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Effects of Suction/Injection on Transient Couette Basic Gaseous Fluctuating-Micro-Flow

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Abstract: *The transient Couette fluctuating-micro-flow where the upper plate moves with sinusoidal velocity while the bottom plate remained stationary is investigated in this study. The flow problem is modelled and the analytical solution is obtained. The effects of suction/injection and that of other leading flow parameters such as the frequency of the fluctuating driving force, the Knudsen number and the unsteadiness are analyzed, and numerical values for skin friction are obtained. It is found from the study that both suction and injection retard the velocity of the fluid.*

Keywords: *Suction, Injection, Couetteflow,sinusoidal velocity, fluctuating-micro-flow.*

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I. Introduction

Fluid flows in micro-scale devices have several applications in many fields of human endeavor, such as in industrial and medical fields. Micro pumps are used for inkjet printing, environmental testing and electronic cooling. Micro-ducts are found to have played important roles in infrared detectors, diode lasers, and in high frequency fluidic control systems. Micro devices are found useful in accelerators for automobiles airbags, keyless entry systems, blood analyzers, reactor for separating biological cells, and in dense arrays of micro-mirrors for high definition optical displays. In medical field, small pumps are used in controlled delivery and monitoring of minute amount of medication, manufacturing of nano liters of chemicals and in development of artificial pancreas. These numerous applications of micro-fluidic systems make micro-fluidic flow a trending area of research in hydrodynamics[1].

In 1821, Clause Navier and George Gabriel Stokes developed equations named Navier-Stokes equations which can be used to figure out the velocity vector field that applies to a fluid, given some initial conditions. These equations arose from the application of Newton's second law of motion in combination with a fluid stress and pressure term. Later in 1823, Navier, treatise on the movement of fluids where he introduced the linear boundary conditions (this was alsoproposed later by Maxwell[2])[3]. These were based on the assumption of no-slip boundary conditions (that all the three of the solid surfaces are equal to the respective velocity components of the surfaces) and remained the standard characterization of slip conditions used today[4]. Knudsen (1909) was the first person to measure the mass flow rates in micro-channels with slip/no-slip boundary conditions[5]. The slip/no-slip boundary conditions in hydrodynamics has received immense consideration, and was highly debated in the 19th century (the reader may be refer to [6, 7] for historical reviews).However, it is recently found that control experiments with typical dimensions microns have demonstrated an apparent violation of the no-slip boundary conditions for the flow of Newtonian fluids near a solid surface [4].

Arkilic et al carried out investigation on slip flow in micro-channels. Theythoroughly investigated the effect of slip velocity on the mass flow prediction of the Navier-Stokes equations and reported that the Navier-Stokes equation fails to adequately model momentum transferred from the fluid to the channel wall,but whenthey included the slip flow boundary conditions at the wall (which is derived from momentum balance), they accurately modeled the mass flow-pressure relationship[8]. Xu et al carried out molecular dynamics studies of velocity slip phenomena in a nano-channel and found that the velocity slip is largely determined by the fluid temperature and attractive forces of the wall[9].Xie et al introduced second-order velocity slip boundary conditions into the Navier-Stokes equations[10]. Zhou J. et al investigated boundary velocity slip of pressure driven liquid flow in a micron-pipe. Their investigation reveals that a small velocity slip may occur when the pressure gradient is relatively large, and this results to errors in the micron-pipe flow estimates[11].

Haddad and Al-Nimranalyzed the effect of frequency of fluctuating driven force on basic gaseous micro-flows in slip flow regime, where they considered four flowcases, namely; the transient Couette flow, the pulsating Poiseuille flow, the Stokes second problem and natural convention. In their analysis,they found theslip in velocity and jump in temperature to increase with increase in Knudsen number and/or frequency of the driving forces,thoughtheir effect is found to be negligible for sufficiently small frequency of the driving forces and/or the Knudsen number[12].

The first researcher to study the problem of steady flow in an incompressible viscous liquid through a porous channel with rectangular cross-section using low Reynolds number wasBerman [13]. He obtained perturbation solution by assuming normal wall velocity to be equal. Later,Sellars extended the problem studied by Berman using high Reynolds number[14].Yuan worked on the same problem, but assumed different normal velocity at the wall[15].The flows considered by these authors (Berman, Sellars and Yuan above) are symmetrical. Terrill and Shrestha considered asymmetric flow in a channel for walls of different permeability [16]. High order velocity slip boundary conditions for incompressible flow model was developed by Beskok et al [17].Most recently, Hafeez and Ndikilaranalyzed the problem for flow of fluid between two parallel permeable plates subjected to bottom injection and top suction. Considering the flow velocity along the two boundaries to be uniform,they found the velocity to decrease with increase in Reynolds number[18].

