

Influential Codes for Optimum Design Parameters of Wide Span Circular R.C. Halls

Salah El-Din Taher¹, Sherif Elwan² and Ahmed Ehab Abd El-Hameed³

¹Professor of concrete structures, Faculty of Engineering, Tanta University.

²Associate Professor, Civil Engineering Department, Higher Institution of Engineering, Shorouk Academy.

³Lecturer, Civil Engineering Department, Higher Institution of Engineering, Shorouk Academy.

Corresponding Author: Selah El-Din Taher

Abstract: Skeletal wide-span circular halls possess large number of variables that cannot be easily identified unless rigorous three-dimensional analysis is managed. Structural optimization for minimum total cost as an objective function needs subdivision of the mathematical process into subsequent phases in order that discrete-valued solutions of slabs, beams, columns and foundation in RC skeletons. Value engineering implementation to such large projects necessitates the examination of various alternatives for the most effective solution. In this study, two-phase genetic algorithm (GA) technique was operated for the search-find procedure for the best solution fulfilling the design constraints after selective design codes. ACI318-14, Eurocode EC2-8 and Egyptian code ECP203-14 were considered to inspect the impact of different approaches on the overall cost. Reinforcement rebar waste minimization was further integrated in the proposed methodology through a commercially specialized software that provided the best curtailment in workshop documentation for different bar splices. Implementation has been directed to 80.0m diameter two story hall including a circumferential sector of 24.0m open span in the upper floor. The results indicated that the ACI318-14 provided the least cost for 30MPa concrete characteristic cube strength and steel grade 400/600 MPa of 12.1% and 7.2% less than EC2 and ECP203-17, respectively. The optimum solution of the specified architecture led to members dimensioning that behaved as a nonprismatic vierendeel. Near optimum solution saved about 9-11% over the typical design. Although special orders of reinforcement rebars longer than 12.0m yielded the least cost compared with conventional lap splices and mechanical couplers but the difference is minor compared with the overall cost. Finally, the optimum solution predictions were found to be comparable with other alternatives of nonprismatic cylindrical shell and folded plate roofing system.

Keywords: Structural optimization, RC framed structures, ACI 318-14, Eurocode EC2-2008, ECP 203-2007, Vierendeel, cylindrical shell, folded plate, nonprismatic systems.

Date of Submission: 23-07-2018

Date of acceptance: 06-08-2018

I. Introduction

The selection of suitable cross section for beams and columns with minimum cost from many acceptable cross sections is a major problem for engineers. Structural analysis and design usually involve both highly complex procedures and a great number of variables. Consequently, the solution has to be found iteratively while initial values are set to the variables based mainly on designer's sensitivity and experience. In addition, the number of analysis steps is remarkably increased if optimum values are to be found among all possible alternatives. To mathematically describe the physical response of a structure, extreme function values can be found by using optimization techniques.

The optimization problem can be solved by specifying the design variables for the structure, the objective function that needs to be minimized, and the imposed design constraints on the system. The code requirements for the performance (safety, serviceability ...etc) constitute the constraints. Optimization problems of structural frames have been considered by several researchers.

The great development of structural optimization took place in the early 60's by Schmidt [1] when programming techniques were used in the minimization of structures weight. Hussain [2] employed second-order method to analyse and design reinforced concrete (RC) frames. The author formulated the frames using a non-linear programming technique considering ACI 318-83(1998) building code requirements for reinforced concrete. Concrete dimensions and steel areas for columns and beams were the design variables. The objective function was the sum of all the costs for each column and beam. From the study, it was shown that there was a 3.5% reduction in cost while processing time to reach an optimum solution increased by 5%.

Balling and Yao [3] examined the viability of the assumption that optimum concrete section dimensions are insensitive to the number, diameter, as well as longitudinal distribution of the reinforcing bars.

This was achieved by comparing optimum results from a multilevel method that considered the problem as a system optimization problem and a series of individual member optimization problems. From the results, a simplified method was presented and recommended as the most efficient method for the optimization of reinforced concrete frames. Rajeev and Krishnamoorthy [4] applied a simple genetic algorithm (SGA) to the cost optimization of two-dimensional frames. The authors concluded that genetic algorithm-based methodologies provide ideal techniques when further modification such as detailing, placing of reinforcement in beams and columns and other issues related to construction are brought into optimal design model. Bontempi, et al. [5] presented a systematic approach to the optimal design of concrete structures using a combined genetic algorithm and fuzzy criteria. The procedure was oriented to the optimal design of concrete frames but also suitable for other kinds of structures.

In more recent studies, Lee and Ahn [6], and Camp et al. [7] implemented Genetic Algorithms (GA) that searched for discrete-valued solutions of beam and column members in RC frames. Guerra and Kioussis [8] carried out optimization design of multi-storey and multi-bay reinforced concrete frames. It was found that the optimal design results in cost savings for 8m and 24m spans were 1% and 17% respectively. Babiker et al. modeled an Artificial Neural Networks (ANN) to optimize cost of simply supported beams [9]. During the last two decades, several researches [10-16] provided deeper insight on optimization tools and knowledge applied to RC frames.

In the present study, two-phase genetic algorithm technique is used as the optimization tool of circular halls of wide span. The sectorial nature of such structures imposes variable constraints on the structural elements depending on their radial coordinates. Members proportioning limits are set forth according to three codes' provisions for the relevant design variables. Because of the conventional rebars length limitation, lap and mechanical splices are considered. The main supporting elements of the static system are examined for the optimum solution of several architectural configurations.

II. Proposed Methodology

The proposed methodology aims at practical dimensioning and detailing of wide-span RC roofing systems with non-prismatic geometry of circular halls in value engineering framework that fulfills the following requirements:

- i. Maintain automated CAD integrated through commercial software packages for structural analysis, quantity surveying and optimum design.
- ii. Afford minimum total cost in terms of concrete volume, steel reinforcement weight and surface area of formwork.
- iii. Provide least steel waste in workshop detailing of reinforcement.
- iv. Satisfy international codes provisions where ACI 318-14 [17], EC2-08 [18] and ECP 203-17 [19] are considered for strength requirements.

The methodology consists of two main phases. Several structural systems are proposed conforming to aesthetic and functional prerequisites then input to the proposed solution routine in the outer loop of Phase I as depicted in Fig. (1). For each system, the control program uses the architectural module to build the problem initialization module that defines the variables of the each system (e.g. dead load, live loads, assigned code provisions, concrete grade, steel reinforcement type, foundation-super structure interface and bearing capacity of soil). With the help with the constraint-screening regime that implements each code provisions, the control program manages the optimization parameters generator module. The parameters include geometrical design parameters of the roofing surface, concrete dimensions of each structural element, longitudinal steel reinforcements and their coupling for rebars longer than 12.0m and shear reinforcements. These have been considered for slabs, girders, columns and foundation along the entire hall.

In the beginning, nonlinear programming was tested to solve the problem but it was found inappropriate because of the numerous variables and complexity of the problem. Later on, a commercial mathematical software was used in the form of sequential quadratic programming algorithm, which searches for continuous valued optimal solutions, then rounding them to discrete, constructible design values. However, this provided better convergence but still in long time. Finally, genetic algorithm was employed with simplicity through a commercial software [20] that proved better performance and was decided to be used through the entire research. The next Phase II conducts routine structural analysis using conventional finite element solvers then carrying out structural design in an iterative scheme of the inner loop shown in Fig. (1) to achieve the optimum and practical design.

Data Exchange Files (DXF) were used to transfer data among programs and Excel worksheets were formulated to control data migration besides simple batch file and structural input file generator and output diagnostic reader. The commercial software package constituting the proposed methodology were: (i) AutoCad then Revit softwares have been employed for the system identification, (ii) 3-D Structural finite element analysis software employing shell and frame elements is used, (iii) Structural optimization software that utilized Genetic

Algorithm solution, (iv) Autodesk Structural detailing software has been applied for steel drawing and (v) Nest 1-D Rebar Optimizer software has been activated for minimizing steel reinforcement waste in workshop detailing.

