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# Application of Shear Deformation Theory for Analysis of CCCS and SSFS Rectangular Isotropic Thick Plates.

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***Abstract:*** *This paper presents the application of shear deformation theory for pure bending analysis of CCCS and SSFS rectangular isotropic using polynomial shape function. The theory herein accounts for shear deformation and no shear correction factor is required.The principle of elasticity is adopted for the formation of the total potential energy equation of a thick plate. The governing equations for determination of displacement coefficients were derived by subjecting the total potential energy equation to direct variation. Numerical studies of three edges clamped with one edge simply supported and three edges simply supported with one edge single free of support thick plate were carried out. The results obtained herein for in plane and transverse displacements and stresses were compared with those from previous works and observed that they have the same behavioral trend and are quite close. It is alsoobserved that at span to depth ratio of 100 the values of the obtained results herein coincides with that of Classical Plate Theory (CPT). This satisfied the assumption that at span to depth ratio of 100, a plate is consider to be thin plate and thin plate theory can be used for the analysis.*

***Keywords:****shear deformation, shear correction factor, vertical shear stress, deflection, displacement, potential energy*

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## I. Introduction

The effect of shear deformation has been the basis for thick plate theory. Refined plate theory is mostly used by previous researchers for thick plate analysis. This required the use of trigonometry displacement functions which involved the use of double Fourier series. The complexity of using double Fourier series for thick plate analysis has made most engineers resorted to thin plate analysis in the face of its (double Fourier) numerous challenges.The idealization of a thick plate as a thin plate by most engineers because the difficulty of handling double Fourier series of thick plate analysis always underestimates the stresses in the plate. The consequences of using these erroneous stresses in design and construction are structural failure and sometimes total collapse. Previous researchers have delved into different aspects of thick plate analysis such as: pure bending (Ghugal& George, 2010; Sayyad&Ghugal, 2012; Sayyad et al., 2016; etc), buckling (Avalos &Larondo, 1995; Wang et al., 2001; Kim al., 2009; Ibearugbulem et al., 2014; etc), free vibrations (Guruwamy& Yang, 1979; Gupta & Ansari, 1998; Wu & Liu; 2001; Sayyad&Ghugal, 2012; etc), isotropic plates (Raju & Rao, 1996; Sayyad, 2011; Sayyad&`Ghugal 2012, etc), orthotropic plates (Gupta & Lai, 1985; Shimpi& Patel, 2006; Chikathanker et al., 2013; etc), anisotropic plates (Krishna, 1984; Setoodeh&Karami, 2004; Azhari&Kassaei, 2004; etc), graded laminated plates (Karama&Mistov, 2003; Goswami& Becker, 2013; Daouadji et al., 2013, Reddy, 2014; etc). One common observation is that most of these works are based mainly on trigonometric displacement functions. In the course of the development of refined plate theory, the assumption that the shear deformation line does not vary linearly with the depth of the plate was introduced. This according to many scholars helps to ensure that the vertical shear stress across the plate section does not remain constant, but varies parabolically with zero values at both the top and bottom surfaces (Ambartsumian, 1958; Murty, 1984; Touratier, 1991; Karama&Mistou, 2003). They came up with different shear deformation line functions, here-in-after called F(z). However, their F(z) functions were not strictly based on the vertical shear stress mathematical formulation. If we follow the work of Timoshenko and Woinowsky-krieger, (1970), we shall note that maximum shear stress occurs at the mid surface (where z = 0) and the value of the maximum shear stress is one and half of nominal vertical shear stress. With most of the F(z) functions from the literature, we may obtain good profile (curve) for the deformation line and shear stress distribution across the section, but the midsurface value of shear stress may not coincide with that by Timoshenko and Woinowsky-krieger, (1970).Thus, the present study came up with these specific objectives:

i. To develop a direct governing simultaneous equations for thick plate analysis
ii. To formulation of a polynomial F(z) mathematical in line with works of Timoshenko

iii. To use polynomial displacement functions, variation calculus analysis.

## II. Theoritical Formulation

The plate under considerationoccupies in x-y-z Cartesian coordinate system region:
$0 \leq x \leq a; 0 \leq y \leq b; -t/2 \leq z \leq t/2$
The dimensions (lengths) along x, y and z axes are a, b and t respectively.

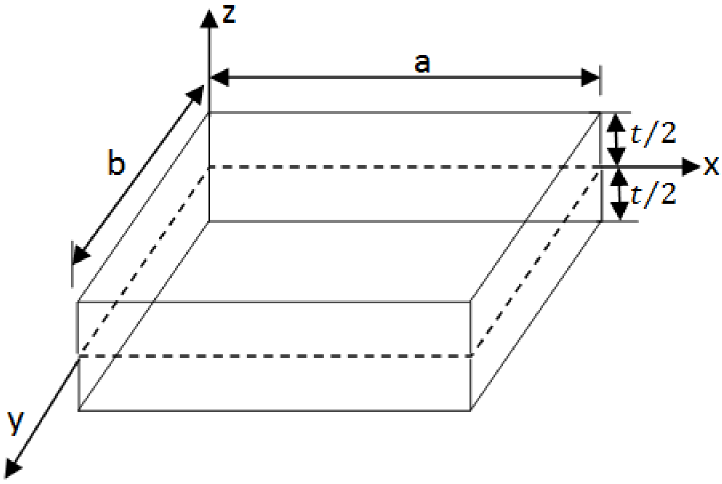


**Figure 1:** Geometry of a rectangular thick plate under load

To achieve the specific objectives of this study, we used the under-listed assumptions:

i. The displacement components u and v are the in-plane displacements in x and y directions respectively and w is the transverse displacement in z-direction. These displacements are small when compared with plate thickness.
ii. The in-plane displacements, u and v are differentiable with respect to x, y and z coordinates, while the transverse displacement (deflection), w is only differentiable with respect to x and y coordinates. This means that the first derivative of w with respect to z is zero. Consequently, $\varepsilon_z = 0$.
iii. The effect of the transverse normal stress on the gross response of the plate is small when compared with other stresses. Thus, it can be neglected. That is $\sigma_z = 0$.
iv. The vertical line that is initially normal to the middle surface of the plate before bending is no longer straight nor normal to the middle surface after bending. That is $\phi \neq \theta_c$. Where $\phi$ is the total rotation of the middle surface in this case, $\theta_c$ is the classical plate theorem rotation of the middle surface.

## III. Kinematic Relations

The in-plane displacements composed of classical and shear deformation parts in line with the fourth assumption as stated herein. Ibeargbulem et al. (2016) gave the classical in-plane displacements ($u_c$ and $v_c$) and shear deformation in-plane displacements ($u_s$ and $v_s$) as:

$$u_c = -z\theta_{cx} = -z\frac{dw}{dx} \qquad (1)$$

$$v_c = -z\theta_{cy} = -z\frac{dw}{dy} \qquad (2)$$

$$u_s = F(z)\theta_{sx} \qquad (3)$$

$$v_s = F(z)\theta_{sy} \qquad (4)$$

Where F(z) is the shear deformation profile of the vertical line, which was earlier (before bending) straight and normal to the middlle surface, but after bending refused to be straight nor being normal to the middle surface. For classical part of deformation F(z) remained straight and normal to the middle surface. That is F(z) is the same as z for classical part of dewformation. Ibearugbulem et al. (2016) defined the shear deformation profile line as:

$$F(z) = \frac{3z}{2}\left(1 - \frac{4}{3}\left[\frac{z}{t}\right]^2\right) \qquad (5a)$$

This is written in non dimensional form as:

$$F(S) = \frac{3St}{2}\left(1 - \frac{4}{3}S^2\right) \qquad (5b)$$

Where S = z/t. Adding equations (1) and (2) gave the in-plane displacement of thick plate along x direction as:

$$u = -z\frac{dw}{dx} + F(z)\theta_{sx} \qquad (6)$$

Similarly, adding equations (2) and (3) gave the in-plane displacement of thick plate along y direction as:

$$v = -z\frac{dw}{dy} + F(z)\theta_{sy} \qquad (7)$$

Let the deflection (transverse displacement), w be defined as:

$$w = c_1 h \qquad (8)$$

Where c1 and h are the yet to be determined coefficient of deflection and shape function of deflection respectively. Substituting equation (8) into equations (6) and (7) respectively gave:

$$u = (-c_1 z + c_2 F(z))\frac{dh}{dx} \qquad (9)$$

$$v = (-c_1 z + c_3 F(z))\frac{dh}{dy} \qquad (10)$$

Where $c_2$ and $c_3$ are coefficients of shear deformation rotations along x and y directions ($\theta_{sx}$ and $\theta_{sy}$).

## IV. Strain - Displacement Relations

Based on the second assumption herein, that deflection is not differentiable with respect to z, it follows that $\varepsilon_z$ is equal to zero. Conseuently, the normal stress along z axis is also taken to be zero. Thus, engineering strain components remain only five with five coreesponding stress components. Thefive engineering strain components are defined as:

$$\varepsilon_x = \frac{du}{dx} = [-c_1 z + c_2 F(z)]\frac{d^2h}{dx^2} \qquad (11)$$

$$\varepsilon_y = \frac{dv}{dy} = [-c_1 z + c_3 F(z)]\frac{d^2h}{dy^2} \qquad (12)$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} = [-c_1 z + c_2 F(z)]\frac{d^2h}{dxdy} + [-c_1 z + c_3 F(z)]\frac{d^2h}{dxdy}\ .\ That\ is:$$

$$\gamma_{xy} = [-2c_1 z + c_2 F(z) + c_3 F(z)]\frac{d^2h}{dxdy} \qquad (13)$$

$$\gamma_{xz} = \frac{du}{dz} + \frac{dw}{dx} = \left[-c_1 + c_2\frac{dF(z)}{dz}\right]\frac{dh}{dx} + c_1\frac{dh}{dx} = c_2\frac{dF(z)}{dz}\frac{dh}{dx} \qquad (14)$$

$$\gamma_{yz} = \frac{dv}{dz} + \frac{dw}{dy} = \left[-c_1 + c_3\frac{dF(z)}{dz}\right]\frac{dh}{dy} + c_1\frac{dh}{dy} = c_3\frac{dF(z)}{dz}\frac{dh}{dy} \qquad (15)$$

