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Forced Vibration Analysis of Low Suspended Reinforced Concrete Highway Viaducts Pavement using split deflection method

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Abstract : *This paper presents forced vibration analysis of low suspended reinforced concrete highway viaducts pavement using split deflection method. The split-deflection approach was used in formulating the total potential energy functional for a suspended thin rectangular reinforced concrete Pavementsubjected forced vibration. This functional was subjected to both general and direct variations by minimizing it with respect to deflection function and coefficient of deflection respectively. General variation gave the governing equation from where the expression for deflection was obtained. In the same way, direct variation gave the formula for determining the coefficient of deflection of the pavement. The obtained deflection function is of polynomial family. The boundary condition for pavement with four sides simply supported was satisfied in the deflection equation to obtain the exact deflection equation for the pavement. This function was used in numerical forced vibration analysis of the pavement. Theresults show that the maximum percentage differences recorded for pure bending analysis of ssss and cscs plates of present study with previous results are 4.86% and 4.88%. It is seen that the pavement becomes dynamic when forced frequency gets up to 30% of the fundamental natural frequency.*

Keywords: *Forced vibration, split-deflection, total potential energy functional, direct variation, general variation, trigonometric family, concrete pavement, pavement*

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I. Introduction

Most of the earlier scholars based their Classical plate theory (CPT) vibration analysis on the Naviar's and Levy's approaches ([1], [2], [3]). These methods present difficulty in satisfying the boundary conditions of plates. This difficulty led many scholars to resort to energy methods such as Raleigh, Raleigh-Ritz, minimum potential energy, Galerkin, work-error etc. ([1], [3], [4]). Use of single orthogonal deflection functionscharacterizes analyze by energy methods. Hence, the reason why most scholars use assumed deflection functions as against integrating the governing equation. However, one can still integrate the governing equation while using the energy approach. The major difficulty encountered is to separate single orthogonal deflection function into two distinct and independent components. Based on the literature reviewed, most earlierstudies on forced vibration analyses of thin plates used this single orthogonal deflection function ([5], [6], [7], [8], [3], [9], [10], [11], [12]). This problem was addressed by Reference [13]when they introduced the use of two independent and distinct functions to replace the single orthogonal function ($w = w_x \times w_y$). This approached worked. However, their work was based on assumed deflection function. Besides energy approach, another popular approach is the numerical approach. This include finite element, finite strip, Runge-Kuta, finite difference etc. ([14], [15]). To overcome the gaps presented herein, this present study tried to use split deflection approach and integrate the governing equation to arrive at exact non-assumed deflection function. In doing this, the present work relied on the split-deflection equation given by Reference [13]as:

$$w = w_x . w_y \quad \text{1a}$$

$$w = A_x h_x . A_y h_y = A h_x . h_y \quad \text{1b}$$

w_x and w_y are the two distinct and independent components of deflection along x and y directions respectively. Their respective shape functions are h_x and $h_y$and A is deflection coefficient

II. Theoretical approach

2.1Total potential energy

The strain energy, U is defined using the stresses and strains as:

$$U = \frac{1}{2}\int_0^a \int_0^b \int_{-t/2}^{t/2} (\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy})\, dxdy\, dz \qquad 2$$

The normal stresses along x and y directions are denoted with σ_{xx} and σ_{yy}, while the normal strains along x and y directions are ε_{xx} and ε_{xx}. The shear stress and strain within the x-y plane are respectively denoted as τ_{xy} and γ_{xy}. The strains, which are the ratios of displacements to original lengths, are defined as:

$$\varepsilon_{xx} = \frac{du}{dx} = -Z\frac{d^2w}{dx^2} \qquad 3$$

$$\varepsilon_{yy} = \frac{dv}{dy} = -Z\frac{d^2w}{dy^2} \qquad 4$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} - 2Z\frac{d^2w}{dxdy} \qquad 5$$

From constitutive relations, the stresses are defined as:

$$\sigma_{xx} = \frac{E}{1-\mu^2}\left[\varepsilon_{xx} + \mu\varepsilon_{yy}\right] = \frac{-EZ}{1-\mu}\left[\frac{d^2w}{dx^2} + \mu\frac{d^2w}{dy^2}\right] \qquad 6$$

$$\sigma_{yy} = \frac{E}{1-\mu^2}\left[\mu\varepsilon_{xx} + \varepsilon_{yy}\right] = \frac{-EZ}{1-\mu}\left[\mu\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2}\right] \qquad 7$$

$$\tau_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xy} = \frac{-EZ(1-\mu)}{(1-\mu^2)} \cdot \frac{d^2w}{dxdy} \qquad 8$$

