

- Discretize the continuous plant, either the state-space model or the transfer function, to obtain a DT system. Use that DT system to design a DT controller.

- Design a CT controller, and then discretize that to obtain a DT controller.

The objective of this paper is to provide smooth movement and zero steady state speed error for a selected speed by minimizing the overshoot and optimizing the settling time through turning the controller

Here we will focus on the second method. Given a continuous-time controller, designed by any technique (PID). We will show how to convert it to a digital controller. The controller will be designed by a Root Locus method.

A digital DC motor model can obtain from conversion of analog DC motor model. A proportional–integral–derivative controller (PID controller) is a generic control loop feedback mechanism (controller) widely used in industrial control systems; The PID controller algorithm involves three separate constant parameters, and is accordingly sometimes called three-term control: the proportional, the integral and derivative values, denoted P, I, and D.

1.1 Physical System Background

Typical models for a DC motor contains one differential equation for the electric part, one differential equation for the mechanical part, and their interconnections. This is valid for the PMDC motors as well.

The motor torque, T, is related to the armature current, i, by a constant factor Kt. The back emf, e, is related to the rotational velocity by the following equations:

$$\begin{aligned} T &= K_t i \\ E &= K_e \Theta' \end{aligned}$$

In SI units (which we will use), Kt (armature constant) is equal to Ke (motor constant).

From the figure above we can write the following equations based on Newton's law combined with Kirchhoff's law:

$$\begin{aligned} J \Theta'' + b \Theta' &= K i \\ L \frac{di}{dt} + Ri &= V - K \Theta' \end{aligned}$$

1.2 Transfer Function

Using Laplace Transforms, the above modeling equations can be expressed in terms of s.

$$\begin{aligned} S (Js+b) \Theta(s) &= KI(s) \\ (Ls+R) I(s) &= V - Ks\Theta(s) \end{aligned}$$

By eliminating I(s) we can get the following open-loop transfer function, where the rotational speed is the output and the voltage is the input.

$$\frac{\Theta}{v} = \frac{K}{(Js + b)(Ls + R) + K^2}$$

1.3 State-Space Equations

In the state-space form, the equations above can be expressed by choosing the rotational speed and electric current as the state variable and the voltage as an input.

The output is chosen to be the rotational speed.

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} \dot{\Theta} \\ i \end{bmatrix} &= \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \dot{\Theta} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v \\ \dot{\Theta} &= [1 \quad 0] \begin{bmatrix} \dot{\Theta} \\ i \end{bmatrix} \end{aligned}$$

2.1 Digital Controller Design Methodology

In practice controllers are nowadays almost exclusively implemented digitally. This means that the controller operates in discrete time, although the controlled systems usually operate in continuous time.

Digital controllers are far more convenient to implement on microprocessors than are continuous-time controllers. Continuous-time controllers must be implemented either using analog circuitry. Discrete-time controllers, on the other hand, are easily implemented using difference equations, i.e. simple computer software.

Two methods are available for design of digital controllers:

- Discretize the CT plant, either the state-space model or the transfer function, to obtain a DT system. Use that DT system to design a DT controller.
- Design a CT controller, then discretize that to obtain a DT controller.

Here we will focus on the second method. Given a continuous-time controller, designed by any technique (root locus, PID, lead, lag, etc.), and show how to convert it to a digital controller.

2.2 Discretization of Continuous-Time Controllers Methodology

By any of a variety of techniques, one may design a continuous-time compensator $K(s)$. This may be converted to digital form $K(z)$ using several techniques, among the most direct of which is the bilinear transformation (BLT).

The relation between the Laplace transform variable s and the Z-transform variable z is $z=e^{sT}$, with T the sampling period. However, using this to transform $K(s)$ to $K(z)$ will give non-polynomial transfer functions. Note that

$$e^{sT} \approx \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}}$$

Therefore define the BLT by

$$z = \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}}$$

and its inverse

$$S = \frac{2Z - 1}{TZ + 1}$$

To convert a continuous transfer function $G(s)$ to a discrete transfer function using sample period T .