Suction or injection of fluid through the bounding surfaces, as for example, in mass transfer, cooling can change the flow field and as a consequence affects the rate of heat transfer from the bounding surfaces. In general, suction tends to increase the skin friction and heat transfer co-efficients, whereas injection acts in the opposite manner. Injection/suction of fluid through porous heated/cooled surfacesare of general interest in practical problems involving film cooling, control of boundary layers, etc. This can lead to enhanced heating/cooling of the system and can help to delay the transition from laminar to turbulent flow [19]. In this work, the effect of suction/injection on the transient Couette flow discussedby Haddad et al in [12] isinvestigated.

II. Formulation of the problem

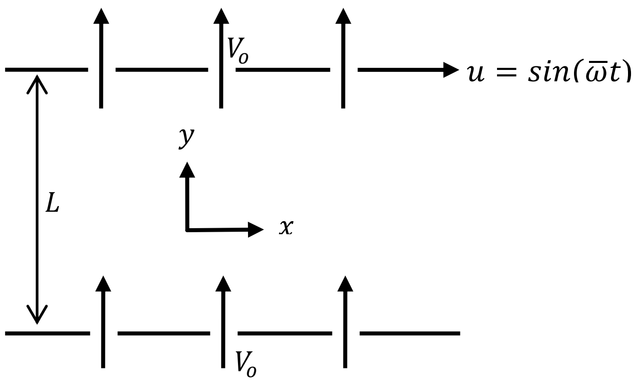

Fig. 1: *Schematic diagram of the flow problem*

Consider the fully developed Couette flow of a basic micro-gas, in the slip flow regime, between horizontal micro-channel separated by two parallel permeable plates of infinite lengths,where the upper plate moves with sinusoidal velocity (i.e. $u = \sin(\varpi t)$) in its plane, while the lower plate remains stationery.The distance between the plates is L. A coordinate system is chosen such that the x-axis is parallel and the y-axis is orthogonal to the plates. Furthermore, fluid is being injected into the flow region through the plate at $y = 0$ and in order to conserve the mass of the fluid in the micro-channel, fluid being sucked out of the channel at the same rate through the plate at $y = L$.

The governing equation for this flow case is as follows:

$$\rho\left(\frac{\partial u}{\partial t}+V_o\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2} \qquad 2.1$$

Boundary conditions with slip applied on both walls:

$$u(t,0)=\frac{2-\sigma_v}{\sigma_v}\lambda\left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad 2.2$$

$$u(t,L)-\sin(\varpi t)=-\frac{2-\sigma_v}{\sigma_v}\lambda\left.\frac{\partial u}{\partial y}\right|_{y=L} \qquad 2.3$$

Considering the following dimensionless variables:

$$\tau=\frac{t}{t_r},\ \omega=\frac{\varpi}{\omega_r},\ Y=\frac{y}{L},\ Kn=\frac{\lambda}{L},\ \ U=\frac{u}{u_o},\ S=\frac{V_oL}{\upsilon}, \qquad 2.4$$

The above governing equations and boundary conditions are transformed as follows:

$$\frac{\partial U}{\partial \tau}+S\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2} \qquad 2.5$$

Boundary conditions with velocity slip applied on both walls:

$$U(\tau,0)=\frac{2-\sigma_v}{\sigma_v}Kn\left.\frac{\partial U}{\partial Y}\right|_{Y=0} \qquad 2.6$$

$$U(\tau,1)-\sin(\omega\tau)=-\frac{2-\sigma_v}{\sigma_v}Kn\left.\frac{\partial U}{\partial Y}\right|_{Y=1} \qquad 2.7$$

Where Kn is the Knudsen number, which is the ratio of the characteristic length of the flow gradients (L) and the mean free path (λ, i.e. the average distance travelled by molecules between collisions). Knudsen number determines the degree of flow rarefaction or degree of scaling of the flow problem. It's also used to classify fluid flow into different regimes [20].