The Young's modulus of elasticity and the Poisson's ratio of the plateare denoted with E and μ respectively. The work on the plate due to the applied lateral and inertia loads is given as:

$$V = \frac{1}{2}\int_x \int_y \left[qw + m\theta^2 w^2\right]\, dxdy \qquad 9$$

Adding equations 2 and 9 algebraically gives the total potential energy functional as:

$$\Pi = \frac{1}{2}\int_0^a \int_0^b \int_{-t/2}^{t/2} (\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy})\, dxdy\, dz - \frac{1}{2}\int_x \int_y \left[qw + m\theta^2 w^2\right]\, dxdy \qquad 10$$

Substituting Equations 1, 3, 4, 5, 6, 7 and 8 into Equation 10and carrying out the integration with respect to z coordinate gives:

$$\Pi = \frac{D}{2}\int_x \int_y \left[\left(\frac{d^2w_x}{dx^2}\right)^2 {w_y}^2 + 2\left(\frac{dw_x}{dx}.\frac{dw_y}{dy}\right)^2 + \left(\frac{d^2w_y}{dy^2}\right)^2 {w_x}^2 - \frac{q}{D}w_x w_y - \frac{m\theta^2}{D}\left(w_x w_y\right)^2\right] dxdy \qquad 11$$

In a similarly manner, substituting Equations 2, 3, 4, 5, 6, 7 and 8 into Equation 10 and carrying out the integration with respect to z coordinate gives:

$$\Pi = \frac{A^2 D}{2}\int_x \int_y \left[\left(\frac{d^2h_x}{dx^2}\right)^2 {h_y}^2 + 2\left(\frac{dh_x}{dx}.\frac{dh_y}{dy}\right)^2 + \left(\frac{d^2h_y}{dy^2}\right)^2 {h_x}^2 - \frac{q}{AD}h_x h_y - \frac{m\theta^2}{D}\left(h_x h_y\right)^2\right] dxdy \qquad 12$$

The flexural rigidity, D of the plate defined as:

$$D = \frac{Et^3}{12(1-\mu^2)} \qquad 13$$

Where t is the thickness of plate.

It is more expedient representing the coordinates x, y and z with their non-dimensional forms R, Q and S respectively. R is ratio of x to a (that is R = x/a) while Q is the ratio of y to b (that is Q = y/b). Equations 11 and 12 were written in terms of the non dimensional coordinates as:

$$\Pi = \frac{D}{2a^4}\int_0^1\int_0^1\left[\left(\frac{d^2w_x}{dR^2}\right)^2 {w_y}^2 + \frac{2}{p^2}\left(\frac{dw_x}{dR}.\frac{dw_y}{dQ}\right)^2 + \frac{1}{p^4}\left(\frac{d^2w_y}{dQ^2}\right)^2 {w_x}^2 - \frac{2qa^4}{D}w_x w_y - \frac{m\theta^2a^4}{D}{w_x}^2{w_y}^2\right]abdRdQ \qquad 14$$

$$\Pi = \frac{A^2D}{2a^4}\int_0^1\int_0^1\left[\left(\frac{d^2h_x}{dR^2}\right)^2 {h_y}^2 + \frac{2}{p^2}\left(\frac{dh_x}{dR}.\frac{dh_y}{dQ}\right)^2 + \frac{1}{p^4}\left(\frac{d^2h_y}{dQ^2}\right)^2 {h_x}^2 - \frac{2qa^4}{AD}h_x h_y - \frac{m\theta^2a^4}{D}{h_x}^2{h_y}^2\right]ab\,dRdQ \qquad 15$$

The aspect ratio, p is defined as the ratio of b to a (p = b/a).

2.2 Determination of the split deflection functions

Since the total potential energy functional is in terms of the distinct split deflection, w_x and w_y, it will be wise to rearrange it. Thus, rearranging equation 14 gives:

$$\Pi = \frac{D}{2a^4}\int_0^1\int_0^1\left[\left\{\left(\frac{d^2w_x}{dR^2}\right)^2 {w_y}^2 - \frac{2qa^4}{D}w_x w_y - \frac{m\theta^2a^4}{D}{w_x}^2{w_y}^2\right\} + \left\{\frac{1}{p^4}\left(\frac{d^2w_y}{dQ^2}\right)^2 {w_x}^2 + \frac{2}{p^2}\left(\frac{dw_x}{dR}.\frac{dw_y}{dQ}\right)^2\right\}\right]abdRdQ \qquad 16$$