Then, one simply replaces all occurrences of s by

The continuous-time PID controller can be written in the form

$$G_c(s) = Kc[1 + \frac{1}{\tau_I s} + \tau_D s]$$

where T_i is the integration time constant or 'reset time', T_d is the derivative time constant,

To convert this to digital form using the BLT, write

$$G_c(z) = Kc[1 + \frac{1}{\tau_I \left(\frac{2z-1}{Tz+1}\right)} + \tau_D \left(\frac{2z-1}{Tz+1}\right)]$$

This may be simplified to obtain

$$G_c(z) = Kc[1 + \frac{T(Z+1)}{\tau_I(2(Z-1))} + \tau_D \left(\frac{2Z-1}{TZ+1}\right)]$$

Then

$$G_c(z) = Kc[1 + \frac{T(Z+1)}{\tau_{ID}(z-1)} + \frac{\tau_{DD}}{T} \left(\frac{z-1}{z+1}\right)]$$

where the digital integral and derivative time constants are

$$\begin{aligned} \tau_{ID} &= 2\tau_I \\ \tau_{DD} &= 2\tau_D \end{aligned}$$

3.1 Planet modeling

We can express the DC speed motor system by the following diagrams

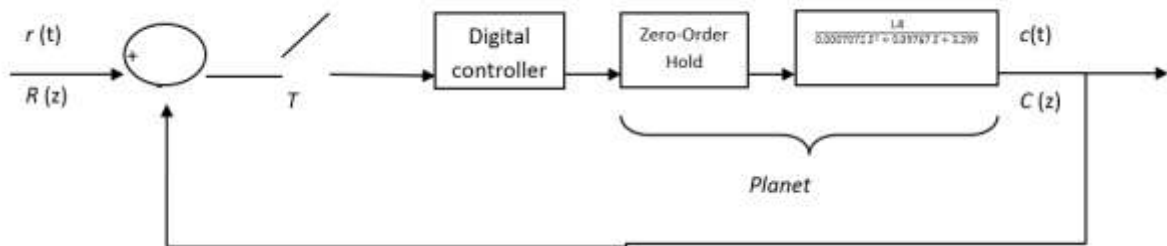


Fig 2: Digital Control System of the Motor

3.2 Design requirements

The most basic requirement of a motor is that it should rotate at the desired speed; the steady-state error of the motor speed should be less than 1%. The other performance requirement is that the motor must accelerate to its steady-state speed as soon as it turns on. In this case, we want it to have a settling time of 2 seconds. Since a speed faster than the reference may damage the equipment, we want to have an overshoot of less than 5%. If we simulate the reference input (r) by a unit step input, then the motor speed output should have:

- Settling time less than 2 seconds
- Overshoot less than 5%
- Steady-state error less than 1%

3.3 Planet Transfer Function in Continues and Discrete (open loop)

$$G(s) = \frac{1.8}{0.0007072s^2 + 0.09767s + 3.299}$$

To find $G(z)$ using ZOH cascaded with the planet

$$G(Z) = \zeta \left\{ \frac{1 - e^{st}}{s} \cdot \frac{1.8}{0.0007072s^2 + 0.09767s + 3.299} \right\}$$

$G(Z) =$

$$(1 - z^{-1}) \zeta \left\{ \frac{1.8}{s(0.0007072s^2 + 0.09767s + 3.299)} \right\}$$

The first part is $\frac{(z-1)}{z}$

From the table we can get the

$$\zeta \left\{ \frac{1.8}{s(0.0007072s^2 + 0.09767s + 3.299)} \right\}$$

Then after simplification:

$$G(z) = \frac{0.4607z^4 + 1.161z^3 + 0.9688z^2 + 0.2974z + 0.02894}{z^4 + 0.52z^3 - 0.9372z^2 - 0.52z - 0.06285}$$

3.4 Continuous Time PID Controller for continues signal Design Procedure

More practical approach is to specify the closed loop transfer function so the realistic setting time is achieved. Design of continuous PID controller for continuous signal, If the system of second order as

$$G(s) = \frac{K}{(\tau_1 S + 1)(\tau_2 S + 1)}$$

$$G_c(s) = \frac{\tau_1 + \tau_2}{K\tau_c} \left[1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} s \right]$$

Comparing with

$$G_c(s) = Kc \left[1 + \frac{1}{\tau_I s} + \tau_D s \right]$$

Where

$$Kc = \frac{\tau_1 + \tau_2}{K\tau_c}$$

$$\tau_I = (\tau_1 + \tau_2)$$

$$\tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

Comparing the used motor transfer function to the previously mentioned TF.

