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A Strength Evaluation of Dam Concrete by Using Equivalent Stress Method

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Abstract: *The zone nearby the dam's heel is the most important part in strength evaluation of dam concrete. However, there occurs stress concentration phenomena in the zone nearby the dam's heel, where the strength of dam concrete cannot be evaluated by using the material mechanics method or finite element method. The paper proposes a method for evaluating the strength of dam concrete according to various strength criteria by using equivalent stress method and demonstrates its feasibility through the numerical experiments on the practical concrete gravity dam and concrete arch dam.*

Keywords: *concrete strength, dam heel, equivalent stress method, strength criterion, William-Warnke criterion*

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I. Introduction

The zone nearby the dam heel is the most important part in the design, analysis, construction and operation of the dam. The problem of evaluating the accurate stress state here is the focus of ensuring the safety of the dam.

The stress in the zone nearby the dam's heel and toe, which are the stress concentration points of the dam, can not be determined by using the material mechanics method or the finite element method (FEM). The reason is that the material mechanics method can not reflect the influence of the foundation, and the finite element method cannot overcome the obstacles that the stress values in the zones nearby the heel and toe increase infinitely as the mesh size decreases [1-2]. To overcome these obstacles, numerous equivalent stress methods (ESM) have been proposed to convert the finite element stress to equivalent stress satisfying the mechanical conditions on the boundary between the dam and foundation.

Ref. [3-6] proposed a linear global ESM by which FEM stresses on global boundary are synthesized with the section interior force and converted into corresponding linear equivalent stresses. Ref. [7-8] suggested a nonlinear global ESM, considering actual stress curve is nonlinear. Ref. [9] introduced the linear local ESM of vertical normal stress by which linear equivalent treatment is applied using the moment equivalent condition in the local area of the vicinity of dam's heel and toe.

Ref. [10], the last paper in the field of ESM, proposed a nonlinear local ESM to estimate the stress state by setting the approximate function as a three order polynomial so that the equivalent stress in the local area satisfies four mechanical conditions of moment equivalent condition, the continuity condition and the gradient continuity condition of stress curve, and the force equivalent condition.

If these ESMs are applied to the finite element analysis, more accurate solutions of dam structure analysis can be achieved.

The analysis of concrete dam by FEM includes statistical analysis [11], slide stability analysis [12-13], concrete strength analysis [14-15], optimal design of dam structure [16], seismic response analysis [17-19], construction process analysis [16], temperature field analysis [16], damage analysis [20] and crack analysis [21], representatively. Most of them are concerned with the strength stability of the dam concrete at the heel. But they are usually focused on the predicting intendancy of the FEM solution according to a referenced element size because the stress values in the zone nearby the heel and toe increase infinitely as the mesh size decreases due to stress concentration.

Therefore, it is clear that the FEM can not accurately evaluate the strength of the dam concrete, and it is necessary to convert the finite element stress to the equivalent stress and apply the concrete strength criterion to the obtained equivalent stress state.

Based on the above research results, the paper proposes a method for evaluating the strength of dam concrete according to various strength criteria by using equivalent stress method and demonstrates its feasibility through the numerical experiments on the practical concrete dams.

II. Comparison of the concrete strength of virtual dam according to various concrete strength criteria by using ESM

The virtual model has the height of 100m, the upper width of 6.5m, the bottom width of 65m and the upstream water depth of 100m, and its uplift pressure is not considered. (The bottom width of model is assumed smaller than the practical dams in order to simulate the failure state of dam heel.)

The elasticity modulus and Poisson's ratio of dam concrete and foundation rock are 20GPa and 0.2. The concrete strength in the zone nearby the dam heel is compared according to various strength criteria.There are several concrete strength criteria used in uniaxial stress state, biaxial stress state and triaxial stress state. The strength criteria of Rankine, Drucker-Prager and William-Warnkke are commonly used in the design and analysis of dam concrete.

1.1 Strength evaluation of dam concrete according to the strength criterion of Rankine

Concrete has tension strength much smaller than compression strength. The Rankine's criterion (1976) means that if the maximum principal stress σ_1 at any point of material is over the tension strength f_t, the failure occurs in the material. That is

$$F_s = \sigma_1 - f_t \qquad (1)$$

, where F_s is the function of strength.

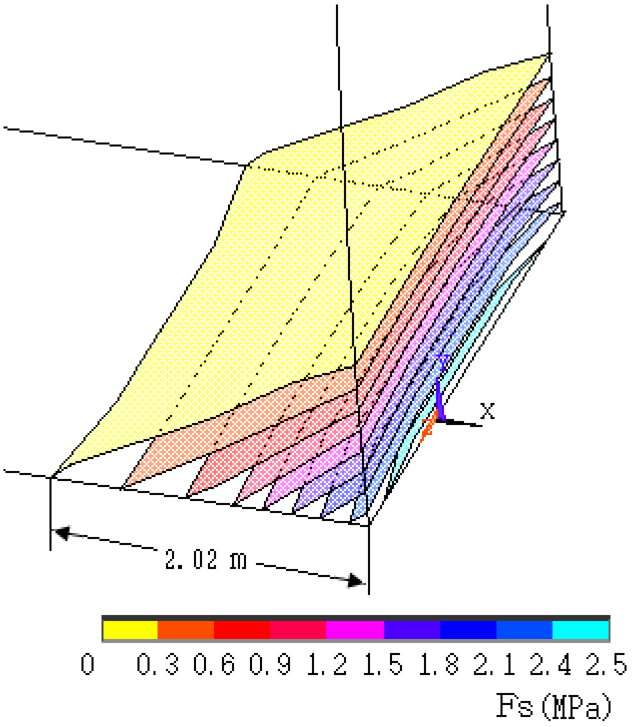

Fig. 1 Hazard district according to the Rankine criterion

Fig. 1 shows the hazard district (the area where the F_s is over 0) in the zone nearby the dam heel, when the tensile strength of concrete is 1.5MPa and the element size is 2m.