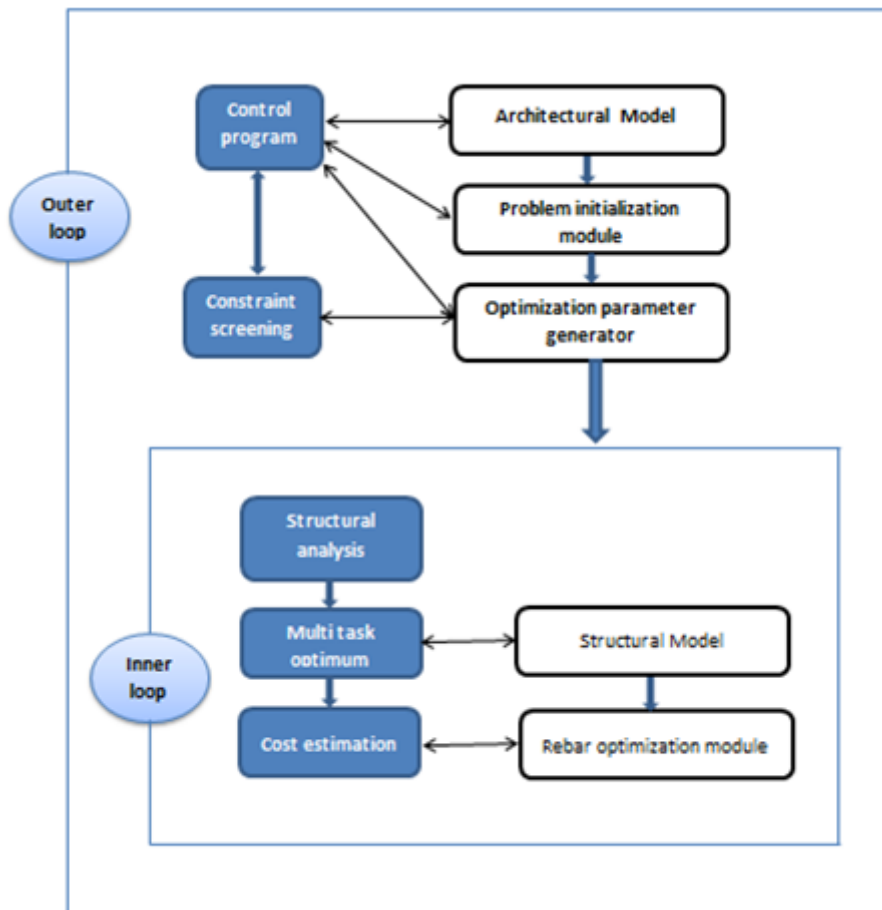


Figure (1): Workflow block diagram of the proposed methodology.

III. Codes Identification

In order that optimum design dependence on design procedures be figured out, three RC codes of practice have been selected in this research: (i) ACI 318-14 that is based on strength design approach not accounting for any material safety factor but considers strength reduction factors for the ultimate nominal capacity depending on the load combination under consideration [17, 21]; (ii) Eurocode EC2-08 that is based on limit states design approach contemplating partial safety factors for concrete and steel for ultimate limit state [18, 21] and (iii) Egyptian Code of Practice ECP203-17[19] that is based on limit states design approach with slight differences from EC2-08. Serviceability conditions for deflection and cracking in all of the three codes should be further considered per se. The salient features of the design parameters of each code are summarized hereafter.

3.1 ACI 318-14[17]

The factored ultimate load shall be the greater of $(1.2D+1.6L)$ and $1.4D$. In all equations, yield stress of steel f_y and specific concrete strength f'_c based on 150x300mm cylinders which is equivalent to 0.8 cube strength are considered. The ultimate concrete strain $\epsilon_{cu} = 0.003$ for flexure and 0.002 for concentric compression is used. The idealized stress strain curve of concrete is parabolic-rectangular; i.e. parabolic up to the latter strain till reaching the concrete strength f'_c then remains constant up to the former strain. The equivalent rectangular stress block is permitted for flexure at concrete stress of $0.85 f'_c$ and of height ratio to the neutral axis depth from the extreme compression soffit equals to $\beta_1 = 0.85 - 0.008(f'_c - 30) \geq 0.65$. On the other hand, elastic perfectly plastic stress strain curve for steel reinforcement may be used. To ensure ductile failure, the maximum reinforcement ratio ρ_{max} is kept at lower levels than $\frac{3}{4}$ the balanced ratio ρ_b for flexure. The strength reduction factor $\phi = 0.65 + (\epsilon_t - 0.002)(250/3) \leq 0.9$ for tied members in which the steel strain $\epsilon_t \leq 0.002$ for compression controlled failure and $0.002 < \epsilon_t < 0.005$ for transition failure and $\epsilon_t \geq 0.005$ for tension controlled failure. Moment

redistribution is limited to tensile steel strain of at least 0.0075. P- Δ effect is permitted for eccentric compressive loading. For shear and torsion, the strength reduction factor $\phi=0.85$. The critical section for shear and torsion is at distance equal to the effective depth of the section, d_b , and is resisted by the web of the section.

3.2 Eurocode EC2-2008[18]

The factored ultimate load shall be $(1.35D+1.5L)$. In all equations, the characteristic yield stress of steel f_{yk} and characteristic concrete strength f_{ck} based on 150x300mm cylinders related to cube and tensile strength as listed in Table (1) are used. Partial materials safety factors γ_c and γ_s for concrete and steel reinforcement, respectively, are considered in the research only for persistent and transient design situations. The idealized stress strain curve of concrete may be considered as parabolic, parabolic-rectangular or bilinear. The ultimate concrete strain $\epsilon_{cu} = 0.0035$ for $f_{ck} \leq 50\text{MPa}$ or $\epsilon_{cu} = 0.0028 + 0.027 * \{(90 - f_{ck})/100\}^4$ otherwise) for flexure and $\epsilon_{c1} = 0.00175 + 0.00055 * (f_{ck} - 50)/40$ for concentric compression is used for the bilinear idealization. The equivalent rectangular stress block is permitted for flexure at concrete stress of $\eta \alpha_{cc} f_{ck} / \gamma_c$ ($\alpha_{cc} = 1$ for short term effects, $\gamma_c = 1.5$, $\eta = 1$ for $f_{ck} \leq 50\text{MPa}$ or $\eta = 1 - (f_{ck} - 50)/200 \geq 0.8$ otherwise) and of height ratio of the neutral axis depth from the extreme compression soffit equals to $\lambda = 0.8$ for $f_{ck} \leq 50\text{MPa}$ or $\lambda = 0.8 - (f_{ck} - 50)/400 \geq 0.7$ otherwise. On the other hand, elastic perfectly plastic or bilinear upto ultimate strain limit stress strain curves for steel reinforcement may be used at f_{yk} / γ_s , $\gamma_s = 1.15$. To ensure ductile failure, the maximum reinforcement ratio ρ_{max} requires that the compressed depth should not exceed 0.6 the depth. P- Δ effect is permitted for eccentric compressive loading. Unless variable strut method is used for shear design, the critical section for shear and torsion is at distance equal to the effective depth of the section, d_b , and is resisted by the web of the section.

Table (1): Eurocode EC2-2008 concrete strength parameters

Symbol	Description	Properties (MPa)							
		16	25	30	35	40	45	50	
f_{ck}	Characteristic cylinder strength	16	25	30	35	40	45	50	
$f_{ck, cube}$	Characteristic cube strength	20	30	37	45	50	55	60	
f_{ctm}	Mean tensile strength	1.9	2.2	2.9	3.2	3.5	3.8	4.1	

3.3 ECP203-2017[19]

The factored ultimate load shall be $(1.4D+1.6L)$. In all equations, the yield stress of steel f_y and characteristic concrete strength f_{cu} based on 150mm cubes are considered. Partial materials safety factors γ_c and γ_s for concrete and steel reinforcement, respectively, are taken into account. The ultimate concrete strain $\epsilon_{cu} = 0.003$ for flexure and 0.002 for concentric compression is used for the bilinear idealization. Minimum eccentricity ratio $e/t = 0.05$ for axial loading (eccentricity e and section thickness t). Axial compression may be neglected if less than $0.004 f_{cu} b t$ (section breadth b). The equivalent rectangular stress block is permitted for flexure at concrete stress of $0.67 f_{cu} / \gamma_c$ ($\gamma_c = 1.5 \{7/6 - e/3t\} \geq 1.5$ for eccentric compression) and of height ratio to the neutral axis depth from the extreme compression soffit equals to $\lambda = 0.8$. On the other hand, elastic perfectly plastic or bilinear upto ultimate strain limit stress strain curves for steel reinforcement may be used at f_y / γ_s , ($\gamma_s = 1.15 \{7/6 - e/3t\} \geq 1.15$). To ensure ductile failure, the maximum reinforcement ratio ρ_{max} is kept at lower levels than 2/3 the balanced ratio ρ_b for flexure. The critical section for shear and torsion is at distance equal to half the effective depth of the section, $d_b/2$, and is resisted by the web of the section.

IV. Design variables of the System with Radial Vierendeels

The circular hall of 80.0m diameter shown in Fig. (2) depicts the 3-D representation of the main structural components. The inner part of radius $R_i = 16.0\text{m}$ has the highest clear level of (+12.0m) and is covered by dome roof of 5.0m rise. The outer hall, which of the present optimization concern, has an exterior radius $R_o = 40.0\text{m}$ that consists of two floors. The ground floor is a conventional commercial/administrative spaces at 3.0m height and the architectural design allowed for columns supporting flat slab system according to the given partitions and is not demonstrated in the optimization framework. Interior columns are not permitted in the upper floor, which is an open space of 3.0m story height. Side windows along the radial sides are specified for ventilation/light purposes. With reference to Figs (2) and (3), it has to be noted that the lower level is horizontally flat at (+6.0m) while the upper level connects two levels at (+8.0m) and (+10.m).

For structural analysis module, the live load on the roof was taken 1.0 kN/m^2 and the dead load included own weight plus finishing weight of 2.0 kN/m^2 . Live load was taken 4.0 kN/m^2 for the first floor, while the dead load including own weight plus finishing weight of 1.5 kN/m^2 along with wall load as specific in the architectural drawing using masonry brickwork of unit weight of 18 kN/m^3 was considered. Such architectural shaping of the roof necessitated formulation of an automated optimization system for the tremendous number of design variables that included the following:

- Three codes provisions as pre-mentioned;
- Two variables for the material properties of concrete and steel reinforcements grades.