$$G(s) = \frac{K}{(\tau_1 S + 1)(\tau_2 S + 1)}$$

$$G(s) = \frac{K}{\tau_1 \tau_2 S^2 + (\tau_1 + \tau_2)S + 1}$$

$$G(s) = \frac{1.8}{0.0007072S^2 + 0.09767S + 3.299}$$

Divide by 3.299 the numerator and denominator to make the coefficient of the lowest power is 1 in order to use the controller equation mentioned above.

$$G(s) = \frac{\frac{1.8}{3.299}}{\frac{0.0007072}{3.299} S^2 + \frac{0.09767}{3.299} S + 1}$$

Comparing the two G(s) equations

$$K = 1.8 / 3.299 = 0.54562$$

$$\tau_1 \tau_2 = \frac{0.0007072}{3.299} = 2.14368 * 10^{-4}$$

$$\tau_1 + \tau_2 = \frac{0.09767}{3.299} = 2.96059 * 10^{-2}$$

$$Kc = \frac{\tau_1 + \tau_2}{K\tau_c} = \frac{0.029605}{0.54562\tau_c} = 1 \text{ when } \tau_c = \frac{0.54562}{0.029605} = 18.43$$

Where τ_c is design parameter

$$\tau_I = (\tau_1 + \tau_2) = 0.029605$$

$$\tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} = \frac{0.0214368}{0.029605} = 7.2404 * 10^{-3}$$

$$G_c(s) = [1 + \frac{1}{0.029605s} + (7.2404 * 10^{-3})S]$$

3.4 Discretize the system using Bilinear Transformation

To convert this to digital form using the BLT, write

$$G_c(z) = Kc[1 + \frac{T(Z + 1)}{\tau_{ID}(z - 1)} + \frac{\tau_{DD}}{T}(\frac{z - 1}{z + 1})]$$

where the digital integral and derivative time constants and the sampling period are

$$\tau_{ID} = 2\tau_I = 2 * 0.029605 = 0.05921$$

$$\tau_{DD} = 2\tau_D = 2 * 7.2404 * 10^{-3} = 0.0144808$$

$$T = 0.05$$

$$Kc = 1$$

Then

$$G_c(z) = [1 + \frac{0.05(Z + 1)}{0.05921(z - 1)} + \frac{0.0144808}{0.05}(\frac{z - 1}{z + 1})]$$

After Simplification the controller Transfer Function:

$$G_c(z) = \frac{2.134z^2 + 1.11z + 0.1341}{z^2 - 1}$$

3.5 The Closed Loop Discrete Time System

$$C(z) = \frac{G_c(z)G(z)}{1 + G_c(z)G(z)}R(z)$$

We would like to see what the closed-loop response of the system looks like when no controller is added. First, we have to close the loop of the transfer function by using the feedback command. After closing the loop, let's see how the closed-loop stair step response performs by using the stepand stairs commands. The step command will provide the vector of discrete step signals and stairs command will connect these discrete signals.

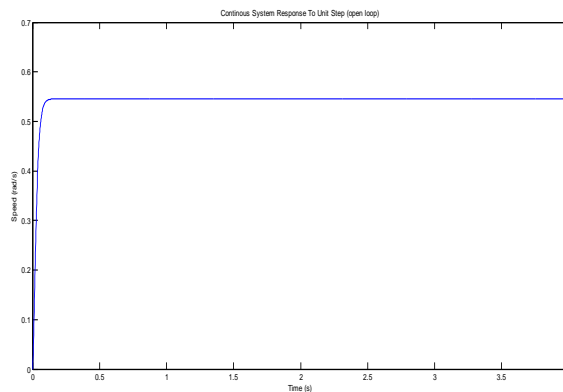


Fig 3: The Continues System Response to unit step (open system)

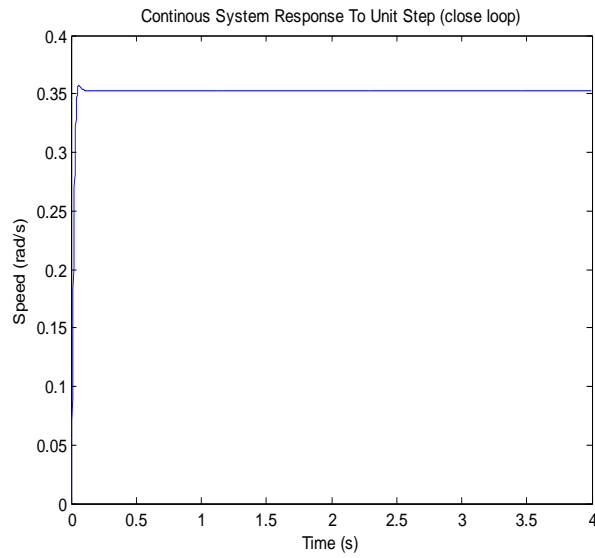


Fig 4: The Continues System Response to unit step (Close system)