Where:

$$n_x + n_y = 1 \qquad 17$$

In a simpler form, equation 16 is written as:

$$\Pi = \Pi_x + \Pi_y \qquad 18$$

Where:

$$\Pi_x = \frac{D}{2a^4}\int_0^1\int_0^1\left[\left(\frac{d^2w_x}{dR^2}\right)^2 {w_y}^2 - \frac{2qa^4}{D}w_x w_y - \frac{m\theta^2a^4}{D}{w_x}^2{w_y}^2\right]abdRdQ \qquad 19$$

$$\Pi_y = \frac{D}{2a^4}\int_0^1\int_0^1\left[\frac{1}{p^4}\left(\frac{d^2w_y}{dQ^2}\right)^2 {w_x}^2 + \frac{2}{p^2}\left(\frac{dw_x}{dR}.\frac{dw_y}{dQ}\right)^2\right]abdRdQ \qquad 20$$

Thus, minimizing Equation 19 with respect to w_x gives:

$$\frac{d\Pi_x}{dw_x} = \frac{D}{2a^4}\int_0^1\int_0^1\left[2\frac{d^4w_x}{dR^4}{w_y}^2 - \frac{2qa^4}{D}w_y - 2\frac{m\theta^2a^4}{D}w_x{w_y}^2\right]abdRdQ = 0$$

That is:

$$\int_0^1\frac{d^4w_x}{dR^4}dR\,.\int_0^1{w_y}^2\,dQ - \int_0^1\frac{qa^4}{D}dR\,.\int_0^1 w_y\,dQ - \int_0^1\frac{m\theta^2a^4}{D}w_x\,dR\,.\int_0^1{w_y}^2\,dQ = 0 \qquad 21$$

In a similarly way, minimizing Equation 20 with respect to w_y gives:

$$\frac{d\Pi_y}{dw_y} = \frac{D}{2a^4}\int_0^1\int_0^1\left[\frac{2}{p^4}\frac{d^4w_y}{dQ^4}{w_x}^2 + \frac{4}{p^2}.\frac{d^2w_y}{dQ^2}\left(\frac{dw_x}{dR}\right)^2\right]abdRdQ = 0.$$ That is:

$$\int_0^1{w_x}^2dR\,.\int_0^1\frac{d^4w_y}{dQ^4}dQ + 2p^2.\int_0^1\left(\frac{dw_x}{dR}\right)^2dR\,.\int_0^1\frac{d^2w_y}{dQ^2}dQ = 0 \qquad 22$$

Carrying out the integration of Equation 21 with respect to Q and rearranging the outcome gives:

$$\int_0^1\left[\frac{d^4w_x}{dR^4} - \frac{w_3}{w_4}\frac{qa^4}{D} - \frac{m\theta^2a^4}{D}w_x\right]dR = 0 \qquad 23$$

Where w_3 and w_4 are constants defined mathematically as:

$$w_3 = \int_0^1 w_y \, dQ; \; w_4 = \int_0^1 {w_y}^2 \, dQ$$

Carrying out the integration of Equation 22 with respect to R and rearranging the outcome gives:

$$\int_0^1 \left[\frac{d^4 w_y}{dQ^4} + \frac{2p^2 w_2}{w_1}\frac{d^2 w_y}{dQ^2}\right] dQ = 0 \qquad 24$$

Where w_1 and w_2 are constants defined mathematically as:

$$w_1 = \int_0^1 {w_x}^2 \, dR; \; w_2 = \int_0^1 \left(\frac{dw_x}{dR}\right)^2 dR$$

For the case of pure bending (that is in the absence of inertia force), equations23 become:

$$\int_0^1 \left[\frac{d^4 w_x}{dR^4} - \frac{w_3}{w_4}\frac{qa^4}{D}\right] dR \; = 0 \qquad 25$$

In the same way, for the case of free vibration (that is in the presence of only the inertia force), equations 23 become:

$$\int_0^1 \left[\frac{d^4 w_x}{dR^4} - \frac{m\theta^2 a^4}{D} w_x\right] dR \; = 0 \qquad 26$$

The ready solutions for the equations 25 and 26 for pure bending and free vibration respectively are:

$$w_x = a_0 + a_1 R + a_2 R^2 + a_3 R^3 \; + a_4 R^4 \qquad 27$$

$$w_x = c_1 e^{g_1 R} + c_2 e^{-g_1 R} + c_3 e^{jg_1 R} + c_4 e^{-jg_1 R} \qquad 28$$