## V. Constitutive Relations

The five stress components are defined in terms of strains as:

$$\sigma_x = \frac{E}{1-\mu^2}[\varepsilon_x + \mu\varepsilon_y] \qquad (16)$$

$$\sigma_y = \frac{E}{1-\mu^2}[\mu\varepsilon_x + \varepsilon_y] \qquad (17)$$

$$\tau_{xy} = \frac{E(1-\mu)}{1-\mu^2}\gamma_{xy} \qquad (18)$$

$$\tau_{xz} = \frac{E(1-\mu)}{1-\mu^2}\gamma_{xz} \qquad (19)$$

$$\tau_{yz} = \frac{E(1-\mu)}{1-\mu^2}\gamma_{yz} \qquad (20)$$

## VI. Stress – Displacement Equations

Substituting equations (11) to (13) into equations (14) to (18) where appropriate gave:

$$\sigma_x = \frac{E}{1-\mu^2}\left[[-c_1 z + c_2 F(z)]\frac{d^2h}{dx^2} + \mu[-c_1 z + c_3 F(z)]\frac{d^2h}{dy^2}\right] \qquad (21)$$

$$\sigma_y = \frac{E}{1-\mu^2}\left[\mu z\left(c_1 + \frac{F(z)}{z}B_2\right)\frac{d^2h}{dx^2} + [-c_1 z + c_3 F(z)]\frac{d^2h}{dy^2}\right] \qquad (22)$$

$$\tau_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}[-2c_1 z + c_2 F(z) + c_3 F(z)]\frac{d^2h}{dxdy} \qquad (23)$$

$$\tau_{xz} = \frac{E(1-\mu)}{2(1-\mu^2)} c_2 \frac{dF(z)}{dz}\frac{dh}{dx} \quad (24)$$

$$\tau_{yz} = \frac{E(1-\mu)}{2(1-\mu^2)} c_3 \frac{dF(z)}{dz}\frac{dh}{dy} \quad (25)$$

## Total Potential Energy

Total potential energy is the summation of strain energy, U and external work, V. that's

$$\Pi = U + V \quad (26)$$

Let's define external work as:

$$V = -q\int_x \int_y w\,dxdy \quad (27)$$

Let's also define strain energy mathematically as: $U = \int_x \int_y \left[\int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma.\varepsilon dz\right] dxdy.\ That\ is:$

$$U = \int_x \int_y \left[\int_{-\frac{t}{2}}^{\frac{t}{2}} \left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right) dz\right] dxdy \quad (28)$$

Applying equations (11) and (21), (12) and (22), (13) and (23), (14) and (24), and (15) and (25) respectively gave:

$$\sigma_x\varepsilon_x = \frac{E}{1-\mu^2}\Bigg[[z^2{c_1}^2 - 2c_1c_2zF(z) + {c_2}^2F(z)^2]\left(\frac{d^2h}{dx^2}\right)^2 + \mu[z^2{c_1}^2 - c_1c_2zF(z) - c_1c_3zF(z) + c_2c_3F(z)^2]\left(\frac{d^2h}{dxdy}\right)^2\Bigg] \quad (29)$$

$$\sigma_y\varepsilon_y = \frac{E}{1-\mu^2}\Bigg[[z^2{c_1}^2 - 2c_1c_3zF(z) + {c_3}^2F(z)^2]\left(\frac{d^2h}{dy^2}\right)^2 + \mu[z^2{c_1}^2 - c_1c_2zF(z) - c_1c_3zF(z) + c_2c_3F(z)^2]\left(\frac{d^2h}{dxdy}\right)^2\Bigg] \quad (30)$$

$$\tau_{xy}.\gamma_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}[4{c_1}^2z^2 - 4c_1c_2zF(z) - 4c_1c_3zF(z) + {c_2}^2F(z)^2 + 2c_2c_3F(z)^2 + {c_3}^2F(z)^2]\left(\frac{d^2h}{dxdy}\right)^2 \quad (31)$$

$$\tau_{xz}.\gamma_{xz} = \frac{E(1-\mu)}{2(1-\mu^2)}{c_2}^2\left[\frac{dF(z)}{dz}\right]^2\left(\frac{dh}{dx}\right)^2 \quad (32)$$

$$\tau_{yz}.\gamma_{yz} = \frac{E(1-\mu)}{(1-\mu^2)}{c_3}^2\left[\frac{dF(z)}{dz}\right]^2\left(\frac{dh}{dy}\right)^2 \quad (33)$$