As shown in the figure, the hazard length HL (the length of exceeding over the tension strength from the heel) is about 2m by using FEM.

By using the ESM proposed in Ref. [10], the finite element stress in the boundary between dam and foundation is converted to the equivalent stress, and the concrete strength state of these equivalent stress is evaluated according to the strength criterion of Rankine and compared with the result of FEM.

Fig. 2 shows F_s curves according to the Rankine's criterion using FEM and ESM.

As shown in the figure, according to the Rankine's criterion, the hazard lengths from the heel on the boundary are $HL_{FEM} = OB = 2m$ in the case of FEM (element size - 2m) and $HL_{ESM} = OA = 0.5m$ in the case of ESM.

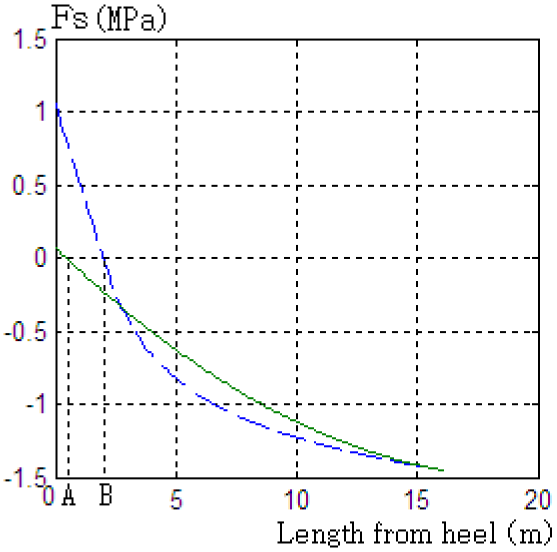

Fig. 2 F_s curves according to Rankine's criterion (- -: FEM, -: ESM)

1.2 Strength evaluation of dam concrete according to the strength criterion of Drucker-Prager

Drucker-Prager's criterion is used to model the elasto-plastic behavior of nonmetallic materials such as concrete, rock and soil [22-23].

In 1952, Drucker and Prager modified the strength criterion of Von Mises to obtain a smooth strength surface closely similar to the Mohr-Coulomb surface.

$$f(I_1, J_2) = \alpha I_1 + \sqrt{J_2} - k = 0 \qquad (2)$$

, where α and k are material constants. When α is 0, the Drucker-Prager's criterion equals to the Von Mises's criterion. There are several ways to obtain the Mohr-Coulomb's hexagonal lattice approximate to the Drucker-Prager cone, and the cone size is controlled by α and k.

The stress of Drucker-Prager is

$$\sigma_e = 3\beta\sigma_m + [\frac{1}{2}\{s\}^T[M]\{s\}]^{\frac{1}{2}} \qquad (3)$$

, where $\sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$ is the mean normal stress, $\{s\}$ is the deviator stress,

$[M] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$, $\beta = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)}$ is material constants and φ is the angle of internal friction.

The flow parameters of the material, σ_y is denoted by cohesion force as below.

$$\sigma_y = \frac{6C\sin\varphi}{\sqrt{3}(3-\sin\varphi)} \qquad (4)$$

Therefore, the strength function is

$$F_s = 3\beta\sigma_m + [\frac{1}{2}\{s\}^T[M]\{s\}]^{\frac{1}{2}} - \sigma_y \quad (5)$$

As can be seen in Eq. (5), the Drucker-Prager's criterion can determine the flow state of the material using two parameters – the angle of internal friction and cohesion.

Fig. 3 shows the hazard district in the zone nearby the dam heel, when the angle of internal friction is 40°, cohesion is 1.2MPa and the element size is 2m.

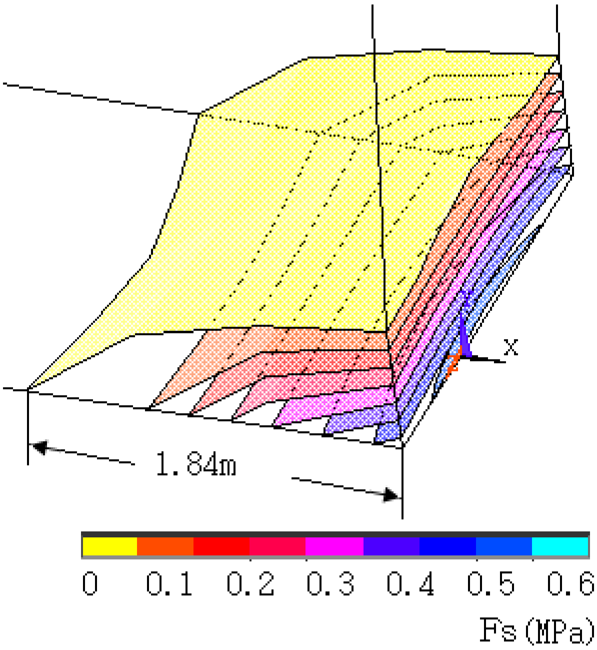

Fig. 3 Hazard district according to the Drucker-Prager criterion

As shown in the figure, the hazard length HL is about 1.84m by using FEM.