Where d_0, d_1, d_2, d_3, d_4, c_1, c_2, $c_3$and c_4 are integration constants, and

$${g_1}^4 = \frac{m\theta^2 a^4}{D} \qquad 29$$

Transforming Equation 28 in trigonometric form gives:

$$w = a_1 \cos g_1 R + a_2 \sin g_1 R + a_3 \cosh g_1 R + \; a_4 \sinh g_1 R \qquad 30$$

Where: $a_1 = [c_3 + c_4]; \; a_2 = [jc_3 - jc_4]; \; a_3 = [c_1 + c_2]; \; a_4 = [c_1 - c_2]$

In similar way, the ready solutions for the equations 24 is

$$w_y = d_0 + d_1 Q + d_2 e^{jg_2 Q} + d_3 e^{-jg_2 Q} \qquad 31$$

Where d_0, d_1, d_2, $d_3$and $d_4$are integration constants, and

$${g_2}^2 = \frac{2p^2 w_2}{w_1} \qquad 32$$

Transforming Equation 32 in Polynomial form gives:

$$w_y = b_0 + b_1 Q + b_2 Q^2 + \frac{b_3}{3!} Q^3 + \frac{b_4}{4!} Q^4 + . \; . \; . \qquad 33$$

Where: $b_0 = c_1 + c_2 + c_3 + c_4; \; b_1 = jc_1 - jc_2 + c_3 - c_4;$

$$\frac{b_m}{m!} = {g_2}^m [j^m c_1 - j^m c_2 + c_3 - c_4]; \; \frac{b_{m+1}}{[m+1]!} = {g_2}^{m+1} [j^{m+1} c_1 + j^{m+1} c_2 + c_3 + c_4]$$

$$m = 1, 3, 5, \; . \; . \; . \; , \; \infty$$

Substituting Equation 27 and Equation 33 into Equation 1 gives:

$$w = (a_0 + a_1 R + a_2 R^2 + a_3 R^3 \; + a_4 R^4).(b_0 + b_1 Q + a_2 Q^2 + a_3 R^3 \; + a_4 Q^4) \qquad 34$$

2.3 Determination of the formula for calculating coefficient of deflection for pavement under forced vibration

Formula for calculating coefficient of deflection is obtained when the total potential energy functionalis minimized with respect the coefficient of deflection. After minimizing Equation 15 with respect to deflection coefficient the following was obtained:

$$\frac{d\Pi}{dA}=\frac{AD}{a^4}\int_0^1\int_0^1\left[\left(\frac{d^2h_x}{dR^2}\right)^2 {h_y}^2+\frac{2}{p^2}\left(\frac{dh_x}{dR}.\frac{dh_y}{dQ}\right)^2+\frac{1}{p^4}\left(\frac{d^2h_y}{dQ^2}\right)^2 {h_x}^2-\frac{qa^4}{AD}h_xh_y-\frac{m\theta^2a^4}{D}{h_x}^2{h_y}^2\right]abdRdQ=0 \qquad 35$$

Rearranging and rewriting equation 35 gives:

$$\frac{d\Pi}{dA}=[k_{xR}.k_{xQ}]+\frac{2}{p^2}[k_{xyR}.k_{xyQ}]+\frac{1}{p^4}[k_{yR}.k_{yQ}]-\frac{qa^4}{AD}[k_{qR}.k_{qQ}]-\frac{m\theta^2a^4}{D}[k_{yR}.k_{xQ}]=0 \qquad 36$$

Where:

$$k_{xR}=\int_0^1\left(\frac{d^2h_x}{dR^2}\right)^2 dR;\ k_{xQ}=\int_0^1 {h_y}^2\,dQ;\ k_{xyR}=\int_0^1\left(\frac{dh_x}{dR}\right)^2 dR;\ k_{qR}=\int_0^1 h_x dR$$

$$k_{xyQ}=\int_0^1\left(\frac{dh_y}{dQ}\right)^2 dQ;\ k_{yR}=\int_0^1 {h_x}^2\,dR;\ k_{yQ}=\int_0^1\left(\frac{d^2h_y}{dQ^2}\right)^2 dQ;\ k_{qQ}=\int_0^1 h_y\,dQ$$

Rearranging Equation 36 gives:

$$\frac{AD}{qa^4}=\frac{k_{qR}.k_{qQ}}{k_T-\frac{m\theta^2a^4}{D}k_{yR}.k_{xQ}} \qquad 37$$