Substituting equations (39) to (33) into equation (28) gave:

$$U = \frac{D}{2}\int_x \int_y [g_1{c_1}^2 - 2g_2c_1c_2 + g_3{c_2}^2]\left(\frac{d^2h}{dx^2}\right)^2$$
$$+\left[2g_1{c_1}^2 - 2g_2c_1c_2 - 2g_2c_1c_3 + \frac{1}{2}g_3{c_2}^2 + g_3c_2c_3 + \frac{1}{2}g_3{c_3}^2\right]\left(\frac{d^2h}{dxdy}\right)^2$$
$$+\mu\left[g_3c_2c_3 - \frac{1}{2}g_3{c_2}^2 - \frac{1}{2}g_3{c_3}^2\right]\left(\frac{d^2h}{dxdy}\right)^2$$
$$+[g_1{c_1}^2 - 2g_2c_1c_3 + g_3{c_3}^2]\left(\frac{d^2h}{dy^2}\right)^2 + (1-\mu)\frac{\alpha^2}{2}g_4{c_2}^2\left(\frac{dh}{dx}\right)^2 + (1-\mu)\frac{\alpha^2}{2}g_4{B_3}^2\left(\frac{dh}{dy}\right)^2]\,dxdy \quad (34)$$

$$Where: \overline{D} = \frac{t^3}{12}$$

$$g_1 = \frac{\left(\int_{-\frac{t}{2}}^{\frac{t}{2}} z^2dz\right)}{\overline{D}} = 1 \quad (35)$$

$$g_2 = \frac{\left(\int_{-\frac{t}{2}}^{\frac{t}{2}} zF(z)dz\right)}{\overline{D}} \qquad (36)$$

$$g_3 = \frac{\left(\int_{-\frac{t}{2}}^{\frac{t}{2}} F(z)^2 dz\right)}{\overline{D}} \qquad (37)$$

$$\alpha^2 g_4 = \frac{\left(\int_{-\frac{t}{2}}^{\frac{t}{2}} \left[\frac{dF(z)}{dz}\right]^2 dz\right)}{\overline{D}} \qquad (38)$$

The flexural rigidity of the plate is:

$$D = \frac{E}{1-\mu^2} * \overline{D} = \frac{Et^3}{12(1-\mu^2)} \qquad (39)$$

Let's define the span-to-depth ratio as

$$\propto = \frac{a}{t} \qquad (40)$$

Let define non dimensional coordinates R and Q and the span-span aspect ratio, P as:

$$R = \frac{x}{a} \Rightarrow x = aR \qquad (41)$$

$$Q = \frac{y}{b} \Rightarrow y = bQ \qquad (42)$$

$$P = \frac{b}{a} \Rightarrow b = aP \qquad (43)$$

Substituting equations (27), (34) and (41) to (43)into equation (26) gives:

$$\Pi = \frac{abD}{2a^4}\int_0^1\int_0^1 [\, g_1 {c_1}^2 - 2g_2c_1c_2 + g_3{c_2}^2] \left(\frac{d^2h}{dR^2}\right)^2$$

$$+\frac{1}{P^2}[2g_1{c_1}^2 - 2g_2c_1c_2 - 2g_2c_1c_3]\left(\frac{d^2h}{dRdQ}\right)^2$$

$$\frac{(1+\mu)}{P^2} g_3c_2c_3\left(\frac{d^2h}{dRdQ}\right)^2$$

$$+\frac{(1-\mu)}{2P^2}[g_3{c_2}^2 + g_3{c_3}^2]\left(\frac{d^2h}{dRdQ}\right)^2$$

$$+\frac{(1-\mu)\alpha^2}{2} g_4{c_2}^2\left(\frac{dh}{dR}\right)^2$$

$$+\frac{(1-\mu)\alpha^2}{2P^2} g_4{c_3}^2\left(\frac{dh}{dQ}\right)^2 ]\, dRdQ$$

$$-ab\int_0^1\int_0^1 FF\, dRdQ \qquad (44)$$

## VII. Direct Governing Equations

This total potential energy contains three unknown coefficients ($c_1$, $c_2$ and $c_3$) for deflection, rotation in x axis and rotation in y axis. Differentiating total potential energy equation with respect to $c_1$, $c_2$ and $c_3$ in turn will give three simultaneous equations.

$$\frac{d\Pi}{dc_1} = \frac{d\Pi}{dc_2} = \frac{d\Pi}{dc_3} = 0 \qquad (45)$$

Substituting equation (44) into equation (45) gave in matrix form:

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{12} & r_{22} & r_{23} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \frac{qa^4}{D}\begin{bmatrix} F_{rq} \\ 0 \\ 0 \end{bmatrix} \quad (46a)$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{12} & r_{22} & r_{23} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} F_{rq} \\ 0 \\ 0 \end{bmatrix} \quad (46a)$$

Where

$$B_i = \frac{c_i}{\left(\frac{qa^4}{D}\right)} = c_i\left(\frac{D}{qa^4}\right); \quad c_i = B_i\left(\frac{qa^4}{D}\right)$$