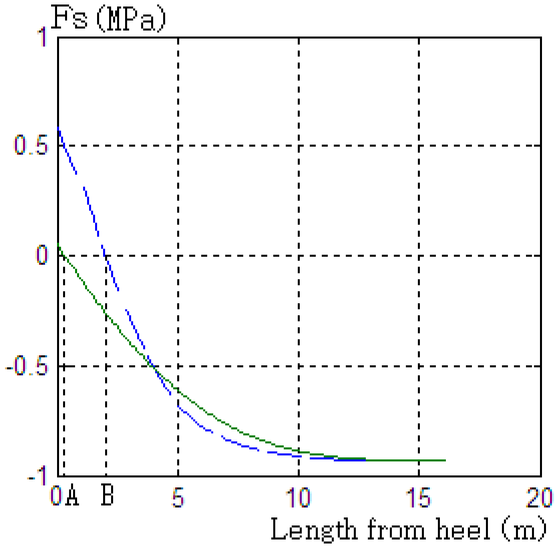

Fig. 4 F_s curves according to Drucker-Prager's criterion (- -: FEM, -: ESM)

Fig. 4 shows F_s curves according to the Drucker-Prager's criterion using FEM and ESM.

As shown in the figure, according to the Drucker-Prager's criterion, the hazard lengths are $HL_{FEM} = OB = 1.84m$ in the case of FEM (element size - 2m) and $HL_{ESM} = OA = 0.33m$ in the case of ESM.

1.3 Strength evaluation of dam concrete according to the strength criterion of William-Warnke

The progress of research on the strength criteria of concrete shows that the failure of concrete in the triaxial stress state depends more than two parameters.

It is impossible to explain the concrete failure surface in the triaxial stress state only with one or two parameters of the strength criteria corresponding to the uniaxial stress state or the biaxial stress state.

Therefore, lots of researchers proposed multi-parameter strength criteria considering several parameters in the triaxial stress state to explain the failure characteristics of concrete [24-27].

There are three kinds of strength criterion of 3 parameters, 4 parameters and 5 parameters in the multi-parameter strength criteria.

The criteria of 5 parameters include William-Warnke's criteria (1975), Kotsovos's criteria (1979) and so on.

The previous strength criteria, such as Rankine, Von Mises, and Drucker-Prager, can be represented as a special case of William-Warnke's criterion.

The failure surface of the SOLID65 element of ANSYS is based on the improved William-Warnke's strength criterion of 5 parameters, which is defined as below [28].

f_t - uniaxial tensile strength, f_c - uniaxial compression strength, f_{bc} - biaxial compression strength, σ_h^a - hydrostatic pressure, f_2 - biaxial compression strength under the σ_h^a and f_1 - uniaxial compression strength under the σ_h^a.

William-Warnke criterion uses the inequality below for evaluating the material strength.

$$\frac{F}{f_c} - S \geq 0 \qquad (6)$$

, where F is the stress function and S is the failure surface.

In order to describe the failure behavior of concrete under different stress conditions, the William-Warnke criterion uses four kinds of failure surface of the concrete as below:

$0 \geq \sigma_1 \geq \sigma_2 \geq \sigma_3 \Rightarrow$ compression-compression-compression

$\sigma_1 \geq 0 \geq \sigma_2 \geq \sigma_3 \Rightarrow$ tension-compression-compression

$\sigma_1 \geq \sigma_2 \geq 0 \geq \sigma_3 \Rightarrow$ tension-tension-compression

$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0 \Rightarrow$ tension- tension- tension

, when the every definition of failure surface and F_s is different from each other [28] (Eq. (7)).