Where:

$$k_T=k_{xR}.k_{xQ}+\frac{2}{p^2}k_{xyR}.k_{xyQ}+\frac{1}{p^4}k_{yR}.k_{yQ}$$

Under free – vibration only, the denominator of Equation 37 shall be zero and the vibration frequency shall become the natural frequency, λ. This gives:

$$k_T-\frac{m\lambda^2a^4}{D}k_{yR}.k_{xQ}=0 \qquad 38$$

Upon rearrangement of equation 38 the following equation is obtained:

$$\frac{m\lambda^2a^4}{D}=\frac{k_T}{k_{yR}.k_{xQ}} \qquad 39$$

The forced frequency of the pavement ordinarily ranges from zero to a maximum value of the natural frequency. Thus:

$$0\le(\theta=n.\lambda)\le\lambda \qquad 40$$

Substituting the condition given in equation 40 into equation 39 gives:

$$\frac{m\theta^2a^4}{D}=n^2.\frac{k_T}{k_{yR}.k_{xQ}} \qquad 41$$

By substituting equation 41 into equation 37 the following equation is obtained:

$$\frac{AD}{qa^4}=\frac{k_{qR}.k_{qQ}}{k_T-\left(n^2.\frac{k_T}{k_{yR}.k_{xQ}}\right)k_{yR}.k_{xQ}}.\ \text{That is:}$$

$$\frac{AD}{qa^4}=\left(\frac{k_{qR}.k_{qQ}}{k_T}\right).\frac{1}{[1-n^2]}=\beta \qquad 42$$

The formula for calculating the non-dimensional coefficient of deflection for pavement under forced vibration is as presented on equation 42.

Rearranging equation 42 gives the following equations:

$$\frac{AD}{a^2}=\beta qa^2 \qquad 43$$

$$\frac{AD}{a^3}=\beta qa \qquad 44$$

Substituting the equation 2 into the traditional equations for bending moment and shear forces of rectangular pavements, the following equations are obtained:

$$m_x=-\frac{AD}{a^2}\left(\frac{\partial^2h_x}{\partial R^2}.h_y+\frac{\mu h_x}{p^2}.\frac{\partial^2h_y}{\partial Q^2}\right) \qquad 45$$

$$m_y = -\frac{AD}{a^2}\left(\mu h_y.\frac{\partial^2 h_x}{\partial x^2} + \frac{h_x}{p^2}.\frac{\partial^2 h_y}{\partial y^2}\right) \quad 46$$

$$V_x = -\frac{AD}{a^3}\left(h_y.\frac{\partial^3 h_x}{\partial R^3} + \frac{(2-\mu)}{p^2}.\frac{\partial h_x}{\partial R}.\frac{\partial^3 h_y}{\partial Q^2}\right) \quad 47$$

$$V_y = -\frac{AD}{a^3}\left(\frac{(2-\mu)}{p}.\frac{\partial^2 h_x}{\partial R^2}.\frac{\partial h_y}{\partial Q} + \frac{h_x}{p^3}\frac{\partial^3 h_y}{dQ^3}\right) \quad 48$$

Substituting equations 43 into the equations 45 and 46 gives:

$$m_x = -\beta q a^2\left(\frac{\partial^2 h_x}{\partial R^2}.h_y + \frac{\mu h_x}{p^2}.\frac{\partial^2 h_y}{\partial Q^2}\right) \quad 49$$

$$m_y = -\beta q a^2\left(\mu h_y.\frac{\partial^2 h_x}{\partial x^2} + \frac{h_x}{p^2}.\frac{\partial^2 h_y}{\partial y^2}\right) 50$$

$$V_x = -\beta q a\left(h_y.\frac{\partial^3 h_x}{\partial R^3} + \frac{(2-\mu)}{p^2}.\frac{\partial h_x}{\partial R}.\frac{\partial^3 h_y}{\partial Q^2}\right) \quad 51$$

$$V_y = -\beta q a\left(\frac{(2-\mu)}{p}.\frac{\partial^2 h_x}{\partial R^2}.\frac{\partial h_y}{\partial Q} + \frac{h_x}{p^3}\frac{\partial^3 h_y}{dQ^3}\right) \quad 52$$

2.4 Numerical analyses

Analyze the classical rectangular thin rectangular isotropic pavements(i) with all the four edges simply supported (ssss) and (ii) with two opposite edges clamped and the other two edges simply supported (cscs) as shown on Figure 1. The Poisson's ratio of the plate is 0.3. Points A (R = 0; Q = 1/2); B (R = 1/2; Q = 0); C (R =1; Q=1/2); D (R =1/2; Q=1); E (R =1/2; Q=1/2).