$$r_{11} = g_1\left(k_1 + 2\frac{k_2}{P^2} + \frac{k_3}{P^4}\right);\; r_{12} = -g_2(k_1 + \frac{k_2}{P^2})$$
$$r_{13} = -g_2\left(\frac{k_2}{P^2} + \frac{k_3}{P^4}\right);\; r_{23} = \frac{(1+\mu)}{2P^2} g_3 k_2$$
$$r_{22} = g_3 k_1 + \frac{(1-\mu)}{2P^2} g_3 k_2 + \frac{(1-\mu) \propto^2}{2} g_4 k_4$$
$$r_{33} = \frac{(1-\mu)}{2P^2} g_3 k_2 + \frac{1}{P^4} g_3 k_3 + \frac{(1-\mu) \propto^2}{2P^2} g_4 k_5$$
$$k_1 = \int_0^1 \int_0^1 \left(\frac{d^2 h}{dR^2}\right)^2 dR dQ$$
$$k_2 = \int_0^1 \int_0^1 \left(\frac{d^2 h}{dR dQ}\right)^2 dR dQ$$
$$k_3 = \int_0^1 \int_0^1 \left(\frac{d^2 h}{dQ^2}\right)^2 dR dQ;$$
$$k_4 = \int_0^1 \int_0^1 \left(\frac{dh}{dR}\right)^2 dR dQ$$
$$k_5 = \int_0^1 \int_0^1 \left(\frac{dh}{dQ}\right)^2 dR dQ;\; F_{rq} = \int_0^1 \int_0^1 h\, dR dQ$$

## VIII. Definition Of Some Quantities

From equation (8) it is gathered that:

$$w = c_1 h = \left[B_i\left(\frac{qa^4}{D}\right)\right].h \Longrightarrow w = B_i h\left(\frac{qa^4}{D}\right)$$
$$w = \bar{w}\left(\frac{qa^4}{D}\right) = B_1 h\left(\frac{qa^4}{D}\right)$$
$$u = \bar{u}\left(\frac{qa^4}{D}\right) = \frac{1}{\propto}(-B_1 S + B_2 F(S))\frac{dh}{dR}.\left(\frac{qa^4}{D}\right)$$
$$v = \bar{v}\left(\frac{qa^4}{D}\right) = \frac{1}{P\propto}(-B_1 S + B_3 F(S))\frac{dh}{dQ}\left(\frac{qa^4}{D}\right)$$
$$\sigma_x = \overline{\sigma_x}\,.q = \frac{E}{[1-\mu^2]}\left\{\frac{[-B_1 S + B_2 F(S)]}{\propto a}\frac{d^2 h}{dR^2} + \mu\frac{[-B_1 S + B_3 F(S)]}{P^2 \propto a}\frac{d^2 h}{dQ^2}\right\}\left(\frac{qa^4}{D}\right)$$

That is:

$$\sigma_x = \overline{\sigma_x}\,.q = 12q \propto^2 \left\{[-B_1 S + B_2 F(S)]\frac{d^2 h}{dR^2} + \frac{\mu}{P^2}[-B_1 S + B_3 F(S)]\frac{d^2 h}{dQ^2}\right\}$$

Similarly;

$$\sigma_y = \overline{\sigma_y}\,.q = 12q \propto^2 \left\{\mu[-B_1 S + B_2 F(S)]\frac{d^2 h}{dR^2} + \frac{1}{P^2}[-B_1 S + B_3 F(S)]\frac{d^2 h}{dQ^2}\right\}$$
$$\tau_{xy} = \overline{\tau_{xy}}\,.q = \frac{6q \propto^2}{P}\left\{[-2B_1 S + B_2 F(S) + B_3 F(S)]\frac{d^2 h}{dR dQ}\right\}(1-\mu)$$
$$\tau_{xz} = \overline{\tau_{xz}}.q = 6q \propto^3 \left\{B_2 \frac{dF(S)}{dS}\frac{dh}{dR}\right\}(1-\mu)$$
$$\tau_{yz} = \overline{\tau_{yz}}.q = 6q \propto^3 \left\{\frac{B_3}{P}\frac{dF(S)}{dS}\frac{dh}{dQ}\right\}(1-\mu)$$

From the foregoing definitions, it was gathered that:

$$\bar{w} = B_1 h \quad (47)$$
$$\bar{u} = \frac{1}{\propto}(-B_1 S + B_2 F(S))\frac{dh}{dR} \quad (48)$$
$$\bar{v} = \frac{1}{P\propto}(-B_1 S + B_3 F(S))\frac{dh}{dQ} \quad (49)$$
$$\overline{\sigma_x} = 12 \propto^2 \left\{[-B_1 S + B_2 F(S)]\frac{d^2 h}{dR^2} + \frac{\mu}{P^2}[-B_1 S + B_3 F(S)]\frac{d^2 h}{dQ^2}\right\} \quad (50)$$

$$\overline{\sigma_y} = 12 \propto^2 \left\{\mu[-B_1S + B_2F(S)]\frac{d^2h}{dR^2} + \frac{1}{P^2}[-B_1S + B_3F(S)]\frac{d^2h}{dQ^2}\right\} \quad (51)$$

$$\overline{\tau_{xy}} = \frac{6 \propto^2}{P}(1-\mu)\left\{[-2B_1S + B_2F(S) + B_3F(S)]\frac{d^2h}{dRdQ}\right\} \quad (52)$$

$$\overline{\tau_{xz}} = 6 \propto^3 (1-\mu)\left\{B_2\frac{dF(S)}{dS}\frac{dh}{dR}\right\} \quad (53)$$

$$\overline{\tau_{yz}} = 6 \propto^3 (1-\mu)\left\{\frac{B_3}{P}\frac{dF(S)}{dS}\frac{dh}{dQ}\right\} \quad (54)$$