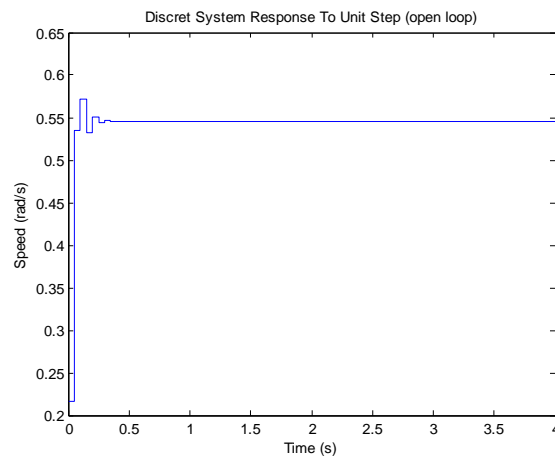


Fig 5: The Discrete System Response to unit step (Open loop system)

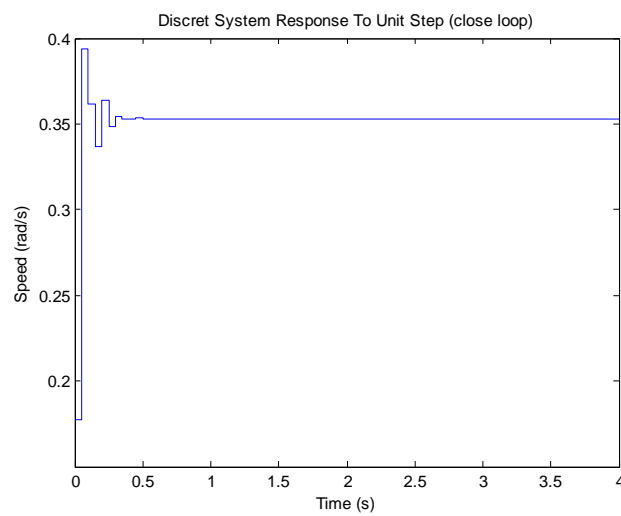


Fig 6: The Discrete System Response to unit step (Close loop system)

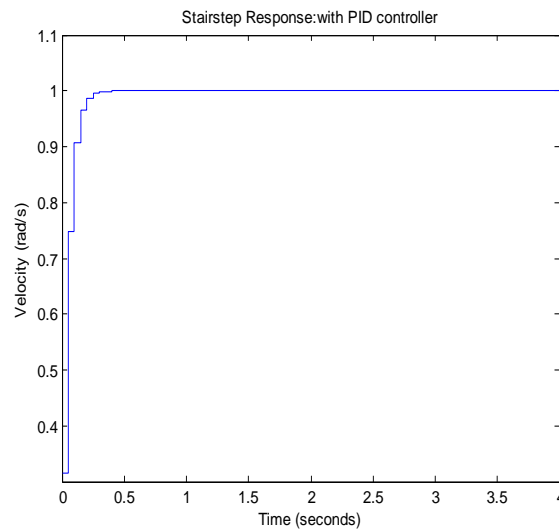


Fig 7: The Discrete System Response to unit step (Close system with PID controller)