- Two variables for the even number of divisions for the sectorial arrangement of exterior columns on the outer sector (n_s) and for the radial arrangement of intermediate circular beams (n_r).
- Subdividing the optimization process into three subsequent modules, a set of 78 design variables are listed in Table (2-a,b) for concrete dimensions and reinforcements of slabs, circular beams, radial girders, vertical hangers, interior and exterior columns/footings. As illustrated in Fig. (3), Zone 2 is particularized for the outer part of the circular hall from circular beams of radius greater than the mid-radius $R_m=(R_i+R_o)/2$ whereas Zone 1 is within remaining inner part.
- Five variables including curvature and slope inclination of the upper roof: three geometries of the radial top chord as circular, parabolic or straight besides the two roof configurations shown in Fig.(4). Configuration (1) with outward sloping, Fig. (4-a), has higher rise at the inner radius than the outer radius to allow for rainfall drainage to the outer perimeter. Configuration (2) with inward sloping, Fig. (4-b), has higher rise at the outer radius than the inner radius to provide higher structural stiffness proportional to the loading intensity.
- Three rebars connectivity for longitudinal steel longer than 12.0m. These include:(i) special orders of the required length at 4% additional cost, (ii) conventional lap splices and (iii) mechanical splices using threaded coupler at 15% additional cost of piece relative to cage cost. The latter two splices are illustrated in Fig. (4-c).
- Two base conditions as hinged or fixed support and their impact on conventional (non-optimal) dimensioning of foundation with net allowable bearing capacity of soil of 150kN/m^2 .

It has to be noted that Model (1) of the radial girders is supported on columns at the ends while Model (2) is supported on exterior column from one side and on radial ring beam on the other. The latter situation is used to be idealized as hinge support as the ring beam supports the system vertically and prevents relative deformation horizontally due to axisymmetrical nature of the geometry. This requires caution during the construction and execution process of not removing the scaffolds until the ring beam is cast and gains its full strength.

Sensitivity analysis of the GA optimizer has been conducted to ensure the convergence accuracy of the near optimum solution. In addition, arithmetic rounding is important for practical implementation of the optimization predictions into structural design. Slab thickness was assumed in the order of +20mm while other elements were assumed in the order of +50mm. Steel reinforcement diameters were selected according to the commercially available cages for each grade. Minimum concrete cover was considered as specified by each code. The intelligent construction of the proposed methodology enabled detection of the sections of maximum and zero bending moments where development and anchorage lengths were provided for reinforcement curtailment according to each code satisfying the required moment of resistance.

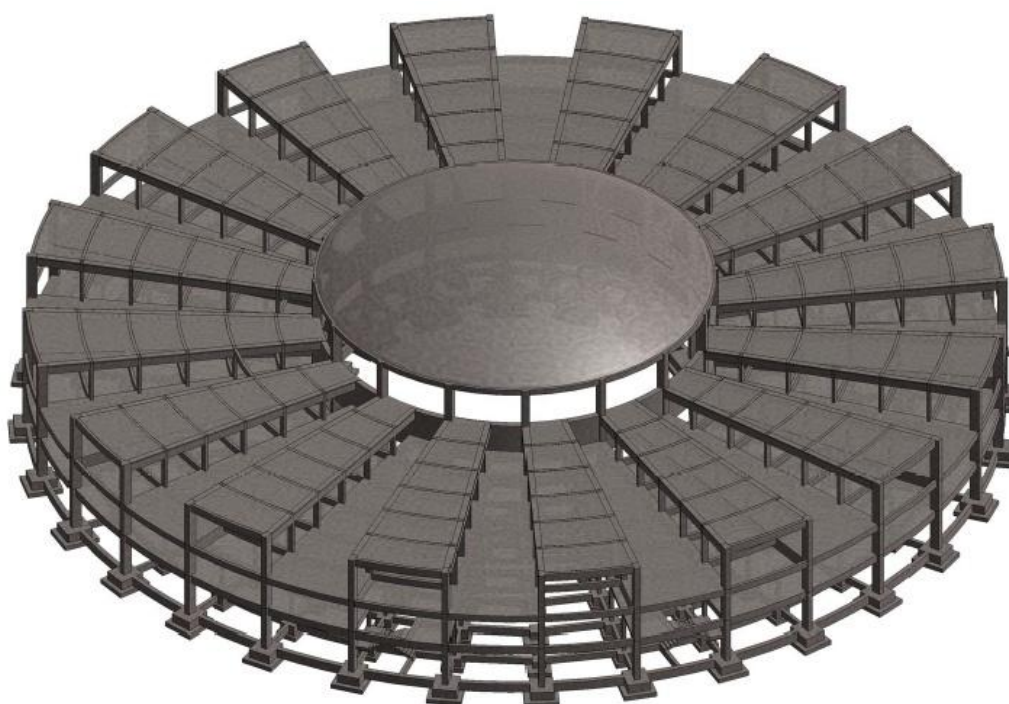


Figure (2): Pictorial view of the main structural components of the project with straight radial top chord-inward sloping (Configuration 2).

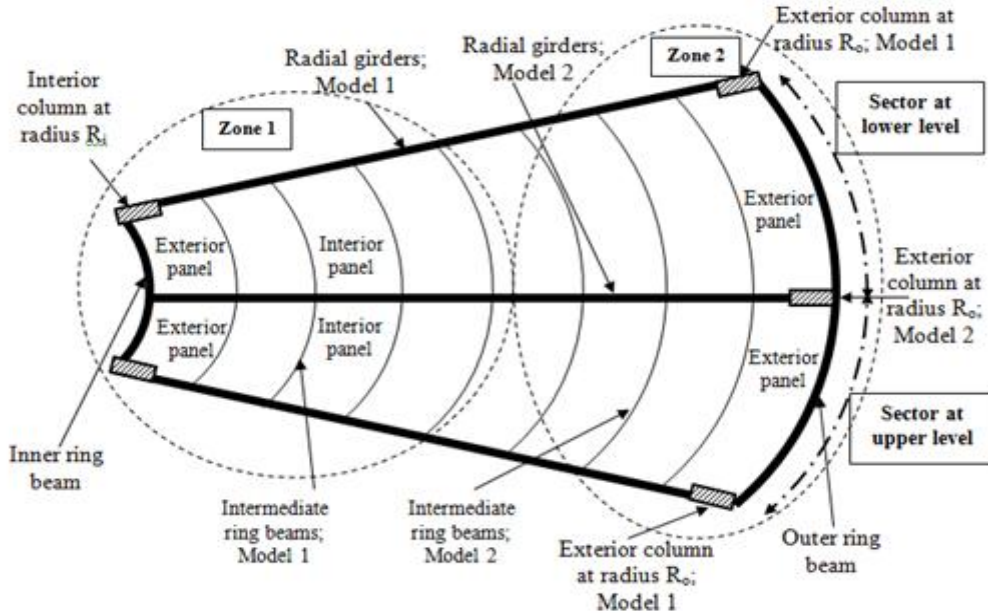


Figure (3): Description of the main design variables for a typical sector of the outer roof

Table (2-a): Definition of the design variables-Optimization Module I

Element	Symbol	Description	Location
Roof slab for inner zone; Model 1	t_{s1}	Slab thickness	Zone 1
	A_{srte1}, A_{srbe1}	Radial top and bottom reinforcements	Exterior panel
	A_{stte1}, A_{stbe1}	Tangential top and bottom reinforcements	Exterior panel
	A_{srbi1}, A_{srbi1}	Radial top and bottom reinforcements in radial direction	Interior panel
	A_{srbi1}, A_{stti1}	Tangential top and bottom reinforcements in tangential direction	Interior panel
Roof slab for outer zone; Model 2	t_{s2}	Slab thickness	Zone 2
	A_{srte2}, A_{srbe2}	Radial top and bottom reinforcements	Exterior panel
	A_{stte2}, A_{stbe2}	Tangential top and bottom reinforcements	Exterior panel
	A_{srbi2}, A_{srbi2}	Radial top and bottom reinforcements in radial direction	Interior panel
	A_{srbi2}, A_{stti2}	Tangential top and bottom reinforcements in tangential direction	Interior panel
Inner ring beam	b_{ri}, t_{ri}	Breadth and thickness of section	
	$A_{sti}, A_{sbi}, A_{sti}$	Longitudinal top&bottom and shear reinforcements	
Outer ring beam	b_{ro}, t_{ro}	Breadth and thickness of section	
	$A_{sto}, A_{sbo}, A_{sto}$	Longitudinal top&bottom and shear reinforcements	
Intermediate ring beams; Model 1	b_{r1}, t_{r1}	Breadth and thickness of section	Zone 1
	$A_{str1}, A_{sbr1}, A_{str1}$	Longitudinal top&bottom and shear reinforcements	
Intermediate ring beams; Model 2	b_{r2}, t_{r2}	Breadth and thickness of section	Zone 2