$$F_s = F_i - f_c S, (i = \overline{1,4}) \qquad (7)$$

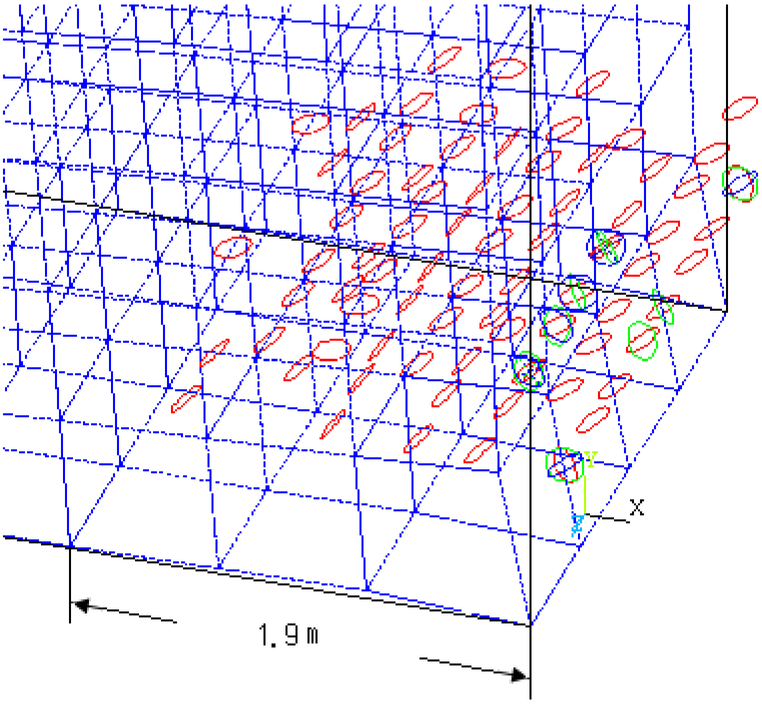

Fig. 5 Hazard district according to the William Warnke criterion

The William-Warnke's criterion can evaluate the F_s and determine the failure state of the material using five parameters as Eq. (8).

$$\begin{cases} f_t = 1.5MPa \\ f_c = 9.0MPa \\ f_{bc} = 10.8MPa \\ f_1 = 13.1MPa \\ f_2 = 15.5MPa \\ \sigma_h^a = 15.6MPa \end{cases} \tag{8}$$

Fig. 5 shows the hazard district in the zone nearby the dam heel, when the 5 parameters equal to Eq. (7) and the element size is 2m.

As shown in the figure, the hazard length HL is about 1.9m by using FEM.

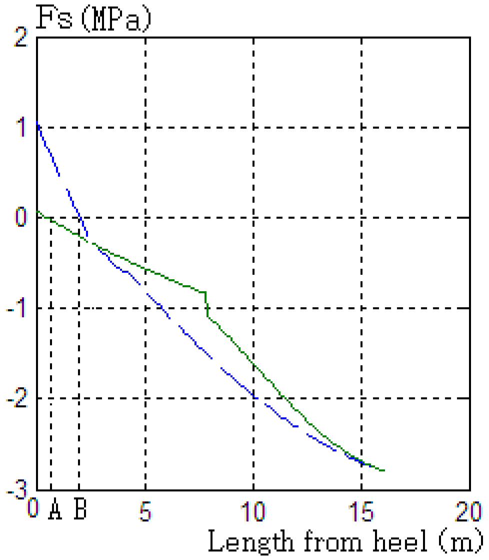

Fig. 6 F_s curves according to William-Warnke's criterion (- -: FEM, -: ESM)

Fig. 6 shows F_s curves according to the William-Warnke's criterion using FEM and ESM. Each stress curve has a inflection point in the middle of the curve at which the stress gradient changes.

As shown in the figure, according to the William-Warnke's criterion, the hazard lengths on the boundary are $HL_{FEM} = OB = 1.9m$ in the case of FEM (element size - 2m) and $HL_{ESM} = OA = 0.4m$ in the case of ESM.

As described above, the hazard length of dam concrete nearby the heel on the boundary depends on the various strength criteria (Table 1).

Table 1. HL according to the calculation method and strength criteria(m)

Calculation Method	Strength Criteria		
	Rankine	Drucker-Prager	William-Warnke
FEM(Element Size: 2m)	2.00	1.84	1.90
ESM	0.50	0.33	0.40

As shown in Figs. 2 and 6, the hazard length and the leading edges before the inflection point on the F_s curves are similar when evaluated according to Rankine criterion and William-Warnke's 5-parameter criterion, because the three principal stresses in this section are all positive, and the strength function Fs of William-Warnke's 5-parameter criterion is the same as one of Rankine's criterion in the tension-tension-tension section.

However, the following edges after the inflection point on the F_s curves are different from those of William-Warnke's 5-parameter criterion and Rankine's criterion, because the stress state is changed from tension-tension-tension state to tension-tension-compression state and the formulas of strength function (F_s) are also changed.

Generally, there often occurs compression principal stress in the zone nearby the heel of dam. In this case, the hazard lengths estimated by Rankine's criterion and William-Warnke's criterion are different from each other. Therefore, in order to consider the influence of the second and third principal stresses in evaluating concrete strength states, the William-Warnke's 5-parameter strength criterion should be used.

2. Evaluation of the concrete strength of practical dam according to William-Warnke's 5-parameter strength criterion by using ESM

2.1 Evaluation of the concrete strength of Ryongrim RCC gravity dam

In Fig. 7, a finite element model of a Ryongrim roller compressing concrete (RCC) gravity dam has the height of 120 m, the upper width of 7 m, the bottom width of 80 m and the upstream water depth of 120 m, and its uplift pressure is not considered. The width and depth of the foundation rock masses are all 1.5 times the height of the dam, and the bottom part of the foundation is assumed to be completely fixed and the horizontal displacement of the vertical boundary lines of foundation is assumed to be fixed. It is a plane deformation problem, where plane 8 nodes isoparametric element PLANE183 is used with ANSYS APDL [28], and the elasticity modulus and Poisson's ratio of dam concrete are 20GPa and 0.167 and those of foundation rock are 20GPa and 0.2. By using the ESM proposed in Ref. [10], the finite element stress in the boundary between dam and foundation is converted to the equivalent stress, and the concrete strength state of equivalent stress is evaluated according to the strength criterion of William-Warnke and compared with the result of FEM.