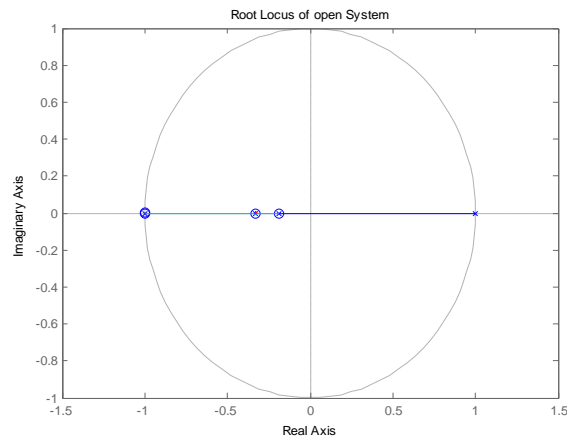


Fig 8: The Root Locus of the Open system

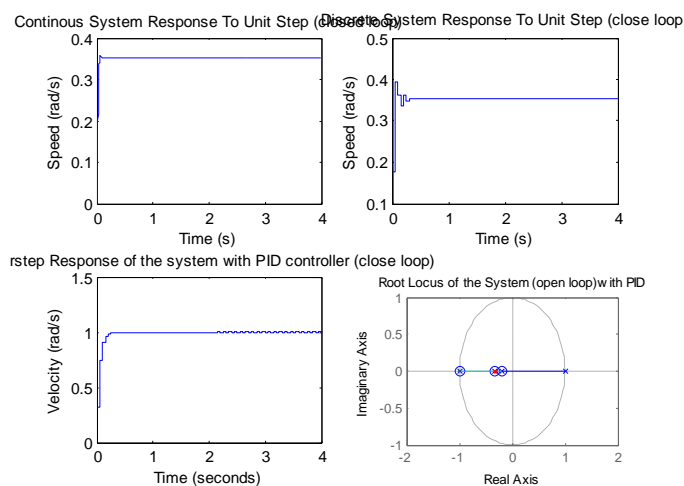


Fig 9: Summarized Results

From the plot original open-loop system performance we see that when 1 volt is applied to the system, the motor can only achieve a maximum speed of 0.1 rad/sec, ten times smaller than our desired speed. Also, it takes the motor 3 seconds to reach its steady-state speed; this does not satisfy our 2 seconds settling time

criterion. The plot above shows that the settling time is less than 2 seconds and the percent overshoot is around 2%. Additionally, the steady-state error is zero. Therefore, this response satisfies all of the given design requirements.

4.3 Simulink PID Design and Simulation

4.3.1 Simulink Block Diagram

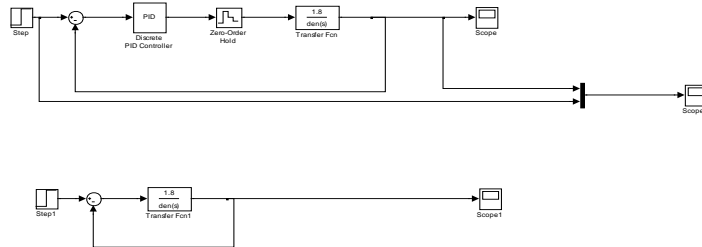


Fig 10: The Simulink Block Diagram of Discrete system with PID controller and the Continuous system

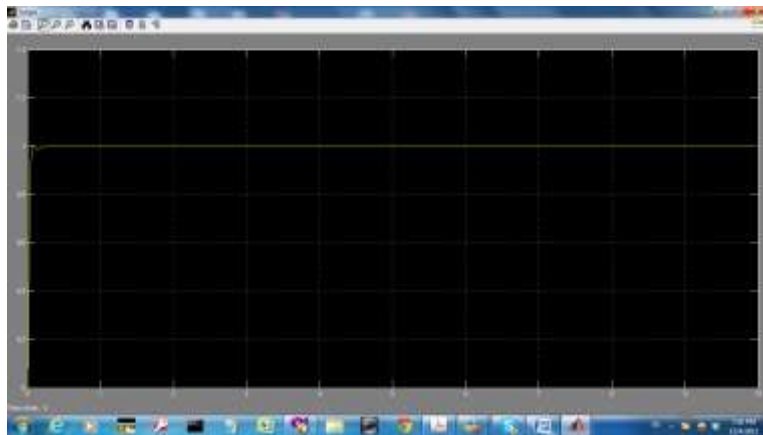


Fig 15: Step Input Response Result with PID controller.

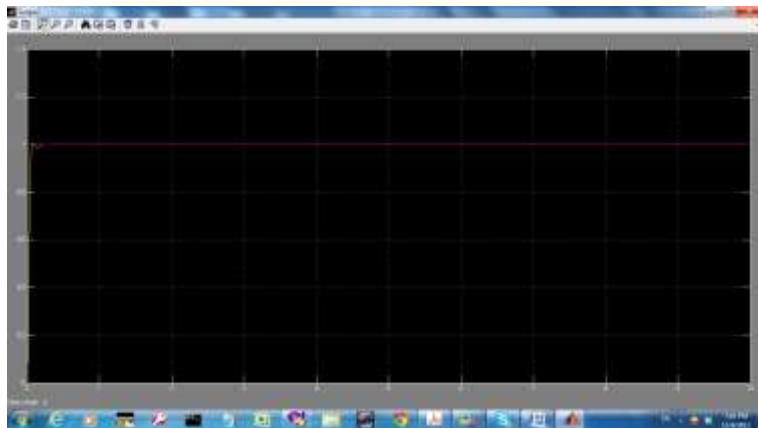


Fig 16: Step Input Response Result with PID controller besides the Step Input.