Table (2-b): Definition of the design variables-Optimization Module II

Element	Symbol	Description	Location
Radial girder-top chord	b_{gt}, t_{gt}	Breadth and thickness of section	
	$A_{stgt1}, A_{sbg1}, A_{stgt1}$	Longitudinal top&bottom and shear reinforcements	Column Model 1
	$A_{stgt2}, A_{sbg2}, A_{stgt2}$		Column Model 2
Radial girder - bottom chord	b_{gb}, t_{gb}	Breadth and thickness of section	
	$A_{stgb1}, A_{sbg1}, A_{stgb1}$	Longitudinal top&bottom and shear reinforcements	Column Model 1
	$A_{stgb2}, A_{sbg2}, A_{stgb2}$		Column Model 2
Interior hangers of Zone 1	b_{ci2}, t_{ci1}	Breadth and thickness of section	
	A_{sci1}, A_{stci1}	Longitudinal and tie reinforcements	
Interior hangers of Zone 2	b_{ci2}, t_{ci2}	Breadth and thickness of section	
	A_{sci2}, A_{stci2}	Longitudinal and tie reinforcements	
Columns at the interior circle	b_{ci}, t_{ci}	Breadth and thickness of section	
	A_{scii}, A_{stcii}	Longitudinal and tie reinforcements	Lower part
	A_{scui}, A_{stcui}	Longitudinal and tie reinforcements	Upper part
Columns at the exterior circle	b_{co}, t_{co}	Breadth and thickness of section	
	A_{sclo1}, A_{stclo1}	Longitudinal and tie reinforcements of column Model 1	Lower part
	A_{scu1}, A_{stcu1}		Upper part
	A_{sclo2}, A_{stclo2}	Longitudinal and tie reinforcements of column Model 2	Lower part
	A_{scu2}, A_{stcu2}		Upper part

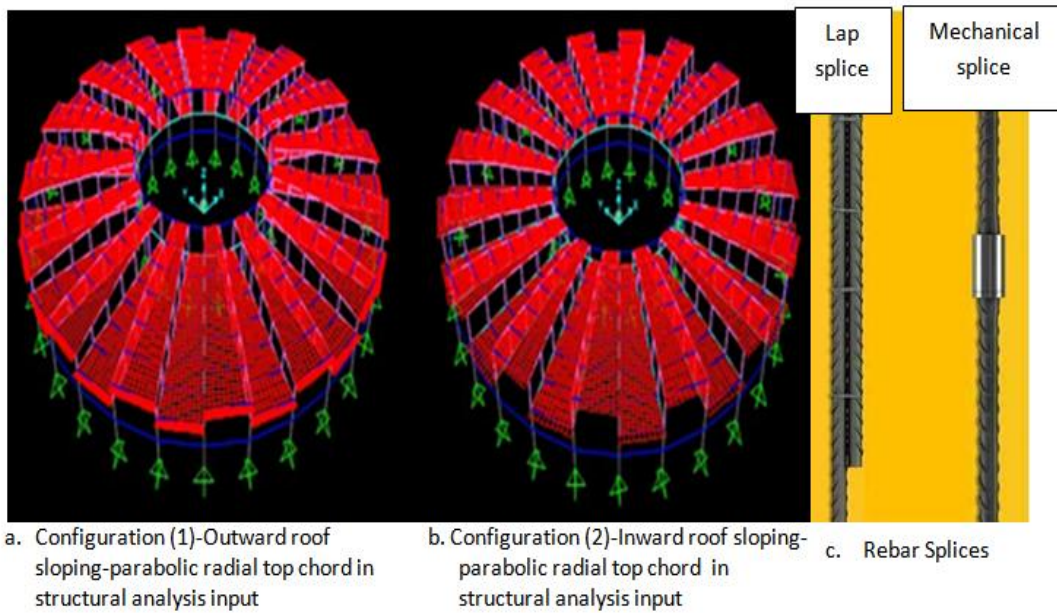


Figure (4): Alternatives of roof configurations and rebars splices

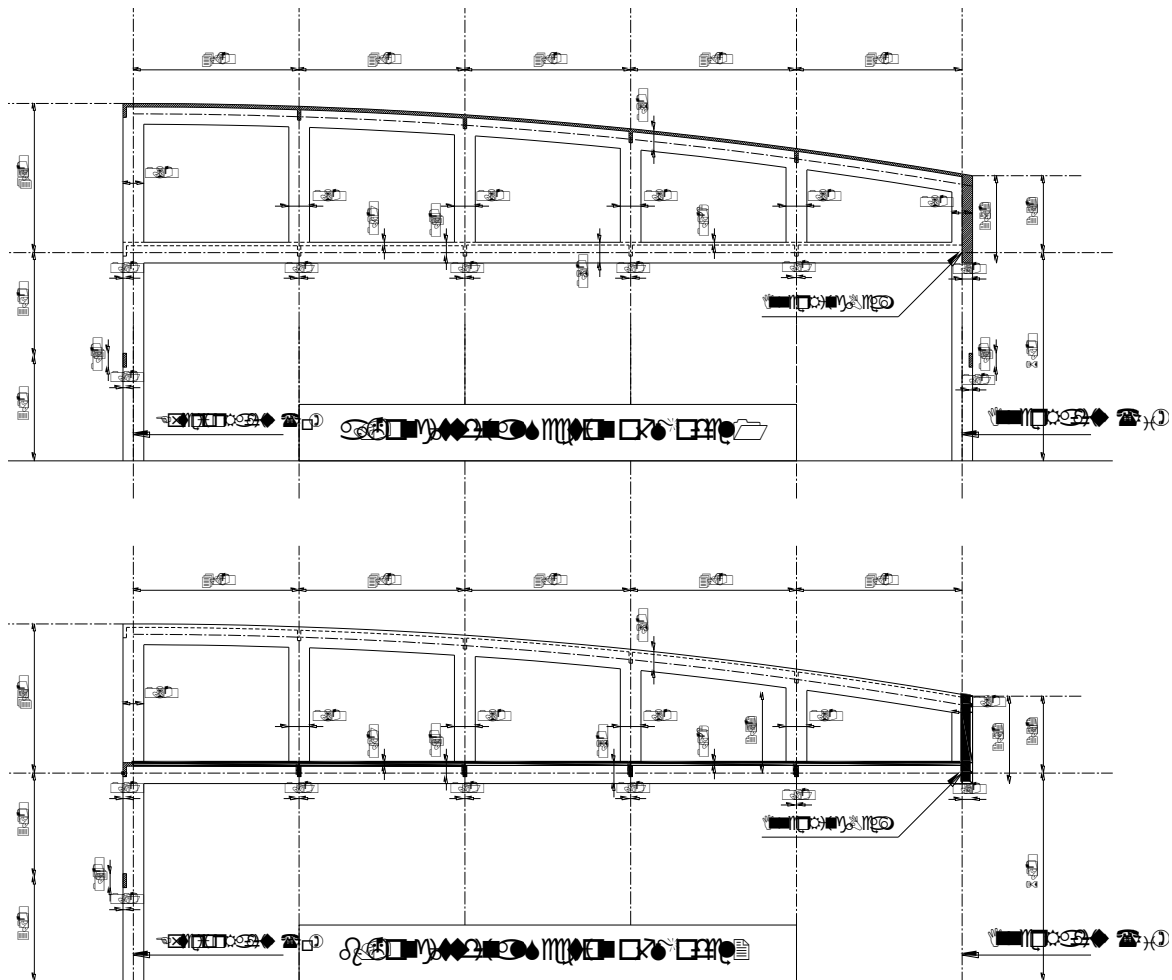


Figure (5): Initialization input of sectional elevations for Configuration (2)

V. Structural Optimization Module

5.1 Objective function

The objective function is formulated for the total cost function is minimized such that

$$\sum C_c(x_i)V_i + C_s(x_j)W_j + C_f(x_k)A_k + C_m(x_l)N_l = \text{minimum} \quad (1)$$

In which

- $C_c(x_i)$ is the cost function of concrete element of volume V_i . The variables x_i represent the concrete grade and the level of member for casting and other manufacturing items.
- $C_s(x_j)$ is the cost function of steel reinforcement of weight W_j . The variables x_j represent the steel grade and the member for reinforcement cage placement.
- $C_f(x_k)$ is the cost function of formwork, shuttering and scaffolding of surface area A_k . The variables x_k represent the type of member, necessity of double shuttering of inclined members and removal conditions.
- $C_m(x_l)$ is the cost function of miscellaneous items of number N_l . The variables x_l represent either type of reinforcement splice or provision of special lead plate hinge support.

RS Means Concrete /Masonry Cost data [22] was incorporated to capture realistic, member size dependent costs. Any other cost items that are not listed therein have been given in the appropriate context.