III. Analytical Solution

An exact solution for these flow problems is possible by assuming the following complex solution:

$$U(\tau,Y)=\mathrm{Im}\{\exp(i\omega\tau)V(Y)\} \qquad 3.1$$

Where $'\mathrm{Im}'$ represents the imaginary part of the complex solution and $i=\sqrt{-1}$. Differentiating Eq. (3.1) and substituting into the governing equations and boundary conditions, we transformed them into ordinary differential equations having the following solutions:

$$V(Y)=\frac{a_2\cosh(Y\delta_1)+\sinh(Y\delta_2)}{\sinh(\delta_2)+a_2[\cosh(\delta_1)+a_1\sinh(\delta_1)+\cosh(\delta_2)]} \qquad 3.2$$

Substituting Eq. (3.2) into Eq. (3.1), we obtained:

$$U(\tau,Y)=\mathrm{Im}\left\{\exp(i\omega\tau\left[\frac{a_2\cosh(Y\delta_1)+\sinh(Y\delta_2)}{\sinh(\delta_2)+a_2[\cosh(\delta_1)+a_1\sinh(\delta_1)+\cosh(\delta_2)]}\right]\right\} \qquad 3.3$$

From Eq. (3.3), the skin friction on the respective walls is obtained as follows:

$$\tau_0=\left.\frac{\partial U}{\partial Y}\right|_{Y=0}=\mathrm{Im}\left\{\exp(i\omega\tau\left[\frac{\delta_2}{\sinh(\delta_2)+a_2[\cosh(\delta_1)+a_1\sinh(\delta_1)+\cosh(\delta_2)]}\right]\right\} \qquad 3.4$$

$$\tau_1 = \left.\frac{\partial U}{\partial Y}\right|_{Y=1} = \mathrm{Im}\left\{\exp(i\omega\tau\left[\frac{\delta_1 a_2 \sinh(\delta_1) + \delta_2 \cosh(\delta_2)}{\sinh(\delta_2) + a_2\left[\cosh(\delta_1) + a_1 \sinh(\delta_1) + \cosh(\delta_2)\right]}\right]\right\} \quad 3.5$$

Where $\delta_1 = \frac{S + \sqrt{S^2 + 4i\omega}}{2}$, $\delta_2 = \frac{S - \sqrt{S^2 + 4i\omega}}{2}$, $a_1 = \frac{2-\sigma_v}{\sigma_v} Kn\delta_1$ and $a_2 = \frac{2-\sigma_v}{\sigma_v} Kn\delta_2$ 3.6

IV. Discussion and Results

In order to get physical insight of the problem, we employed MATLAB programming to generate and analyze results. In order to ensure accuracy, our results were compared with those in [12] and are found in good agreement. When values of suction/injection parameter approached zero, our results reduced to those in [12]. This validates the accuracy of our results.

The effects of suction/injection on the spatial velocity distribution can be observed from Fig. 2. These effects are found more enhanced near the location of the driving force, this is because the fluid near the location of the driving force responds to change with respect to driving force than the farer one.Suction/Injection decreased the velocity of the fluid. The skin friction (from Table 1) was found to have decreased with increasing suction/injection velocity at $y = 0$ and have increased with increasing suction/injection velocity at $y = 1$.

Fig. 3 displays effect of the driving force angular velocity (ω) on the system. Upon increasing(ω), the fluid velocity have decreased, this is because the total energy exerted on the system to drive the flow is inversely proportional to(ω). So, when the frequency is increased, the totalmomentum energy transferred by the wall to the fluid will decrease; resulting to decrease in fluid velocity.

The effect of Knudsen number (Kn) on the fluid's velocity can be comprehended from Fig. 4. The velocity decreases with increase in (Kn),this is because (Kn) increases with slip velocity, and consequently when the slip velocity is increased, the interaction between the fluid and solid surface will decrease, resulting to decrease in the momentum energy transferred to the fluid by the moving wall.

With constant injection velocity, highest velocity is observed at $\omega\tau = \pi$ for the fluid adjacent to the bottom wall, while for those adjacent to the top wall, optimum velocity is at $\omega\tau = 3\pi/2$. With constant suction velocity optimum velocities for the fluid adjacent to the bottom and top wall are respective at $\omega\tau = \pi$ and $\omega\tau = \pi/2$.