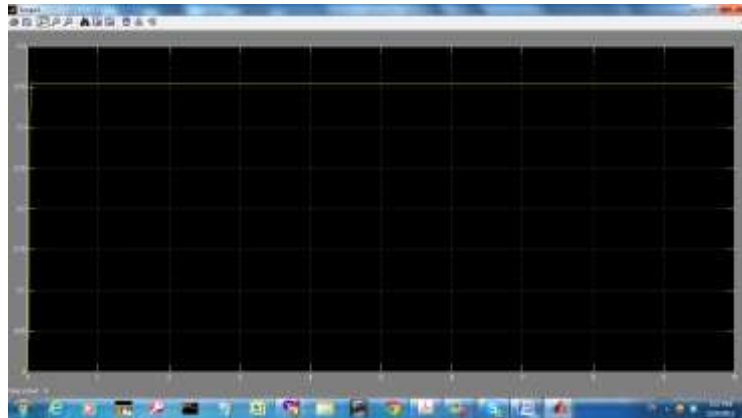


Fig 17: Step Input Response Result without PID controller.

4.3.3 Simulink in simplified form

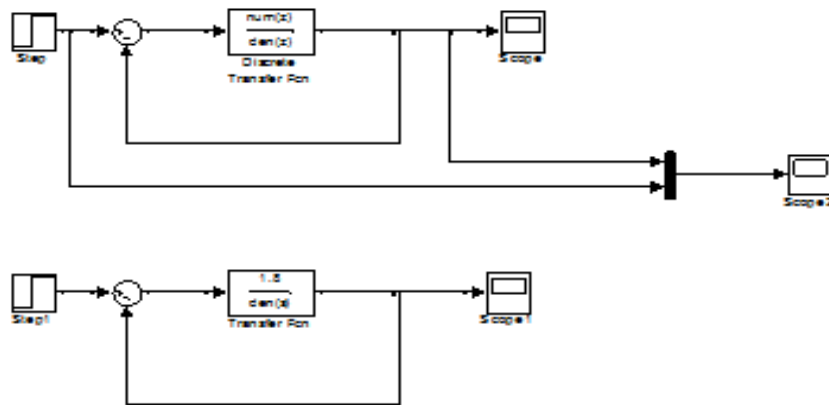


Fig 18: Simulink block Diagram

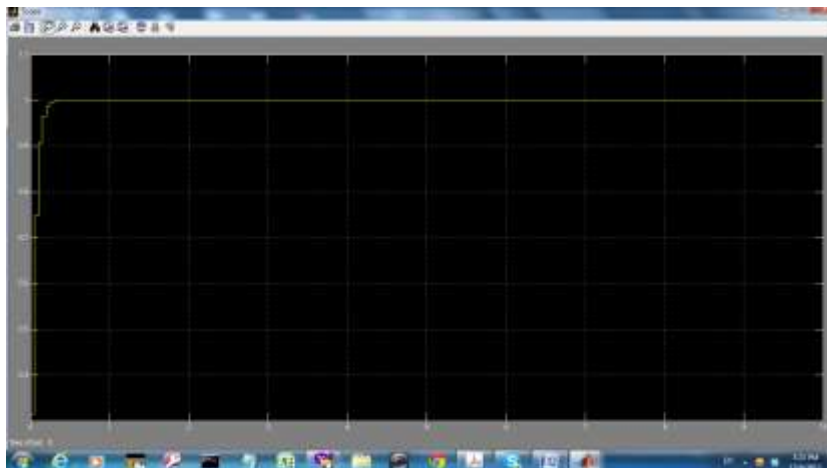


Fig 21: Step Response of the closed system with PID controller.