5.2 Design constraints of the System

I. Columns' constraints:

a- Geometric constraint: the depth of column in plane must be greater than the breadth, i.e.

$$\frac{b}{t} - 1.0 \leq 0.0 \quad (2)$$

The condition is applicable in both design methods.

b- Strength constraint: each column subjected to both axial load and bending moment. Compression control status is frequently encountered [23] and the column sections is necessitated to be adequate for ultimate axial load as follows:

i. According to ACI318-14:

$$\text{ii. } 1 - \frac{0.476 f_c (bt - A_s) + 0.56 A_s f_y}{P_u} \leq 0.0 \quad (3-a)$$

iii. According to EC2-08:

$$\text{iv. } 1 - \frac{\eta \alpha_{cc} f_{ck} / \gamma_c (bt - A_s) + A_s f_{yk} / \gamma_s}{P_u} \leq 0.0 \quad (3-b)$$

v. According to ECP203-17 :

$$\text{vi. } 1 - \frac{0.35 f_{cu} (bt - A_s) + 0.67 A_s f_y}{P_u} \leq 0.0 \quad (3-c)$$

c- Steel area constraint: the column reinforcement should fulfill the eccentric compression requirements according to the appropriate strength interaction diagram.

i. According to ACI 318-14: utilizing the simplified method suggested by Spires and Arora [24,25] and accounting for units conversion, this constraint can be expressed as follows:

$$1.0 - \frac{A_s}{0.007 \frac{M_u}{t} + 0.02 bt} \leq 0.0 \quad (4)$$

i. According to EC2-08 and ECP203-17: Similar to the work of Guerra and Kioussis[8], for a given cross-section, the interaction diagram is typically obtained point-wise by finding numerous combinations (M_u, P_u) that describe failure. Mathematically, if f is a function that describes the interaction diagram, safety requires that for all i members. For the purpose of this study, the interaction diagram is modeled as a cubic spline based on the five pairs shown in Fig. (6) for EC2-08. Minor modification is made to account for ECP203-17 as discussed earlier.

ii. Minimum steel area constraint:

iii. The column reinforcement should fulfill minimum code requirement as the following:

iv. According to ACI 318-14:

$$\text{v. } 0.01 - \frac{A_s}{bt} \leq 0.0 \quad (5-a)$$

vi. According to EC2-08:

$$\text{vii. } 0.002 - \frac{A_s}{bt} \leq 0.0 \quad \text{or} \quad 0.1 \frac{P_u}{f_{yk} / \gamma_s} - \frac{A_s}{bt} \leq 0.0 \quad (5-b)$$

viii. According to ECP203-17

$$0.008 - \frac{A_s}{bt} \leq 0.0 \quad \text{or} \quad 0.06 - \frac{\frac{P_u}{bt} - 0.035 f_{cu}}{0.67 f_y} \leq 0.0 \quad (5-c)$$

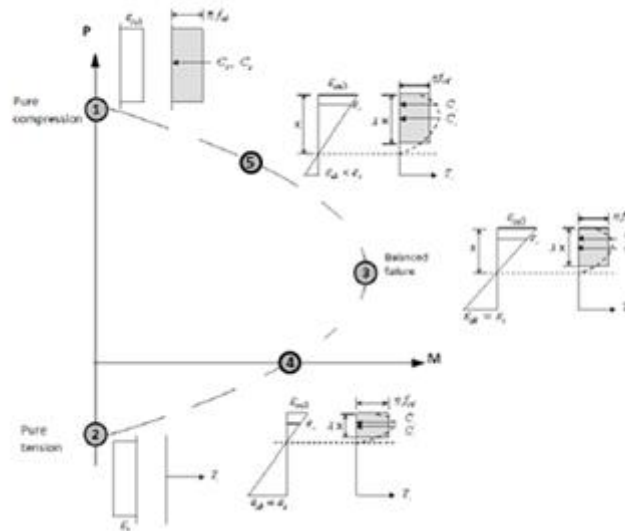


Figure (6): Idealized strength interaction diagram for symmetrical reinforcement based on EC2-08.

d- Maximum steel area constraint:

The column reinforcement should not exceed the maximum reinforcement, $A_{s,max}$, set forth by each code. These constraints can be expressed as the following:

$$\frac{A_{st}}{bt} - A_{s,max} \leq 0.0 \quad (6)$$

- i. According to ACI 318-14: The limiting value of $A_{s,max}$ are 6% and 7% interior and interior columns, respectively.
- ii. According to EC2-08: The limiting value of $A_{s,max}$ is 4%.
- iii. According to ECP203-17: The limiting value of $A_{s,max}$ are 4% and 5% interior and interior columns, respectively.
- iv. In the finite element analysis of the considered optimization problem where iterative scheme was developed, the slenderness effect was accounted for the follows:
- v. According to ACI 318-14, EC2-08 and ECP203-17: the stiffness matrix is modified, as permitted therein, to account for the effect of the axial load in the form :

$$\begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & (\frac{12EI}{L^3})S_1 & (\frac{6EI}{L^2})S_2 & 0 & -(\frac{12EI}{L^3})S_1 & (\frac{6EI}{L^2})S_2 \\ 0 & (\frac{6EI}{L^2})S_2 & (\frac{4EI}{L})S_1 & 0 & -(\frac{6EI}{L^2})S_2 & (\frac{2EI}{L})S_1 \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -(\frac{12EI}{L^3})S_1 & -(\frac{6EI}{L^2})S_2 & 0 & (\frac{12EI}{L^3})S_1 & -(\frac{6EI}{L^2})S_2 \\ 0 & (\frac{6EI}{L^2})S_2 & (\frac{2EI}{L})S_1 & 0 & -(\frac{6EI}{L^2})S_2 & (\frac{4EI}{L})S_1 \end{bmatrix} \quad (7)$$

In which the functions $S_i, i=1,4$ are expressed as

$$S_1 = \frac{(\kappa L)^3 \sin \kappa L}{12\varphi}, \quad S_2 = \frac{(\kappa L)^2 (1 - \cos \kappa L)}{6\varphi}, \quad S_3 = \frac{\kappa L (\sin \kappa L - \kappa L \cos \kappa L)}{4\varphi}, \quad S_4 = \frac{\kappa L (\kappa L - \sin \kappa L)}{2\varphi} \quad \text{and} \quad \kappa = \sqrt{\frac{P}{EI}}, \quad \varphi = 2 -$$

$2 \cos \kappa L - \kappa L \sin \kappa L$ in the case of compression, and $\varphi = 2 - 2 \cosh \kappa L - \kappa L \sin \kappa L$ in the case of tension where P is the column's axial load, L is the length of the member. E is Young's modulus, A is member area and I is the second moment of area. Also the load vector is modified by the F - parameter as $F = \frac{12}{u^2} [1 - u/2 \cot u/2]$ in case of compression and $F = \frac{12}{u^2} [1 - u/2 \coth u/2]$ in case of tension, $u = \kappa L$.

The value of maximum permissible slenderness ratio for unbraced system should not be exceeded, thus imposing the following constraint:

$$\lambda - \lambda_{max} \leq 0.0 \quad (8)$$

in which $\lambda = h_o / \text{side width}$ (b in circumferential direction or t in radial direction).

II. Beams constrains:

a- **Geometric constraint:** the effective depth of a beam d_b ($d_b=t-d'$, d' is the concrete cover to reinforcement) is practically 2 to 4 times the web width b_w i.e.

$$1.0 - \frac{d_b}{2b_w} \leq 0.0 \quad \frac{d_b}{4b_w} - 1.0 \leq 1.0 \quad (9)$$

b- Flexural capacity constraint: consideration of the effective width, b_e , contributing to the flexural resistance of the section depends on the relative location of the connecting slab with respect the acting moment. Thus, T or L section is considered for sagging moment while rectangular section is considered for hogging moment. In case of eccentric axial force, P_u , the moment M_u is replaced by $M_{us}=M_u +P_u(t/2 -d')$ where the positive sign is taken for compression then effect of axial force is incorporated in the form (required reinforcement-selected reinforcement ≤ 0.0)as follows:

i. According to ACI 318-14: $A'_s = A_s - \frac{P_u}{\phi f_y}$

$$ii. \frac{M_{us}/\phi}{\left[A_s f_y \left\{ d_b - \frac{A'_s f_y}{1.7 f_c b_e} \right\} \right]} - \frac{P_u/\phi}{A_s f_y} - 1.0 \leq 0.0 \quad (10- a)$$

iii. According to EC2-08: Considering $A'_s = A_s - \frac{P_u}{f_{yk}/\gamma_s}$

$$iv. \frac{M_{us}}{\left[\frac{A_s f_{yk}}{\gamma_s} \left\{ d_b - \frac{\frac{A'_s f_{yk}}{\gamma_s}}{\eta f_{ck} b_e} \right\} \right]} - \frac{P_u}{\left[\frac{A_s f_{yk}}{\gamma_s} \right]} - 1.0 \leq 0.0 (10- b)$$

v. According to ECP203-17: Considering $A'_s = A_s - \frac{P_u}{f_y/\gamma_s}$

$$vi. \frac{M_{us}}{\left[\frac{A_s f_y}{\gamma_s} \left\{ d_b - \frac{\frac{A'_s f_y}{\gamma_s}}{0.67 f_{cu} b_e} \right\} \right]} - \frac{P_u}{\left[\frac{A_s f_y}{\gamma_s} \right]} - 1.0 \leq 0.0 (10- c)$$

vii. Note that the effect of axial compression may be neglected if less than $0.04f_{cu}bt$

c- Shear strength constraint:following the same practical guidelines outlined by Hassanain [37] that neglected the effect of axial force and postulated that ultimate shear carried by stirrups is about twice the shear capacity of un-cracked concrete, constrains may expressed as follows:

i. According to ACI 318-14:

$$1 - \frac{4.3 \sqrt{f_c} b_w d_b}{V_u} \leq 0.0 \quad (11- a)$$

ii. According to EC2-08:

$$1 - \frac{3v_{Rd,c} b_w d_b}{V_u} \leq 0.0 (11- b)$$

In which the design shear resistance of concrete, $v_{Rd,c}$ is taken the larger of the following two values:

$$[C_{Rd,c} (100 \rho_l)^{1/3} + k_1 \sigma_{cp}] \quad \text{or} \quad [0.035 k^{3/2} f_{ck}]^{1/2} + k_1 \sigma_{cp} \quad (11-b')$$

where,

$C_{Rd,c} = 0.18/\gamma_c$, $k = 1 + \sqrt{200/d_b} \leq 2.0$, $\rho_l = A_{sl}/(b_w d_b) \leq 0.02$, $\sigma_{cp} = P_u/A_c$, $k_1 = 0.15$, A_{sl} is the area of the tensile reinforcement, which extends distance greater than (development length + d_b) beyond the section considered.

