (6), was selected in the MATLAB System Identification Toolbox. In Fig. 9, K corresponds to G_1 , and T_{p1} corresponds to T in Eq. (6). The values of G_1 and T obtained from the MATLAB System Identification Toolbox are 0.9723 and 0.3846, respectively. The comparison of the experimental and simulated response graphs is presented and discussed in section 4 titled “Results and Discussion”.

3.1.3 DC motor parameters identification using MATLAB constraint optimization toolbox

The four parameters of the DC motor and its accessories are related to the two parameters obtained in subsection 3.1.2 by Eqs. (7) and (8). In order to ensure the uniqueness of the parameters, Constraint Optimization Toolbox in MATLAB was used. As stated earlier, the major challenge with system parameters identification is finding a unique solution since there are several combinations of the system’s parameters that could make the error between measured and simulated responses a minimum. In order to ensure the uniqueness of the solution of the optimization problem, the parameter J was estimated by calculating the mass moment of inertia of various components that could be easily calculated (flywheel and shaft). Furthermore, reported values for R , b and k_m [2, 6] were used as starting values (initial guess values) for the constraint optimization algorithm to determine the parameters of the DC servo motor for speed control. The parameters are determined such that the root-mean-square error between the model and measured responses are minimized and the values of J , R , b and k_m satisfy the relationships between parameters of the DC motor and the time constant T and the gain G_1 as expressed in Eqs. (7) and (8).

To accomplish this task, three MATLAB m-files were written. The first m-file is the main file that was used to call the other two m-files. The second file contains the objective function to be minimized, which is basically the MATLAB codes for the root-mean-square error between the model and measured responses. The results of the second m-file is indicated as $f(x)$ in Fig. 10. The third m-file contains the constraint equations, Eqs. (7) and (8), the results of which is shown as “Max Constraint” in Fig. 10. Figure 10 shows a screen shot of the MATLAB files and output window for the parameter identification using constraint optimization technique. The minimized value of $f(x)$ was found to be 0.0678 and the value of the constraint equation was found to be 8.274×10^{-13} . These values indicate that the solution of the constraint optimization algorithm converged. The comparison of the experimental and simulated response graphs is presented and discussed in section 4 titled “Results and Discussion”.