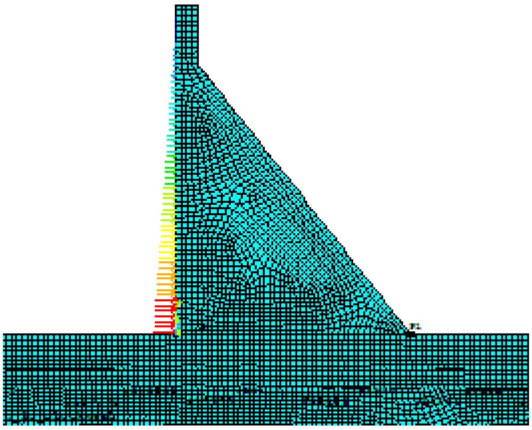

Fig. 7 FEM model of Ryongrim RCC gravity dam

As can be seen in Table 2, the maximum tension stress is 1.43 MPa and acts in the direction of about 147° to the horizontal plane, that is, the maximum tension principal stress at the dam heel does not exceed the tension strength of the concrete. This explains that the concrete of the dam is safe against the tension. according to the Rankine’s criterion.

Table 2. Stress state at the dam heel(MPa)

Calculation Method	Element Size(m)	σ_x	σ_y	τ_{xy}	σ_1	α (°)	β (°)
FEM	16	0.953	-0.000	0.634	1.269	153.48	116.52
	8	1.470	0.285	0.907	1.961	151.58	118.42
	4	2.118	0.673	1.251	2.840	150.00	120.00
	2	3.013	1.254	1.735	4.079	148.44	121.56
	1	4.231	2.104	2.406	5.798	146.92	123.08
	0.5	7.149	4.295	4.035	10.002	144.74	125.26
ESM		0.880	0.118	0.850	1.430	147.07	122.93

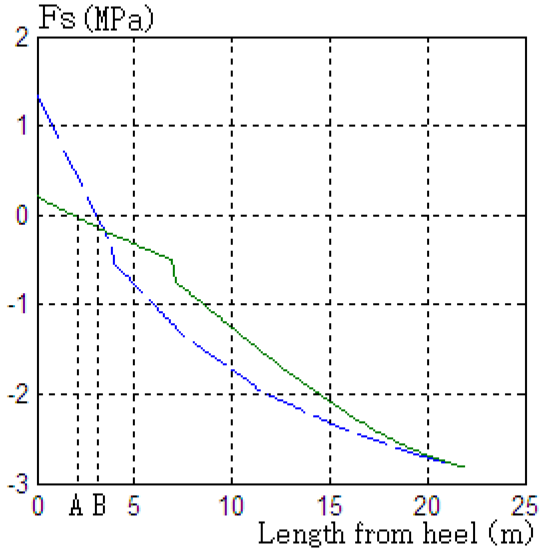

Fig. 8 F_s curves of Ryongrim RCC gravity dam (- -: FEM, -: ESM)

Fig. 8 shows F_s curves in the zone nearby the dam heel according to the William-Warnke's criterion by using FEM and ESM.

As shown in the figure, according to the William-Warnke's criterion, the hazard lengths on the boundary are $HL_{FEM} = OB = 2.95m$ in the case of FEM (element size - 4m) and $HL_{ESM} = OA = 1.77m$ in the case of ESM.

Table 2 and Fig. 8 explain that the concrete of the dam heel seems to be safe according to the Rankine criterion, but the strength function exceeds about 0.2MPa over the allowable value at the dam heel and the hazard length is about 1.77m, when considering the influence of the second and third principal stresses according to William-Warnke's 5-parameter strength standard.

So the water drain turnnel to be constructed on the boundary between the dam body and the foundation should be designed after the hazard length 1.77m.

2.2 Evaluation of the concrete strength of Kumya concrete arch dam

In Fig. 9, a finite element model of a Kumya concrete arch dam has the height of 130 m, the minimum central upper width of 7 m, the maximum bottom width of 48 m and the upstream water depth of 130 m, and its uplift pressure is not considered. The depth of the foundation rock masses is the same as the height of the dam, width of the foundation rock masses is the half of the height of the dam and the bottom part of the foundation is assumed to be completely fixed and the horizontal displacement of the vertical boundary lines of foundation is assumed to be fixed. The concrete element SOLID65 is used with ANSYS APDL [28], and the elasticity modulus and Poisson's ratio of dam concrete are 20GPa and 0.167 and those of foundation rock are 20GPa and 0.2. By using the ESM proposed in Ref. [10], the finite element stress in the boundary between dam and foundation is converted to the equivalent stress, and the concrete strength state of equivalent stress is evaluated according to the strength criterion of William-Warnke and compared with the result of FEM.