For beam section, A_{sl} is considered as the area of the tensile reinforcement provided. For column sections, A_{sl} is used for each side as half of the total area of the longitudinal reinforcement.

iii. According to ECP203-17: After cracking only the shear resistance of concrete is considered, then

$$1 - \frac{0.12 \sqrt{f_{cu}}/\gamma_c b_w d_b}{V_u} \leq 0.0 \quad (11- c)$$

d- Minimum reinforcement constrains: the girder reinforcement should be adequate such that the following constraint may be fulfilled:

i. According to ACI 318-14

$$1.0 - \frac{f_y A_s}{1.4 b_w d_b} \leq 0.0 \text{ or } 1.0 - \frac{4 f_y A_s}{\sqrt{f_c} b_w d_b} \leq 0.0 \quad (12- a)$$

ii. According to EC2-08:

$$1.0 - \frac{f_{yk} A_s}{0.26 f_{ctm} b_w d_b} \leq 0.0 \quad (12- b)$$

iii. According to ECP203-17:

$$1.0 - \frac{f_y A_s}{1.1 b_w d_b} \leq 0.0 \text{ or } 1.0 - \frac{f_y A_s}{0.225 \sqrt{f_{cu}} b_w d_b} \leq 0.0 \quad \text{or}$$

$$1.0 - \frac{A_s}{0.225 \sqrt{f_{cu}} b_w d_b} \leq 0.0 \quad (12-c)$$

e- Maximum reinforcement constrains: It is customary to consider Maximum percentage of reinforcement equal to half the balanced amount ρ_b . This helps satisfying the cracking and deflection requirements of the girder [34, 35]. Thus, this constraint may be expressed as follows:

$$\frac{A_s}{0.75 \rho_b b_w d_b} - 1.0 \leq 0.0 \quad (13)$$

The expression of the balanced amount of reinforcement may be proved to the following form:

i. According to ACI 318-14:

$$\text{ii. } \rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{1}{[1+16.7 f_y * 10^{-4}]} \quad (14- a)$$

iii. According to EC2-08

$$\text{iv. } \frac{3.2 f_{yk} A_s}{f_{ck} b_w d_b} - 1.0 \leq 0.0 \quad (14-b)$$

v. According to ECP203-17

$$\text{vi. } \frac{A_s}{0.67 \rho_b b_w d_b} - 1.0 \leq 0.0 \quad (14-c)$$

$$\rho_b = \frac{0.414 f_{cu}}{f_y} \frac{1}{[1+14.52 f_y * 10^{-4}]} \quad (14- d)$$

III. Slabs' constrains:

a- **Geometric constraint:** the minimum slab thickness $t_{s \text{ min}}$ is 80mm i.e.

$$1.0 - \frac{t_s}{80} \leq 0.0 \quad (15)$$

b- Flexural capacity constraint: considering $b_s=1.0\text{m}$ strip of the slab, the flexural capacity constraint in either circumferential ($d_s=t_s-15\text{mm}$) or radial direction ($d_s=t_s-20\text{mm}$) is formulated for pure moment in the form (ultimate moment-nominal capacity ≤ 0.0) as follows:

i. According to ACI 318-14:

$$1.0 - \frac{\phi}{M_u} \left[A_s f_y \left\{ d_b - \frac{A_s f_y}{1.7 f'_c b_s} \right\} \right] \leq 0.0 \quad (16- a)$$

ii. According to EC2-08:

$$1.0 - \frac{1.0}{M_u} \left[\frac{A_s f_{yk}}{\gamma_s} \left\{ d_b - \frac{A_s f_{yk} / \gamma_s}{2 \eta_{acc} f_{ck} b_s / \gamma_c} \right\} \right] \leq 0.0 \quad (16- b)$$

iii. According to ECP203-17:

$$1.0 - \frac{1.0}{M_u} \left[\frac{A_s f_y}{\gamma_s} \left\{ d_b - \frac{A_s f_y / \gamma_s}{1.33 f_{cu} b_e / \gamma_c} \right\} \right] \leq 0.0 \quad (16- c)$$

c- Shear strength constraint: shear resistance should be entirely resisted by concrete, i.e

i. According to ACI 318-14:

$$1 - \frac{1.43 \sqrt{f'_c} b_s d_s}{V_u} \leq 0.0 \quad (17- a)$$

ii. According to EC2-08:

$$1 - \frac{v_{Rd,c} b_s d_s}{V_u} \leq 0.0 \quad (17- b)$$

In which the design shear resistance of concrete, $v_{Rd,c}$ is taken similar to that of beams.

According to ECP203-17: After cracking only half the shear resistance of concrete is considered, then

$$1 - \frac{0.16 \sqrt{f_{cu}} / \gamma_c b_s d_s}{V_u} \leq 0.0 \quad (17- c)$$

VI. Behavioral Analysis of Vierendeel System

The initial design commenced with dividing the radial and circumferential directions into $n_s=36$ and $n_r=6$ and then the process of optimization continued until getting the near-optimum design. For all cases of design variables, the least cost was achieved at $n_s=40$ and $n_r=5$. Table (3) lists the relative total cost percentage to the optimum solution after ACI 318-14 for $f'_c=25\text{MPa}$, $f_y=400\text{MPa}$, Configuration 1 with parabolic radial top chord and fixed base. The other codes yielded the same number of divisions for the formulated objective function.

Table (3): Relative total cost percentage to the optimum solution after ACI 318-14 for ($f'_c=25\text{MPa}$, $f_y=400\text{MPa}$, Configuration 1-Parabolic radial top chord-Fixed base)

n_s	n_r						
	3	4	5	6	7	8	
24	124	122	119	120	122	123	
28	119	118	115	116	118	119	
32	114	113	110	112	114	116	
36	112	110	108	109*	111	113	
40	109	104	100**	104	108	109	
44	110	111	109	111	112	114	
48	112	113	110	112	113	115	
52	114	114	111	113	114	116	
56	116	115	113	114	115	118	
60	118	117	116	118	119	120	

*Initiation problem **Minimum cost solution

Table (4) summarizes the relative total cost percentage to the optimal solution for $f_{cu}=30\text{MPa}$, $f_y=400\text{MPa}$, $n_r=5$, $n_s=40$ after the three codes with various roof configurations and different geometries. It can be noted that the difference of the outward sloping is minor compared with the inward sloping. However, the inward sloping (Configuration 2) exhibited relatively less deflection (not listed) and implicitly higher stiffness. Systems with fixed support were closer the least cost compared with the hinged support. To have more insight about the code impact on the optimum design, Fig. (7) illustrates the relative total cost percentage to the optimum solution after the three codes for different concrete and steel grades (Configuration 2-Hinged base). It is obvious that ACI 318-14 produced significantly cheaper structures for all cases. Increasing the grade of the steel reinforcement returned less cost. EC2-08 was in the middle while ECP203-2017 was the highest for all cases. On the other hand, f_{cu} in the range 30-35MPa was practically the near-optimum cost design. It is worth noting that albeit parabolic geometry requires more skillful carpentry and hence higher unit cost, it provided the comparatively least cost.

Table (4): Relative total cost percentage to the optimal solution for ($f_{cu}=30\text{MPa}$, $f_y=400\text{MPa}$, $n_r=5$, $n_s=40$)

Slope Configuration	Code	Circular		Parabolic		Straight	
		Hinged	Fixed	Hinged	Fixed	Hinged	Fixed
Outward Sloping-Configuration (1)	ACI 318-14	105.9	102.8	103.9	100.9	106.6	103.5
	EC2-08	113.0	109.7	111.6	108.4	113.5	110.2
	ECP203-17	119.8	117.4	119.7	116.7	119.9	117.9
Inward Sloping-Configuration (2)	ACI 318-14	103.6	101.1	103.1	100.0*	104.2	101.5
	EC2-08	111.4	108.3	111.0	107.8	111.9	108.5
	ECP203-17	119.1	116.9	118.9	116.1	119.3	117.1

*Optimal Solution

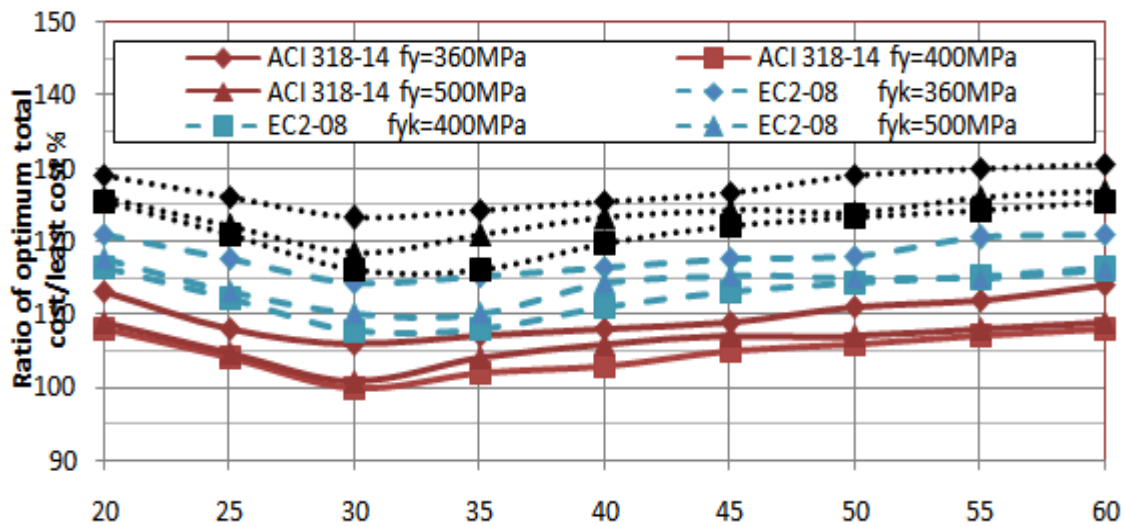


Figure (7): Relative total cost percentage to the optimum solution after the three codes For different concrete and steel grades (Configuration 2-Hinged base)

Fig. (8) depicts the cost distribution among various structural elements with respect to concrete and steel reinforcement quantities. While slabs and foundation represented the highest volume of concrete, vierendeels (1) and (2) got the highest share of steel reinforcement weight. This comprised a major concern for the designer to take into account in the layout of such systems. Of course, this concern may directly influence the total cost that is sensitive to the relative unit cost of components included in the objective function.