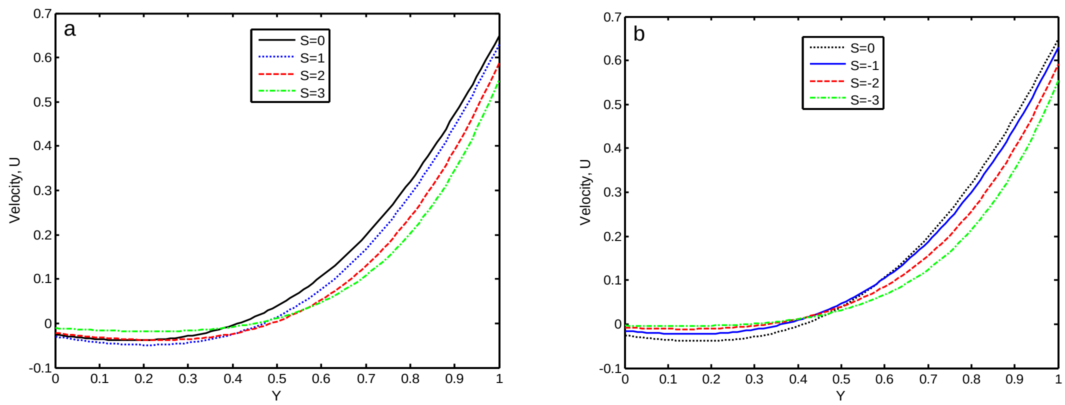

Fig. 2: *Velocity Profiles for* $Kn = 10^{-1}$, $\omega = 10$ *and* $\omega\tau = \pi/2$

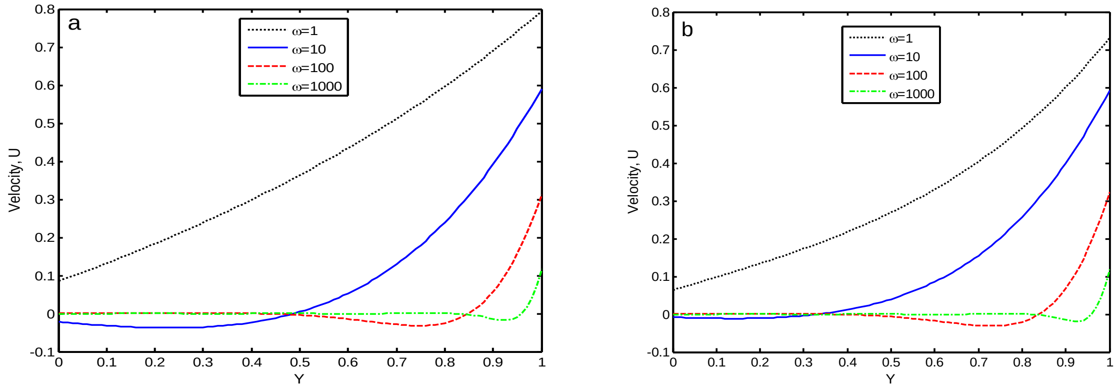

Fig. 3: *Velocity Profiles for*
$Kn = 10^{-1},\ \omega\tau = \pi/2\ and\ \ (a)\ S = 2\ (b)\,S = -2\ (c)\ S = 2\ (d)\,S = -2$