However, Sayyad et al. (2012) defined the non dimensional parameters different from they were defined herein. Their definitions are:

$$\widehat{w} = \frac{100Ew}{qt \propto^4} = 1200(1-\mu^2)\,.\,\overline{w} (55)$$

$$\widehat{u} = \frac{uE}{qt \propto^3} = 12(1-\mu^2)\overline{u} (56)$$

$$\widehat{v} = \frac{vE}{qt \propto^3} = 12(1-\mu^2)\overline{v} (57)$$

$$\widehat{\sigma_x} = \frac{\sigma_x}{q \propto^2} = \frac{\overline{\sigma_x}}{\propto^2} (58)$$

$$\widehat{\sigma_y} = \frac{\sigma_y}{q \propto^2} = \frac{\overline{\sigma_y}}{\propto^2} (59)$$

$$\widehat{\tau_{xy}} = \frac{\tau_{xy}}{q \propto^2} = \frac{\overline{\tau_{xy}}}{\propto^2} (60)$$

$$\widehat{\tau_{xz}} = \frac{\tau_{xz}}{q \propto} = \frac{\overline{\tau_{xz}}}{\propto} (61)$$

$$\widehat{\tau_{yz}} = \frac{\tau_{yz}}{q \propto} = \frac{\overline{\tau_{yz}}}{\propto} (62)$$

## IX. Numerical PROBLEM

Determine the deflection at the center (0.5, 0.5, 0) of cccs and ssfs thick plate. Where (0.5, 0.5, 0) means R = 0.5; Q = 0.5; S = 0. Determine also the in-plane normal stresses at (0.5, 0.5, 0.5), in-plane shear stress at (0, 0, 0.5) and the vertical shear stress ($\tau_{xz}$) at (0, 0.5, 0) of thessss and ssfs plate. Polynomial displacement function shall be used.

The polynomial displacement functions, h is given as:

a).CCCS rectangular thick plate

$h = (R - 2R^3 + R^4).(Q - 2Q^3 + Q^4)$

b). SSFS rectangular thick plate

$$h = (R - 2R^3 + R^4).\left(\frac{7}{3}Q - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right)$$

The stiffness coefficients (k values) for cccs plate are:

$k_1 = 0.002857;\ k_2 = 0.0016327$

$k_3 = 0.006032;\ k_4 = 0.000136$

$k_5 = 0.0001437;\ Frq = 0.0025$

Similarly, the stiffness coefficients (k values) for ssfs plate are:

$k_1 = 4.025782;\ k_2 = 0.6013605$

$k_3 = 0.187457;\ k_4 = 0.407371$

$k_5 = 0.1046604;\ Frq = 0.16667$

## X. Results And Discussions

Non dimensional Center deflection of cccs plate (when multiplied by 100) as obtained in this paper was compared with the ones obtained by Li et al. (2015) as presented on Table 1.They worked on "Symplectic Superposition Method for Benchmark Flexure Solutions forRectangular Thick Plates". The maximum recorded percentage difference obtained is 5.11%. This indicates that at 94% confidence level, the values from the present study shall not differ from the values obtained by Li et al. (2015). The difference should be a result of using different displacement functions. Whereas they used trigonometric displacement functions, the present paper used polynomial displacement function. Hence, the difference should not be attributed to the approach to analysis. Results for other parameters (displacements and stresses) for cccs and ssfs thick plates were presented on Table 2 and Table 3 respectively.

Critical look at Table 2, reveals that for span-to-depth ratio less than 20 the value of vertical shear stress is more than 0.00006when corrected to 5 decimal places. For span-to-depth ratios between 30 and 50 the value of vertical shear stress varies between 0.00001 and 0.00003. For span-to-depth ratios more than 50 the value of vertical shear stress less than 0.00001. This same trend is also evident from Table 3. It can seen from that table that for span-to-depth ratios more than 50, the value of vertical shear stress is less than 0.0000005. When the span-to-depth ratio is between 20 and 50, the vertical shear stress is more than 0.0000005 but less than 0.0000014. However, vertical shear stress is more than 0.0000014 for span-to-depth ratios less than or equal to 20. The observations made could infer that there are three classes of rectangular plate. The plates whose vertical shear stress do not differ much from zero shall be classified as thin plates. The ones that differ very well from zero shall be classified as thick plates. In between the thick plate and thin plate is the class for moderately thick plate. Thus, the span-to-depth ratios for these classes of rectangular plate are: Thick plate - a/t ≤ 20; moderately thick plate - 20 ≤ a/t ≤ 50; thin plate - a/t ≥ 50

**Table 1:** Comparison of values of Non dimensional center deflectionmultiplied by 100 of cccssquare rectangular thick plate obtained herein with those from Li et al. (2015).

| a/t | present | Li et al. (2015) | % Diff |
|---|---|---|---|
| 5 | 0.2434 | 0.2565 | 5.11 |
| 10 | 0.1816 | 0.1833 | 0.93 |
| 20 | 0.166 | 0.166 | 0 |