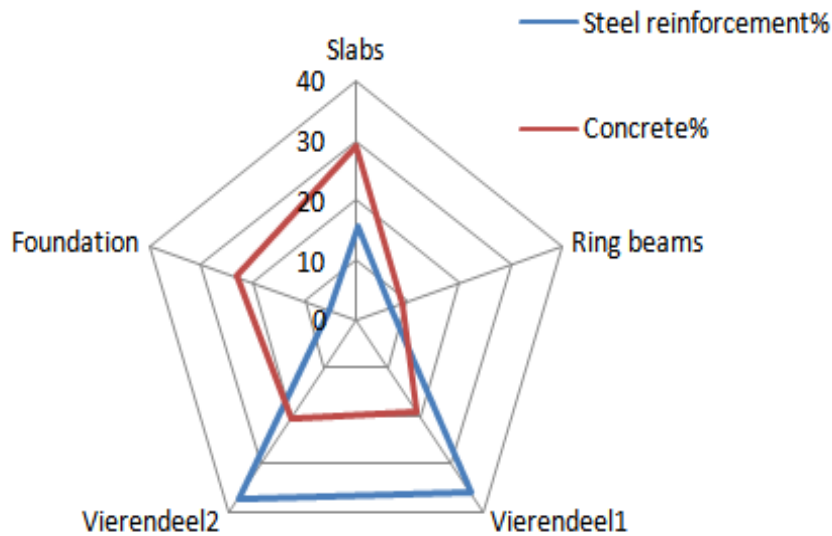


Figure (8): Percentage of the relative reinforcement weight and concrete volume of each element

In as much as steel reinforcement constituted a major component of the cost of the wide-span vierendeels, a scrutiny was devoted to investigate the effect of splice type on the overall reinforcement cost. Fig. (9) shows the comparison of the percentage of cost increase relative to reinforcement cost of radial girders which indicates that special cage length order gave the least cost increase followed by the lap splice. Use of mechanical splices led to the highest cost increase in steel reinforcement due to the cost of couples, threading process, workmanship and additional testing that were included in the unit cost of this item.

The pie chart of Fig. 10 demonstrates the distribution of cost items of the optimum solution of the hinged system with inward top sloping of parabolic geometry. Steel reinforcement represented the major component of the total cost followed by concrete then formwork expenses. Miscellaneous items were minor for the special steel cage splice and special hinge support construction. This component was much less for fixed support and was about 1.1% only.

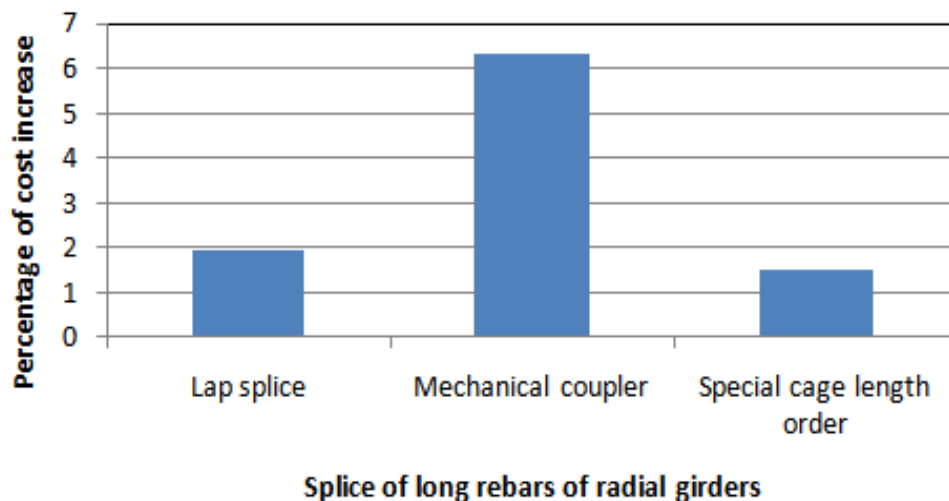


Figure (9): Percentage of cost increase relative to reinforcement cost of radial girders

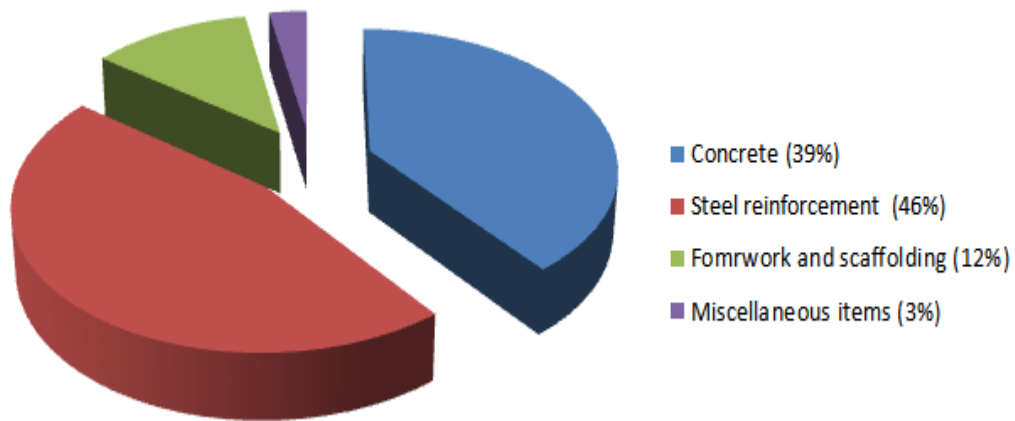


Figure (10): Cost items of the optimum solution of the hinged system with inward top sloping

VII. Comparison versus others systems

Because framed as well as shell roofing systems used to provide the alternatives to the vierendeel system analyzed before with ACI 318-14 for ($f'_c=25\text{MPa}$, $f_y=400\text{MPa}$, Configuration 1-Parabolic radial top chord-Fixed base, their efficiency needed to be investigated in this study. Fig. (11) illustrates the architectural model for three cases: (a) framed system with single chord, (b) nonprismatic folded plate and (c) nonprismatic cylindrical shell for roof of exterior hall. These alternatives were analyzed in a similar optimization framework and indicated that framed system with plane roof resulted in higher total cost than the vierendeel system by about 30%. On the other hand, the nonprismatic folded plate and cylindrical shell roofing were very close and provided an overall cost saving of about 7.6% than the framed system. Cylindrical shell provided higher cost of formwork and scaffolding whose difference was insignificant because of the complexity of the entire system which included many elements for the structural design.

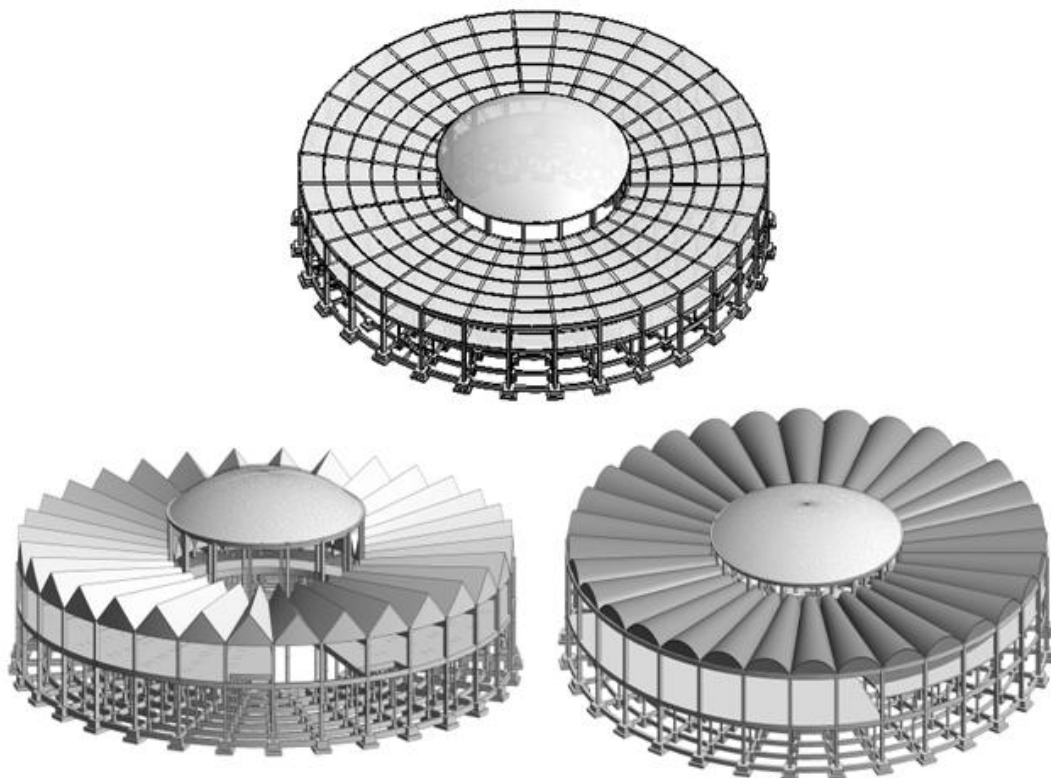


Figure (11) Architectural model for (a) framed system with single chord, (b) nonprismatic folded plate and (c) nonprismatic cylindrical shell for roof of exterior hall