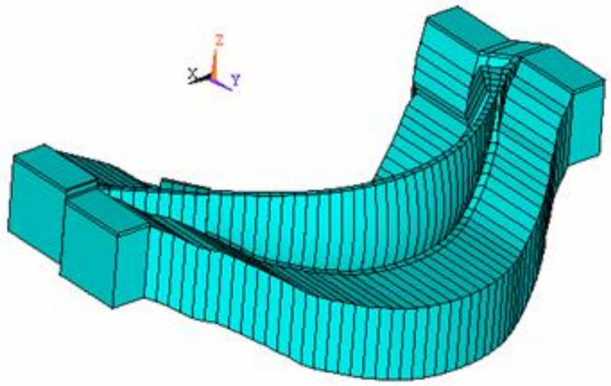

Fig. 9 FEM model of Kumya arch dam

The maximum tension stress appears at the dam heel of the seventh block in the right direction from the central block and at the heel of the eighth block in the left direction from the central block. Since the stress states of two maximum tension stress points are similar, only the stress state in the zone nearby the dam heel of the seventh block in the right direction from the central block will be considered

Table 3 shows the stress results of FEM and ESM at the dam heel where the maximum tension stress appeared.

Table 3. Stress at the dam heel of right 7th block(MPa)

Calculation Method	Element Size(m)	σ_x	σ_y	σ_z	τ_{xy}	τ_{yz}	τ_{xz}	σ_1
FEM	20	-0.024	0.263	0.314	-0.028	-0.005	-0.561	0.731
	10	0.244	0.693	0.857	-0.047	0.015	-0.575	1.205
	5	0.488	1.310	1.410	-0.048	0.025	-0.850	1.919
ESM		0.235	0.794	0.804	-0.035	-0.001	-0.658	1.457

As can be seen in Table 3, the maximum tension stress is 1.46MPa and acts in the direction of about 88.89° to the horizontal direction, 134.78° to the stream direction, 44.80° to the vertical direction, that is, the maximum tension principal stress at the dam heel does not exceed the tension strength of the concrete. This explains that the concrete of the dam is safe against the tension according to the Rankine's criterion.

Fig. 10 shows F_s curves in the zone nearby the dam heel where the maximum tension stress appeared according to the William-Warnke's criterion by using FEM and ESM.

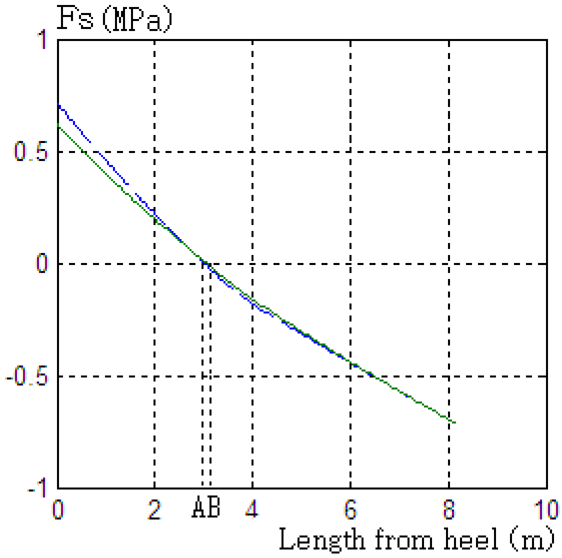

Fig. 10 F_s curves of Kumya arch dam (- -: FEM, -: ESM)

As shown in the figure, according to the William-Warnke's criterion, the hazard lengths on the boundary are $HL_{FEM} = OB = 3.2m$ in the case of FEM (element size - 5m) and $HL_{ESM} = OA = 3m$ in the case of ESM.

Table 3 and Fig. 10 explain that the concrete of the dam heel seems to be safe according to the Rankine's criterion, but the strength function exceeds about 0.7MPa over the allowable value at the dam heel and the hazard length is about 3m, when considering the influence of the second and third principal stresses according to William-Warnke's 5-parameter strength standard.

So the water drain tunnel to be constructed on the boundary between the dam body and the foundation should be designed after the hazard length 3m.

III. Algorithm for the equivalent stress method

In the analysis of concrete dam, the process of equivalent stress treatment and strength evaluation is accompanied by complicated numerical calculation, so it is necessary to combine finite element analysis and equivalent stress calculation to implement the whole process. The paper has integrated these processes with ANSYS, MATLAB, VC++. Figure 11 is the algorithm for equivalent stress treatment and strength evaluation.

The initial input parameters are the geometric dimensions of the dam and the material properties of the dam concrete and foundation rock.

First, ANSYS outputs the necessary result file through the input of initial data, modeling, , meshing, solving and plotting step.

Next, MATLAB [29] calculates the equivalent stress from the ANSYS result file and evaluates the concrete strength.

Visual C++ performs the function of connecting ANSYS and MATLAB and the function of input and output with MFC GUI. If necessary, the special effects of osmotic pressure, temperature field, and dynamic earthquake action and so on can be added in the process of ANSYS APDL.

IV. Conclusions

A method for evaluating the strength of dam concrete, by using equivalent stress method and multi-parameter strength criteria of concrete, has been proposed and proved its feasibility through numerical experiments.

First, the equivalent stress method has been applied to the analysis of concrete strength of gravity dam and arch dam, and more accurate solution of evaluation of concrete strength has been achieved with equivalent stress solution which does not depend on the mesh size of finite element analysis.