**Table 2:**Non dimensional displacements and stresses of cccssquare thick plate

| a/t | $\overline{w}$ | $\bar{u}$ | $\bar{v}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ |
|---|---|---|---|---|---|---|---|
| 2 | 0.006507 | -0.003774 | -0.003347 | 0.022280 | 0.022545 | -0.010241 | 0.007302 |
| 2.5 | 0.004803 | -0.003311 | -0.002968 | 0.019596 | 0.019948 | -0.009027 | 0.004601 |
| 3 | 0.003853 | -0.003024 | -0.002777 | 0.018003 | 0.018562 | -0.008333 | 0.003139 |
| 3.333 | 0.003437 | -0.002888 | -0.002700 | 0.017269 | 0.017978 | -0.008022 | 0.002514 |
| 4 | 0.002889 | -0.002695 | -0.002609 | 0.016252 | 0.017239 | -0.007604 | 0.001712 |
| 5 | 0.002434 | -0.002518 | -0.002544 | 0.015350 | 0.016663 | -0.007246 | 0.001072 |
| 6 | 0.002184 | -0.002412 | -0.002514 | 0.014825 | 0.016365 | -0.007044 | 0.000733 |
| 7 | 0.002032 | -0.002344 | -0.002499 | 0.014493 | 0.016192 | -0.006918 | 0.000533 |
| 8 | 0.001933 | -0.002298 | -0.002490 | 0.014271 | 0.016083 | -0.006836 | 0.000405 |
| 9 | 0.001865 | -0.002265 | -0.002484 | 0.014115 | 0.016010 | -0.006778 | 0.000318 |
| 10 | 0.001816 | -0.002241 | -0.002481 | 0.014002 | 0.015958 | -0.006737 | 0.000257 |
| 11 | 0.001780 | -0.002223 | -0.002478 | 0.013917 | 0.015921 | -0.006706 | 0.000211 |
| 12 | 0.001753 | -0.002209 | -0.002476 | 0.013851 | 0.015892 | -0.006682 | 0.000177 |
| 13 | 0.001731 | -0.002198 | -0.002475 | 0.013800 | 0.015871 | -0.006664 | 0.000151 |
| 14 | 0.001714 | -0.002190 | -0.002474 | 0.013759 | 0.015853 | -0.006649 | 0.000130 |
| 15 | 0.001700 | -0.002183 | -0.002473 | 0.013726 | 0.015840 | -0.006637 | 0.000113 |
| 16 | 0.001689 | -0.002177 | -0.002473 | 0.013699 | 0.015828 | -0.006627 | 0.000099 |
| 17 | 0.001680 | -0.002172 | -0.002472 | 0.013676 | 0.015819 | -0.006619 | 0.000088 |
| 18 | 0.001672 | -0.002168 | -0.002472 | 0.013657 | 0.015811 | -0.006612 | 0.000078 |
| 19 | 0.001665 | -0.002165 | -0.002471 | 0.013641 | 0.015805 | -0.006607 | 0.000070 |
| 20 | 0.001660 | -0.002162 | -0.002471 | 0.013627 | 0.015799 | -0.006602 | 0.000063 |
| 30 | 0.001630 | -0.002146 | -0.002469 | 0.013556 | 0.015771 | -0.006576 | 0.000028 |
| 40 | 0.001620 | -0.002141 | -0.002469 | 0.013531 | 0.015761 | -0.006567 | 0.000016 |
| 50 | 0.001615 | -0.002139 | -0.002469 | 0.013519 | 0.015756 | -0.006563 | 0.000010 |
| 60 | 0.001613 | -0.002137 | -0.002469 | 0.013513 | 0.015754 | -0.006561 | 0.000007 |
| 70 | 0.001611 | -0.002136 | -0.002468 | 0.013509 | 0.015752 | -0.006560 | 0.000005 |
| 80 | 0.001610 | -0.002136 | -0.002468 | 0.013507 | 0.015751 | -0.006559 | 0.000004 |
| 90 | 0.001610 | -0.002135 | -0.002468 | 0.013505 | 0.015751 | -0.006558 | 0.000003 |
| 100 | 0.001609 | -0.002135 | -0.002468 | 0.013504 | 0.015750 | -0.006558 | 0.000003 |

$Legend: \overline{w} = \overline{w}(R = 0.5, Q = 0.5, S = 0.5);\ \bar{u} = \bar{u}(R = 0.2, Q = 0.5, S = 0.5)$
$\bar{v} = \bar{v}(R = 0.5, Q = 0.2, S = 0.5);\ \overline{\sigma_x} = \overline{\sigma_x}(R = 0.5, Q = 0.5, S = 0.5)$
$\overline{\sigma_y} = \overline{\sigma_y}(R = 0.5, Q = 0.5, S = 0.5);\ \overline{\tau_{xy}} = \overline{\tau_{xy}}(R = 0.2, Q = 0.2, S = 0.5)$
$\overline{\tau_{xz}} = \overline{\tau_{xz}}(R = 0.2, Q = 0.5, S = 0);\ *\overline{\tau_{xz}} = \overline{\tau_{xz}}(Present) - \overline{\tau_{xz}} at(a/t = 100)$