After satisfying the boundary condition for ssss and cscs pavement, the deflection components obtained are respectively:

$$w_x = A_x(R - 2R^3 + R^4) and w_y = A_y(Q - 2Q^3 + Q^4) \quad 53$$

$$w_x = A_x(R - 2R^3 + R^4) and w_y = A_y(Q^2 - 2Q^3 + Q^4) \quad 54$$

From Equation53the shape functionfor ssss pavement is:

$$h_x = R - 2R^3 + R^4 and h_y = Q - 2Q^3 + Q^4 \quad 55 .$$

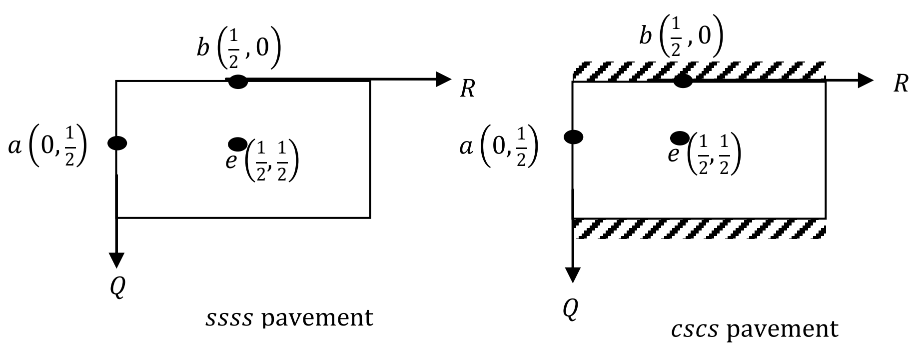

Figure 1: Diagram of rectangular ssss and cscs pavement shown the values of coordinates at various points

From Equation 54the shape functionfor cscs pavement is:
$h_x = R - 2R^3 + R^4$and $h_y = Q^2 - 2Q^3 + Q^4$ 56
Thestiffness coefficients are calculated using the shape functions given in equations 55 and 55. They are tabulated on Table I.

Table .I: Stiffness coefficients for plate with all four edges simply supported

	k_{xR}	k_{xQ}	k_{xyR}	k_{xyQ}	k_{yR}	k_{yQ}
ssss	4.8	$\frac{31}{630}$	$\frac{17}{35}$	$\frac{17}{35}$	$\frac{31}{630}$	4.8
	k_{qR}	k_{qQ}	$k_{xR}.k_{xQ}$	$k_{xyR}.k_{xyQ}$	$k_{yR}.k_{yQ}$	$k_{qR}.k_{qQ}$
	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{124}{525}$	$\frac{222}{941}$	$\frac{124}{525}$	$\frac{1}{25}$
	k_{xR}	k_{xQ}	k_{xyR}	k_{xyQ}	k_{yR}	k_{yQ}
cscs	4.8	$\frac{1}{630}$	$\frac{17}{35}$	$\frac{2}{105}$	$\frac{31}{630}$	$\frac{4}{5}$
	k_{qR}	k_{qQ}	$k_{xR}.k_{xQ}$	$k_{xyR}.k_{xyQ}$	$k_{yR}.k_{yQ}$	$k_{qR}.k_{qQ}$
	$\frac{1}{5}$	$\frac{1}{30}$	$\frac{4}{525}$	$\frac{1}{108}$	$\frac{5}{127}$	$\frac{1}{150}$

For ssss pavement:

$$k_T = \frac{124}{525} + \frac{2}{p^2} \times \frac{222}{941} + \frac{1}{p^4} \times \frac{124}{525} = \frac{124}{525}\left(1 + \frac{1.997703198}{p^2} + \frac{1}{p^4}\right)$$

For cscs pavement:

$$k_T = \frac{4}{525} + \frac{2}{p^2} \times \frac{1}{108} + \frac{1}{p^4} \times \frac{5}{127} = \frac{4}{525}\left(1 + \frac{2.430555556}{p^2} + \frac{5.167323}{p^4}\right)$$