Secondly, the validity of the evaluation method of dam concrete strength according to the William-Warnke 5-parameter strength criterion has been proved through the comparison with Rankine's criterion or Drucker-Prager's criterion.

Third, the validity of the evaluation method of dam concrete strength in the zone nearby the dam heel according to the William-Warnke's criterion by using equivalent stress method is verified by numerical experiments on the currently operating concrete gravity dam and arch dam.

Fourth, the programs for whole process for converting the finite element solution to the nonlinear local equivalent stress and evaluating the strength of the dam concrete have been developed by using ANSYS, MATLAB and VC++.

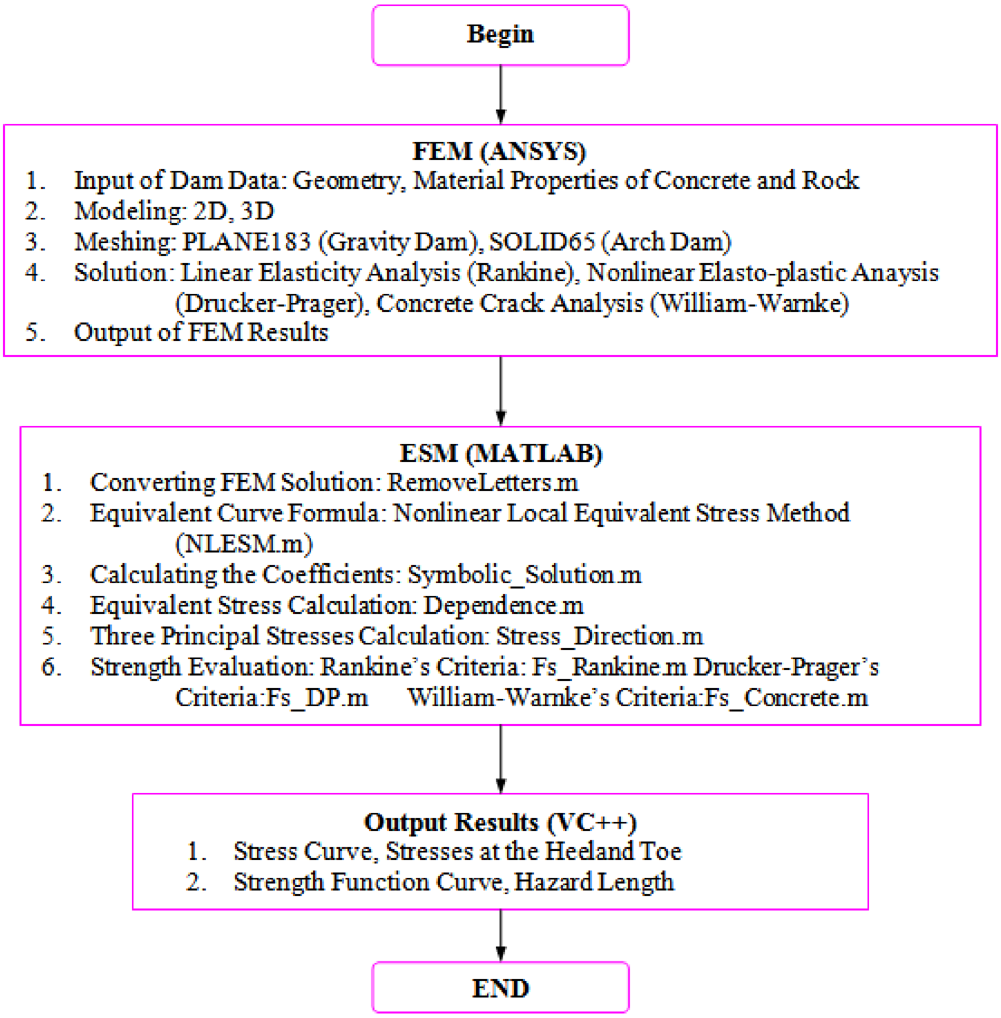

Fig. 11. Algorithm for Equivalent Stress Treatment and Strength Evaluation

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References

[1]. Novak, P., Moffat, A. I. B., Nalluri, C. and Narayanan, R., Hydraulic structures [M] (Taylor & Francis, 2007), 122-191

[2]. Chen, S. H., *Hydraulic Structures* [M] (Springer Press, 2015), 315+426-427.

[3]. Fan S, Chen J, Guo J., Application of finite element equivalent stress method to analyze the strength of gravity dam[J]. *Journal of Hydraulic Engineering, 6*, 2007. 754-759+766, DOI：10.13243/j.cnki.slxb.2007.06.018.

[4]. Jiang, C. Y., Xu, C. C., Li, H. Equivalent stress calculation for gravity dam foundation surface based on ANSYS [J]. *GX Water Resources & Hydropower Engineering, 1*, 2011: 14-16. DOI:10.16014/j.cnki.1003-1510.2011.01.014

[5]. Yang, H. C., Jia, J. S., Zheng, C. Y. Study and application of the improved finite element equivalent stress method to analyze the strength of gravity dams [J]. *Journal of China Institute of Water Resources and Hydropower Research, 11*. 2013. DOI:10.13244/j.cnki.jiwhr.2013.02.009

[6]. Zhao, F., Fan, S. L. Security Analysis of Gravity Dam Based on Finite Element Equivalent Stress Method [J]. *Water Resources and Power, 31,* 2013: 110-112+109.