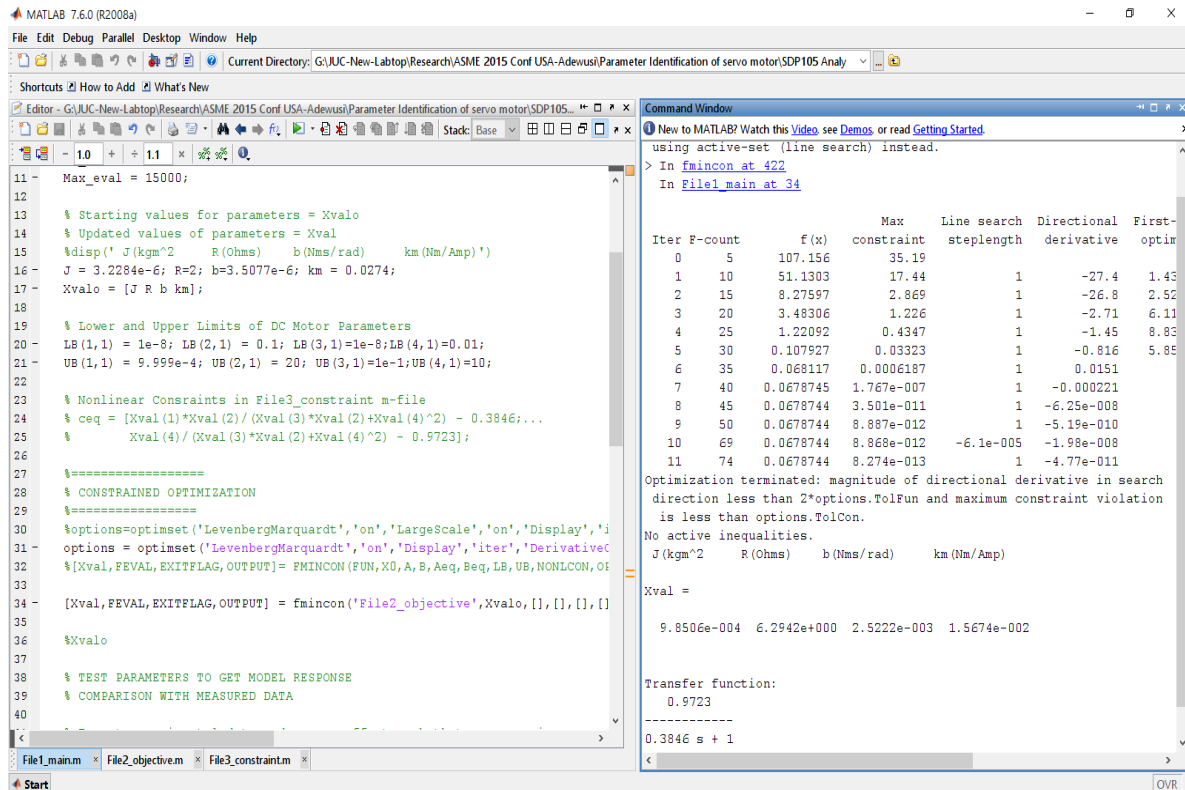


Figure 10: MATLAB files and output windows for constraint optimization.

IV. Results and Discussion

Figure 11 shows the comparison of the simulated and measured responses based on the values of $G_1 = 0.9723$ and $T = 0.3846$ seconds that were obtained from the MATLAB System Identification Toolbox. The two responses are similar except when time is less than 1.5 seconds where there is a slight difference in the measured and simulated responses. The corresponding values of G_1 and T that were directly calculated from the experimental response graph (Fig. 8) are 0.974 and 0.5 seconds, respectively. These values were only used as initial guess values since they are expected to be less accurate than those values obtained from MATLAB System Identification Toolbox, which minimizes the error between the measured and simulated responses. Therefore, the transfer function for the first order model of the DC servo motor is found to be as follows:

$$\frac{\omega(s)}{V(s)} = \frac{G_1}{Ts + 1} = \frac{0.9723}{0.3846 s + 1} \quad (10)$$

It should be noted that the output of the DC servo motor is speed in revolution per minute (RPM) or radians per second but the response graphs in Figure 11 are expressed in voltage (volts) for convenience. The output voltage of the DC motor could be converted to speed in RPM by using a sensitivity factor of 193.75 RPM/volt, which is reported in the user manual for the equipment [13]. The steady-state output response of the DC servo motor is 3.125 volts (Fig. 11), this corresponds to 605.5 RPM.

The comparison of the measured and simulated step responses of the DC motor using the identified parameters from MATLAB Constraint Optimization Toolbox are presented in Figure 12. The figure shows good agreement between the experimental and simulated responses of the system above 1.5 seconds, similar to Fig. 11. A slight deviation between the experimental and simulated responses is observable below 1.5 seconds in both Figs. 11 and 12. This may be attributed to error in simulation when there is a sudden change in the value of step input signal from zero to 3.125 volts (Fig. 8). The mechanical and electrical parameters of the DC motor that were used to obtain the simulated response in Fig. 12 are: mass moment of inertia $J = 9.85 \times 10^{-3} \text{ kgm}^2$, armature resistance $R = 6.29 \text{ Ohms}$, damping $b = 2.52 \times 10^{-3} \text{ Nms/rad}$, and motor constant $k_m = 1.57 \times 10^{-2} \text{ Nm/Amp}$. These values were obtained from the constraint optimization.