Substituting these stiffness coefficients into equation 42 gives:
For ssss pavement:

$$\frac{AD}{qa^4} = \left(\frac{\frac{1}{25}}{\frac{124}{525}\left(1 + \frac{1.997703198}{p^2} + \frac{1}{p^4}\right)}\right) . \frac{1}{[1 - n^2]} = \beta . \text{ That is:}$$

$$\frac{AD}{qa^4} = \left(\frac{21}{124\left(1 + \frac{1.997703198}{p^2} + \frac{1}{p^4}\right)}\right) . \frac{1}{[1 - n^2]} = \beta \qquad 42a$$

For cscs pavement:

$$\frac{AD}{qa^4} = \left(\frac{\frac{1}{150}}{\frac{4}{525}\left(1 + \frac{2.430555556}{p^2} + \frac{5.167323}{p^4}\right)}\right) . \frac{1}{[1 - n^2]} = \beta \text{ . That is:}$$

$$\frac{AD}{qa^4} = \left(\frac{7}{8\left(1 + \frac{2.430555556}{p^2} + \frac{5.167323}{p^4}\right)}\right) . \frac{1}{[1 - n^2]} = \beta$$

The numerical values of the split deflection functions (equations 55 and 56) and their adjuncts at various points on the plate are presented on Table II.
The result of the center deflection of the plate is compared with those from Reference [16]. Simplepercentage difference is the tool used for this comparison. The formula for percentage difference is:

$$\%Diff = abs\left(\frac{w_p - w_E}{w_E}\right) \times 100 \qquad 57$$

“abs”is absolute value, $w_p$is deflection from present study and $w_E$is the earlier scholar’s deflection.

Table. II: Numerical values of functions and their adjuncts at various points on the plate

Function	ssss pavement			cscs pavement		
	Point A	Point B	Point E	Point A	Point B	Point E
h	0	0	25/256	0	0	5/256
$\frac{d^2h}{dR^2}$	0	0	- 15/16	0	0	-0.1875
$\frac{d^2h}{dQ^2}$	0	0	- 15/16	0	0.625	-0.3125
$\frac{d^3h}{dR^3}$	-3.75	0	0	-0.75	0	0
$\frac{d^3h}{dQ^3}$	0	-3.75	0	0	-3.75	0
$\frac{d^3h}{dRdQ^2}$	-3	0	0	-1	0	0
$\frac{d^3h}{dR^2dQ}$	0	-3	0	0	0	0

III. Results and discussions

The Stiffness coefficients for the ssss and cscs rectangular pavements are presented on Table I. They were obtained using the polynomial displacement functions. The pure bending results of the centre deflections, wc (qa4/D) for ssss and cscs pavements are respectively presented on Table III and Table IV. The dynamic bending results of the centre deflections, wc (qa4/D) for ssss and cscs pavements are respectively presented on Table V and Table VI.Maximum recorded absolute difference between the pure bending center deflection from the present study and those of Reference [16] as shown on Table III and Table IV are respectively 4.86% and 4.88% for ssss and cscs pavements. This difference is as a result difference in methods used by the present study and the one used by Reference [16]. Reference [16] adopts a method close to Navier’s and Levy’s approach and the use of Fourier series as the displacement function. However, the present study used method by Reference [13] and Reference [17] and the first mode of deformation polynomial deflection function. The closeness in the results as indicated by the percentage difference shows the sufficiency of the present analysis approach. Moreover, the use of split deflection methods makes the analysis very easy and straight forward. It is devoid of any complexity as normally evident in earlier works.

The dynamic center deflections of the ssss and cscs pavements for various amounts of ratio of forced frequency to fundamental natural frequency and for various aspect ratio as presented on Table V and Table VI were determined. A close and critical look at the tables shows that vibrating pavement deflects more than static pavement. As the pavement is forced to vibrate at higher frequency, the more the deflection increases. This increase in deflection as the pavement is forced to vibrate is gradual when the value of n is in the range of 0.0 and 0.3. The increase is moderate when the value of n is between 0.4 and 0.6. When the value of n is more than 0.6, the increase becomes so rapid. Hence, it is recommended that the engineer should always provide a pavement with high mass per unit area, which reduces the forced frequency of the pavement. The more the mass per unit area of the pavement, the lower the forced frequency of the pavement, and vise-versa. The engineer should also ensure to design the pavement whenever the forced frequency gets up to 30% of the fundamental natural frequency of the pavement. So far, it is evident that the method applied in this present study is very sufficient and less complicated for forced vibration analysis of pavement.