**Table 3:** Non dimensional displacements and stresses of ssfssquare thick plate

| a/t | $\overline{w}$ | $\overline{u}$ | $\overline{v}$ | $\overline{\sigma_x}$ | $\overline{\sigma_y}$ | $\overline{\tau_{xy}}$ | $\overline{\tau_{xz}}$ |
|---|---|---|---|---|---|---|---|
| 2 | 0.000387 | -0.000313 | -0.000374 | 0.001999 | 0.002370 | -0.000977 | 0.0003014 |
| 2.5 | 0.000323 | -0.000300 | -0.000354 | 0.001907 | 0.002252 | -0.000931 | 0.0001938 |
| 3 | 0.000288 | -0.000292 | -0.000344 | 0.001857 | 0.002186 | -0.000905 | 0.0001349 |
| 3.333 | 0.000273 | -0.000289 | -0.000339 | 0.001835 | 0.002157 | -0.000894 | 0.0001094 |
| 4 | 0.000253 | -0.000285 | -0.000333 | 0.001807 | 0.002119 | -0.000879 | 0.0000761 |
| 5 | 0.000236 | -0.000281 | -0.000328 | 0.001784 | 0.002088 | -0.000867 | 0.0000487 |
| 6 | 0.000228 | -0.000280 | -0.000325 | 0.001771 | 0.002071 | -0.000861 | 0.0000339 |
| 7 | 0.000222 | -0.000278 | -0.000323 | 0.001763 | 0.002061 | -0.000857 | 0.0000249 |
| 8 | 0.000219 | -0.000278 | -0.000322 | 0.001758 | 0.002054 | -0.000854 | 0.0000191 |
| 9 | 0.000216 | -0.000277 | -0.000321 | 0.001755 | 0.002050 | -0.000853 | 0.0000151 |
| 10 | 0.000215 | -0.000277 | -0.000321 | 0.001752 | 0.002046 | -0.000851 | 0.0000122 |
| 11 | 0.000213 | -0.000277 | -0.000320 | 0.001750 | 0.002044 | -0.000850 | 0.0000101 |
| 12 | 0.000212 | -0.000276 | -0.000320 | 0.001749 | 0.002042 | -0.000850 | 0.0000085 |
| 13 | 0.000212 | -0.000276 | -0.000320 | 0.001748 | 0.002040 | -0.000849 | 0.0000072 |
| 14 | 0.000211 | -0.000276 | -0.000320 | 0.001747 | 0.002039 | -0.000849 | 0.0000062 |
| 15 | 0.000211 | -0.000276 | -0.000320 | 0.001746 | 0.002038 | -0.000848 | 0.0000054 |
| 16 | 0.000210 | -0.000276 | -0.000319 | 0.001746 | 0.002038 | -0.000848 | 0.0000048 |
| 17 | 0.000210 | -0.000276 | -0.000319 | 0.001745 | 0.002037 | -0.000848 | 0.0000042 |
| 18 | 0.000210 | -0.000276 | -0.000319 | 0.001745 | 0.002036 | -0.000848 | 0.0000038 |
| 19 | 0.000209 | -0.000276 | -0.000319 | 0.001745 | 0.002036 | -0.000847 | 0.0000034 |
| 20 | 0.000209 | -0.000276 | -0.000319 | 0.001744 | 0.002036 | -0.000847 | 0.0000031 |
| 30 | 0.000208 | -0.000276 | -0.000319 | 0.001743 | 0.002034 | -0.000847 | 0.0000014 |
| 40 | 0.000208 | -0.000275 | -0.000319 | 0.001742 | 0.002033 | -0.000846 | 0.0000008 |
| 50 | 0.000208 | -0.000275 | -0.000319 | 0.001742 | 0.002033 | -0.000846 | 0.0000005 |
| 60 | 0.000208 | -0.000275 | -0.000319 | 0.001742 | 0.002032 | -0.000846 | 0.0000003 |
| 70 | 0.000208 | -0.000275 | -0.000319 | 0.001742 | 0.002032 | -0.000846 | 0.0000002 |
| 80 | 0.000207 | -0.000275 | -0.000319 | 0.001742 | 0.002032 | -0.000846 | 0.0000002 |
| 90 | 0.000207 | -0.000275 | -0.000319 | 0.001742 | 0.002032 | -0.000846 | 0.0000002 |
| 100 | 0.000207 | -0.000275 | -0.000319 | 0.001742 | 0.002032 | -0.000846 | 0.0000001 |

$Legend: \overline{w} = \ \overline{w}(R = 0.5, Q = 0.5, S = 0.5);\ \ \overline{u} = \ \overline{u}(R = 0.2, Q = 0.5, S = 0.5)$
$\overline{v} = \overline{v}(R = 0.5, Q = 0.2, S = 0.5);\ \overline{\sigma_x} = \overline{\sigma_x}(R = 0.5, Q = 0.5, S = 0.5)$
$\overline{\sigma_y} = \ \overline{\sigma_y}(R = 0.5, Q = 0.5, S = 0.5);\ \overline{\tau_{xy}} = \overline{\tau_{xy}}(R = 0.2, Q = 0.2, S = 0.5)$
$\overline{\tau_{xz}} = \ \overline{\tau_{xz}}(R = 0.2, Q = 0.5, S = 0);\ *\overline{\tau_{xz}} = \overline{\tau_{xz}}(Present) - \overline{\tau_{xz}}at(a/t = 100)$

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