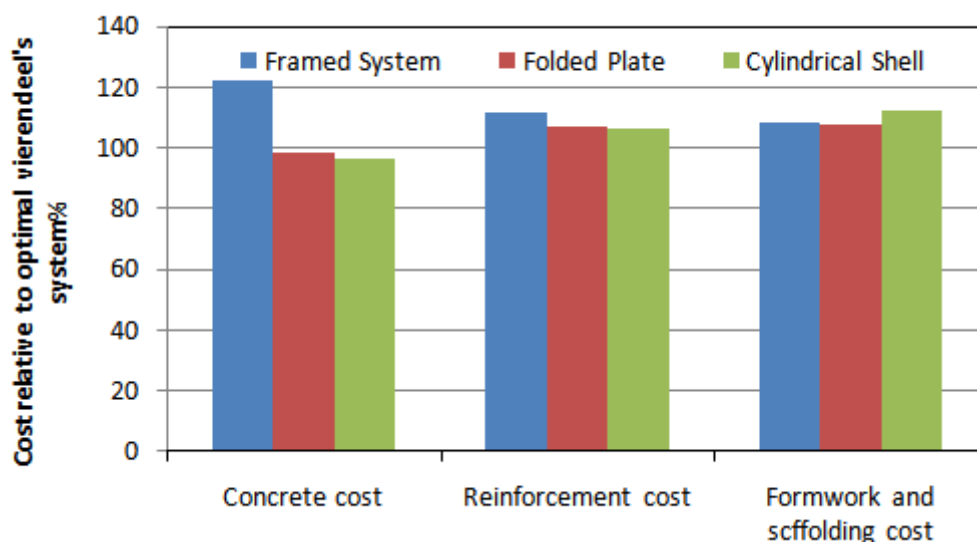


Figure (12): Comparison of different solutions compared with the vierendeel's optimal solution

VIII. Conclusions

From the present study, which is linked to real life practical application, the following conclusions may be drawn:

1. The developed methodology was found suitable for the near optimum design of wide-span circular halls and successfully linked the relevant analysis and design software packages in an integrated framework.
2. Genetic algorithm proved to be suitable for such sophisticated structural optimization problems that included numerous variables and design constraints.
3. Codes' approach had direct impact on the overall cost of the structure. ACI 318-14 yielded the least cost followed by EC2-08 then ECP203-17.
4. Architectural characteristics regarding slope and geometry had not that much significant influence on the cost change.
5. Steel and concrete grades had considerable effect on the cost items.
6. Fixed base provided comparatively less cost than hinged support.
7. Steel cages of special order of the required length indicated less cost compared with lap and mechanical splices. However, its effect on the overall cost is minor.
8. Selecting covering system that is suitable from atheistic prospective might have priority, as the cost difference of vierendeel system was not significantly higher than nonprismatic shell roofing systems. However, framed system resulted in considerably higher cost.

References

- [1]. Schmidt, L.C., 1962, Minimum Weight Layouts of Elastic Statically Determinate Triangulated Frames under Alternative Loads System. *Journal of Mechanics and Physics of Solids*, 10, 139-149.
- [2]. Hassanian , M. A. (1992) "Optimal Design of Reinforced Concrete Frames". M.Sc Thesis , King Fahd University of
- [3]. Balling, R. J. and Yao, X. (1997) "Optimization of Reinforced Concrete Frames". *Journal of Structural Engineering*, Vol 123, No.2. Pp 193-202..
- [4]. Rajeev,S. and Krishnamoorthy, C.S.(1998) "Genetic Algorithm-Based Methodology for Design Optimization of Reinforced Concrete Frames ",*Computer –Aided Civil and Infrastructure Engineering*, Vol 13. Published by Blackwell Publishers, 350 main street, Malden, USA. Pp63-74.
- [5]. Bontempi F., Biondini, F. and Malerba, P.G. (2000)"Fuzzy Based Optimization of Concrete Structures By Using Generic Algorithms", 8th ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability.
- [6]. Lee, C. and Ahn, J, 2003, Flexural Design of Reinforced Concrete Frames by Genetic Algorithm. *Journal of Structural Engineering (ASCE)*, 129, 762-774.
- [7]. Camp, C.V., Pezeshk, S. and Hansson, H, 2003, Flexural Design of Reinforced Concrete Frames Using a Genetic Algorithm. *Journal of Structural Engineering (ASCE)*, 129, 105-115.
- [8]. Guerra A. and Kiouisis P. D., 2006, Design Optimization of Reinforced Concrete Structures,*Computers and Concrete*, Vol. 3, No. 5, pp. 313-334
- [9]. Babiker, S.A., Adam, F.M. and Mohamed, A.E., 2012, Design Optimization of Structure Concrete Beams Using Artificial Neural Network. *International Journal of Engineering Inventions*, 1, 07-13.
- [10]. Govindaraj V. and Ramasamy J. V., 2007, Optimum Detailed Design of RC Frames using Genetic Algorithms, *Engineering Optimization*, Vol. 39, No. 4, pp. 471-494.
- [11]. Taher, S. F., 2001, A Multiphase Expert System-Neural Network Approach for Optimum Design of R.C. Structures, Proc. 6th International Conference on the Application of Artificial Intelligence to Civil and Structural Engineering, 19-21 Sept. Saxe-Coburg Publ., Eisenstadt nr Vienne, Austria.

- [12]. Taher, S. F., 1997, Impact of Codes' Approach on the Optimal Design of R.C. Frames, , Proc. Third Alexandria Conference on Structural and Geotechnical Engineering ACSGE-3, Dec. 27-29, Alexandria, Egypt.
- [13]. Aga A. A. and Adam F. M., 2015, Design Optimization of RC Frames, J. Civil Engineering, Vol. 5, pp. 74-83.
- [14]. Boris C., 2000, Optimum Design of RC Skeletal Systems using Nonlinear Programming Techniques, University of Derby, Ph.D. Thesis.
- [15]. Kripka M., Boito D., Triches J. and de-Medeiros G.F., 2015, Optimization of Reinforced Concrete Frames by Harmony Search Method, 11th World Congress on Structural and Multidisciplinary Optimisation, 07th -12th, June , Sydney Australia
- [16]. KetkukahT.S., Abubakar I. and EjehS.P., 2014, Optimum Design Sensitivity of Reinforced Concrete Frames, International Journal of Advanced Engineering Research and Technology (IJAERT), Vol. 2, No. 5, pp. 144-158.
- [17]. ACI 318-14, 2014, Building Code Reuirements for Reinforced Concrete and Commentary, American Concrete Institute, Detroit.
- [18]. Eurocode 2, 2008, Design of Concrete Structures, DD ENV 1992-1-1.
- [19]. ECP 203, 2017, Design and Execution of Reinforced Concrete Structures, Housing and Building Research Center, Ministry of Housing, Egypt
- [20]. Evolver, 2014, Sophisticated Optimization for Spreadsheets, Version 7, Palisade.
- [21]. Hawileh R.A., MalhasF.A., Rahman A., 2009, Comparison between ACI 318-05 and Eurocode 2 (EC2-94) in flexural Concrete Design, Structural Engineering and Mechanics, Vol. 32, No. 6, pp. 705-724.
- [22]. Construction Publishers and Consultants, 2016, RS Means Concrete/Masonry Cost Data, 34th Ed. Reed Construction Data, Inc.
- [23]. Hassanain , M. A., 1992, Optimal Design of Reinforced Concrete Frames,M.Sc Thesis , King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.
- [24]. Spires D. B. and Arora J. S., 1989, Optimal Design of Tall RC Tube Building, Computer Utilization in Structural Engineering, Structural Congress89, San Francisco, May 1-5, Nelson J. K. Editor, ASCE, pp. 479-489.
- [25]. Spires D. B. and Rojiani K. B., 1983, Reliability Based Optimum Design of RC Frames, J. Structual Eng. Vol. 109, No. 3 March, pp 741-757.

Selah El-Din Taher "Influential Codes for Optimum Design Parameters of Wide Span Circular R.C. Halls." IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) , vol. 15, no. 4, 2018, pp. 54-69