[7]. Yang, Q., F. S. Liu and W. T. Zhou. Analysis of Non-linear Equivalent Stress of Base Surface of Gravity Dam Based on Moment Method [J]. *Water Power, 02,* 2006:23-25+55.

[8]. Han, Y., Ren, Q. W., Xiong, Y., Three-order Equivalent Stress Method for Eliminating Stress Grid Effect of Concrete Dam [J]. *Water Resources and Power, 34,* 2016, 80-90+84.

[9]. Huang, Y. Y., Shen, Z. Z. and Wu, Z. R. Discussion on Local Equivalent Stress Method of Concrete Gravity Dam [J].*Water Power, 05,* 2008:23-25+28.

[10]. Ri N H, Chen Z, Ri Y S, et al. A nonlinear local equivalent stress method in stress analysis of concrete gravity dam[J]. *Results in Physics, 12*, 2019: 1011-1014. DOI: 10.1016/j.rinp.2018.12.073.

[11]. Ren Q W, Jiang Y Z. Ultimate bearing capacity of concrete dam involved in geometric and material nonlinearity[J]. *Science China Technological Sciences, 54(3)*, 2011: 509-515.

[12]. Moftakhar M, Ghafouri H R. Comparison of Stability Criteria for Concrete Dams in Different Approximate Methods Based on Finite Element Analysis[J]. *Procedia engineering, 14,* 2011: 1672-1680.

[13]. Liu X, Han B. Structural Safety Evaluation of Wanjiazhai Concrete Gravity Dam by Monitoring Observations[J]. *Earth and Space*, 2010: 684-696.

[14]. Ren Q W, Xu L Y, Wan Y H. Research advance in safety analysis methods for high concrete dam[J]. Science in China Series E: *Technological Sciences, 50(1)*, 2007: 62-78.

[15]. Wei Z, Xiaolin C, Chuangbing Z, et al. Failure analysis of high-concrete gravity dam based on strength reserve factor method[J]. *Computers and Geotechnics, 35(4),* 2008: 627-636.

[16]. Wang R. Key technologies in the design and construction of 300 m ultra-high arch dams[J]. *Engineering, 2(3)*, 2016: 350-359.

[17]. Arabshahi H, Lotfi V. Earthquake response of concrete gravity dams including dam–foundation interface nonlinearities[J]. *Engineering structures, 30(11),* 2008, 30(11): 3065-3073.

[18]. Qi H, Li Y, Lü X. Practical elasto-plastic damage model for dynamic loading and nonlinear analysis of Koyna concrete dam[J]. *Journal of Central South University, 20(9)*, 2013: 2586-2592.

[19]. Hariri-Ardebili M A, Seyed-Kolbadi S M, Saouma V E, et al. Random finite element method for the seismic analysis of gravity dams[J]. *Engineering Structures, 171*, 2018: 405-420.

[20]. Arici Y, Binici B, Aldemir A. Comparison of the expected damage patterns from two-and three-dimensional nonlinear dynamic analyses of a roller compacted concrete dam[J]. *Structure and Infrastructure Engineering, 10(3)*, 2014: 305-315.

[21]. Paggi M, Ferro G, Braga F. A multiscale approach for the seismic analysis of concrete gravity dams[J]. *Computers & Structures, 122*, 2013: 230-238.

[22]. Xuhua R, Jiaqing S, Nenghui B, et al. Stability analysis of concrete gravity dam on complicated foundation with multiple slide planes[J]. *Water Science and Engineering, 1(3)*, 2008: 65-72.

[23]. Teng-fei B, Miao X, Lan C. Stability analysis of concrete gravity dam foundation based on catastrophe model of plastic strain energy[J]. *Procedia Engineering, 28*, 2012: 825-830.

[24]. Zhu B F. Finite elementwhole course smi ulation and sequential strength reductionmethod for safety appraisal of concrete dams, *Water Resources and Hydropower Engineering Vol. 38 No. 1*, 2007, 1-6. DOI:10.13928 /j.cnki.wrahe.2007.01.001

[25]. Shi L L, Song Y P, Shen L, Wang L C. Effect of dynamic triaxial compressive loading history of dam concrete under dynamic uniaxial compression, *Journal of Dalian University of Technology, 54(5)*, 54(5), 2014, 544-550. DOI:10.7511//dllgxb201405010

[26]. Zhang Y H，Hu X，Yang C，Zhu H D，Jiao J, Statistical analysis of dam concrete strength parameters, *Journal of Hydroelectiric Engineering, 34(6)*, 2015, 169-175

[27]. Wu S B, Wang Z Q. Various concrete strength criteria – based study on range of brittle damage of high arch dam, *Water Resources and Hydropower Engineering Vol. 47 No. 2*, 2016, 24-26+32

[28]. ANSYS Inc., Release 17.0 Documentation for ANSYS (2016).

[29]. MathWorks Inc., Documentation for MATLAB 2016 (2016).

Nam-Hyok Ri. “A Strength Evaluation of Dam Concrete by Using Equivalent Stress Method.” IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) , vol. 16, no. 3, 2019, pp. 39-49