Table III: Centre deflection of ssss plate

b/a	$w_c$present study	w_cReference [16]	Percentage difference
1	0.00414	0.00406	1.897
1.1	0.00496	0.00485	2.280
1.2	0.00576	0.00576	0.060
1.3	0.00653	0.00638	2.371
1.4	0.00726	0.00705	2.911
1.5	0.00793	0.00772	2.729
1.6	0.00856	0.0083	3.086
1.7	0.00913	0.00883	3.425
1.8	0.00966	0.00931	3.773
1.9	0.01015	0.00974	4.165
2	0.010623	0.01013	4.86

Table IV: Centre deflection of cscs plate

b/a	$w_c$present study	w_cReference [16]	Percentage difference
1	0.00199	0.00192	3.525
1.1	0.00261	0.00251	4.139
1.2	0.00330	0.00319	3.426
1.3	0.00402	0.00388	3.701
1.4	0.00477	0.0046	3.626
1.5	0.00551	0.00531	3.788
1.6	0.00624	0.00603	3.515
1.7	0.00695	0.00668	4.011
1.8	0.00762	0.00732	4.115
1.9	0.00826	0.0079	4.516
2	0.00885	0.00844	4.883

Table V; Center deflection, wc (qa4/D) for ssssplate for various aspect ratios and inertia load,

b/a	n = 0	n = 0.1	n = 0.2	n = 0.3	n = 0.4	n = 0.5	n = 0.6	n = 0.7	n = 0.8	n = 0.9
1	0.00414	0.00418	0.00431	0.00455	0.00493	0.00552	0.00646	0.00811	0.01149	0.02177
1.1	0.00496	0.00501	0.00517	0.00545	0.00591	0.00661	0.00775	0.00973	0.01378	0.02611
1.2	0.00576	0.00582	0.00600	0.00633	0.00686	0.00768	0.00901	0.01130	0.01601	0.03033
1.3	0.00653	0.00660	0.00680	0.00718	0.00778	0.00871	0.01021	0.01281	0.01814	0.03438
1.4	0.00726	0.00733	0.00756	0.00797	0.00864	0.00967	0.01134	0.01423	0.02015	0.03819
1.5	0.00793	0.00801	0.00826	0.00871	0.00944	0.01057	0.01239	0.01555	0.02203	0.04174
1.6	0.00856	0.00864	0.00891	0.00940	0.01019	0.01141	0.01337	0.01678	0.02377	0.04503
1.7	0.00913	0.00922	0.00951	0.01004	0.01087	0.01218	0.01427	0.01791	0.02537	0.04807
1.8	0.00966	0.00976	0.01006	0.01062	0.01150	0.01288	0.01510	0.01894	0.02684	0.05085
1.9	0.01015	0.01025	0.01057	0.01115	0.01208	0.01353	0.01585	0.01989	0.02818	0.05340
2	0.01059	0.01070	0.01103	0.01164	0.01261	0.01412	0.01654	0.02076	0.02941	0.05573

Table VI Center deflection, wc (qa4/D)for cscsplate for various aspect ratios and inertia load

b/a	n = 0	n = 0.1	n = 0.2	n = 0.3	n = 0.4	n = 0.5	n = 0.6	n = 0.7	n = 0.8	n = 0.9
1	0.00199	0.00201	0.00207	0.00218	0.00237	0.00265	0.00311	0.00390	0.00552	0.01046
1.1	0.00261	0.00264	0.00272	0.00287	0.00311	0.00349	0.00408	0.00513	0.00726	0.01376
1.2	0.00330	0.00333	0.00344	0.00363	0.00393	0.00440	0.00516	0.00647	0.00916	0.01736
1.3	0.00402	0.00406	0.00419	0.00442	0.00479	0.00536	0.00629	0.00789	0.01118	0.02118
1.4	0.00477	0.00481	0.00497	0.00524	0.00567	0.00636	0.00745	0.00935	0.01324	0.02509
1.5	0.00551	0.00557	0.00574	0.00606	0.00656	0.00735	0.00861	0.01081	0.01531	0.02901
1.6	0.00624	0.00630	0.00650	0.00686	0.00743	0.00832	0.00975	0.01224	0.01734	0.03285
1.7	0.00695	0.00702	0.00724	0.00764	0.00827	0.00926	0.01086	0.01362	0.01930	0.03657
1.8	0.00762	0.00770	0.00794	0.00837	0.00907	0.01016	0.01191	0.01494	0.02117	0.04011
1.9	0.00826	0.00834	0.00860	0.00907	0.00983	0.01101	0.01290	0.01619	0.02294	0.04346
2	0.00885	0.00894	0.00922	0.00973	0.01054	0.01180	0.01383	0.01736	0.02459	0.04659

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