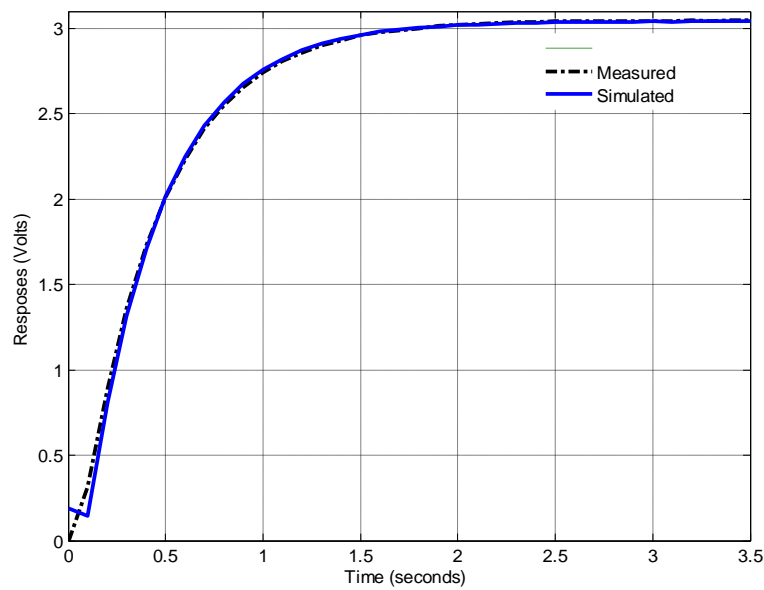


Figure 11: Comparison of experimental and simulated responses using MATLAB System Identification Toolbox for determination of G_1 and T .

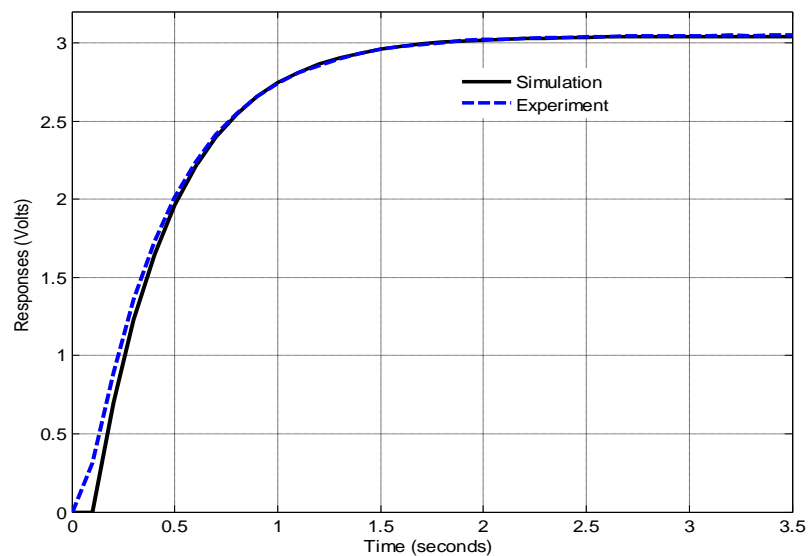


Figure 12: Comparison of experimental and simulated responses using MATLAB constraint optimization toolbox

V. Conclusion

The mechanical and electrical parameters (mass moment of inertia J , damping b , armature winding resistance R and direct current motor constant k_m) of a DC servo motor (TQ CE 110 servo trainer) and its accessories were obtained. These parameters are not reported in the user manual for the device by the manufacturer but the parameters are needed for a proper design and testing of speed controller for the DC motor to enhance students' learning. Laboratory experiments were performed to obtain the step responses of the DC motor and a mathematical model was also derived for the DC motor in terms of the four unknown parameters. The parameters were obtained by minimizing the root-mean-square error between the model response and experimental step response using System Identification and Constraint Optimization Toolboxes in MATLAB. The results of comparison between the experimental and simulated step responses showed excellent agreement and the corresponding mechanical and electrical parameters of the DC motor were found to be mass moment of inertia $J = 9.85 \times 10^{-3} \text{ kgm}^2$, armature resistance $R = 6.29 \text{ Ohms}$, damping $b = 2.52 \times 10^{-3} \text{ Nms/rad}$, and motor

constant $k_m = 1.57 \times 10^{-2}$ Nm/Amp. These parameters were used to prepare laboratory manual on design and simulation of Proportional-Integral controller for speed control, which is a topic in the syllabus for Systems Dynamic and Control course at the Department of Mechanical Engineering, Jubail University College, Saudi Arabia. This work could be extended to study the position control and responses of the TQ CE 110 servo trainer when the clutch is engaged to include the gearbox.

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