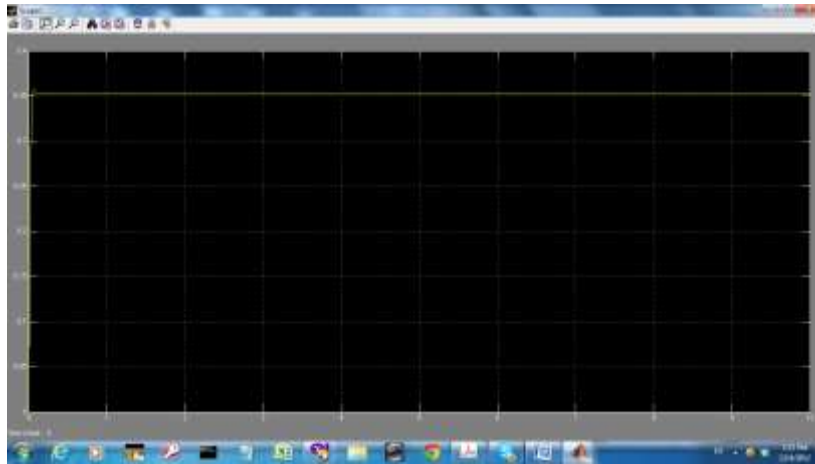


Fig 22: Step Response of the open loop system .

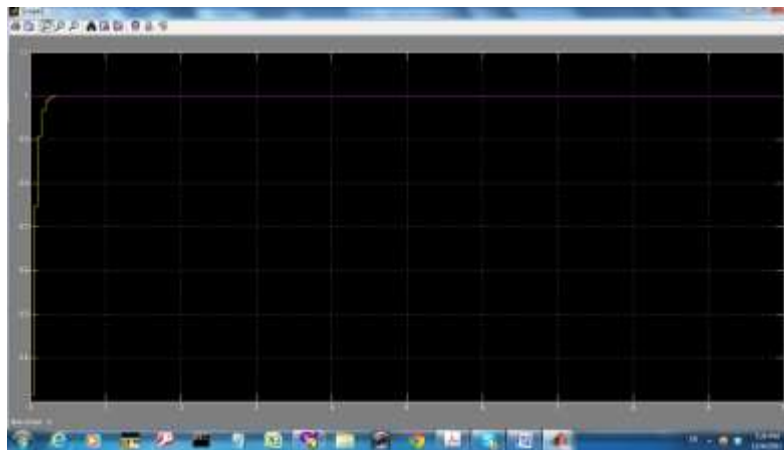


Fig 23: Step Response of the close loop system with step input.

The result of the simplified form is the same as the original system. From the plot original open-loop system performance we see that when 1 volt is applied to the system, the motor can only achieve a maximum speed of 0.1 rad/sec, ten times smaller than our desired speed. Also, it takes the motor 3 seconds to reach its steady-state speed; this does not satisfy our 2 seconds settling time criterion. The plot above shows that the settling time is less than 2 seconds and the percent overshoot is around 2%. Additionally, the steady-state error is zero. Therefore, this response satisfies all of the given design requirements.

II. Conclusion

In this paper we have used MATLAB Simulink to design a digital PID controller to control a DC motor (continuous time transfer function). Electric machines are essential systems in electric vehicles and are widely used in other applications. In particular, permanent magnet direct current (PMDC) motors have been extensively employed in industrial applications such as battery powered devices like wheelchairs and power tools, guided vehicles, welding equipment, X-ray and tomographic systems, CNC machines, etc. PMDC motors are physically smaller in overall size and lighter for a given power rating than induction motors. Because of this controlling such motors is very important. In this paper we used a digital PID controller to control the speed of the motor. Digital controllers are far more convenient to implement on microprocessors than are continuous-time controllers. Continuous-time controllers must be implemented either using analog circuitry (op amps). Discrete-time controllers, on the other hand, are easily implemented using simple computer software. Two methods are available for design of digital controllers. Discretize the continuous plant is the first used method, either the state-space model or the transfer function, to obtain a DT system. Use that DT system to design a DT controller. The second method is to Design a CT controller, and then discretize that to obtain a DT controller. Here we focus on the second method. Given a continuous-time controller, designed by any technique (PID). A proportional-integral-derivative controller (PID controller) is a generic control loop feedback mechanism (controller) widely used in industrial control systems. The PID controller algorithm involves three

separate constant parameters, and is accordingly sometimes called three-term control: the proportional, the integral and derivative values, denoted P, I, and D.

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