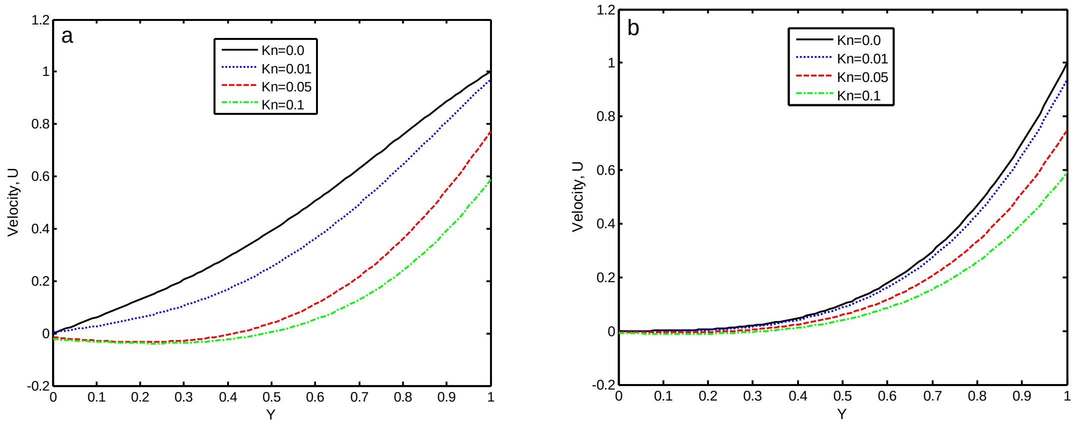

Fig. 4: *Velocity Profiles for* $\omega = 10,\ \omega\tau = \pi/2\ \ and\ \ (a)\ S = 2\ (b)\,S = -2\ (c)\ S = 2\ (d)\,S = -2$

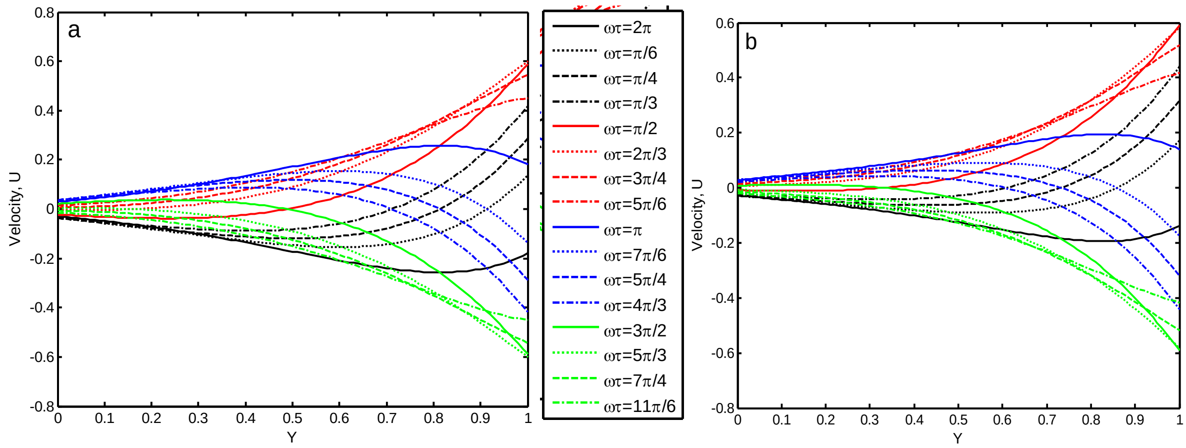

Fig. 5: *Velocity Profiles for* $Kn = 10^{-1},\ \omega = 10\ and\ \ (a)\ S = 2\ (b)\,S = -2\ (c)\ S = 2\ (d)\,S = -2$

Table 1: *Numerical values of skin friction* $(\omega = 10,\ Kn = 10^{-1}\,and\ \ \omega\tau = \pi/2)$

S	τ_0	τ_1		S	τ_0	τ_1
0	0.1355	1.8879		0	0.1355	1.8879
1	0.1600	1.9880		-1	0.0831	1.9961
2	0.1170	2.2121		-2	0.0426	2.1954
3	0.0571	2.4190		-3	0.0172	2.4010

V. Conclusion

The transient Couette fluctuating-micro-flow in slip flow regime, where the upper plate moves with sinusoidal velocity while the bottom plate remained stationary is analyzed. It is concluded from the study that both suction and injection retard the velocity of the fluid.

Nomenclature

Kn	Knudsen number $(=\lambda/L)$
L	Reference length $(=L)$
S	Dimensionless suction/injection parameter
t	Time
t_r	Reference time $(=L^2/\upsilon)$
U	Dimensionless axial velocity $(=u/u_o)$
u	Axial velocity (in x-direction)
u_o	Velocity of the moving wall
V	Complex solution function for velocity
V_o	Suction/injection
X	Dimensionless axial coordinate $(=x/L)$
x	Axial coordinate
Y	Dimensionless transverse coordinate $(=y/L)$
y	Transverse coordinate

Greek symbols

λ	Mean-free-path length
ρ	Density
μ	Dynamic viscosity
υ	Kinematic viscosity
σ_υ	Tangential momentum accommodation coefficient $(=0.7)$
ϖ	Frequency

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