

Orthometric Height Evaluation Considering the Geometrical Arrangement of Satellites and Atmospheric Condition

Kyedong Lee¹

¹(Geo-Information System Research Institute, Panasia, South Korea)

Abstract

Height determination is one of the important factors in determining three-dimensional positions of objects. Of course, depending on the purpose of a survey, height may be ignored, but in most cases, it is considered an essential factor in determining the positions of objects through surveying. Although a direct leveling method is generally used for height determination, there are many difficulties involved in performing direct leveling in mountainous areas. In order to overcome this drawback, an indirect method of height determination through Global Navigation Satellite System (GNSS)/leveling has been used recently. GNSS/leveling can be utilized in mountainous areas where it is difficult to perform direct leveling, and it can also be used in drone surveying, which has recently been used in various fields, and in the surveys for construction design considering the elevations in mountainous areas. Therefore, this study recomputes the elevations of triangulation points through GNSS/leveling by selecting triangulation points where the official height differs by 50 cm or more from the height derived from the national geoid model among triangulation points distributed in and around Jirisan in South Korea. Thus, GNSS observations of the same triangulation points were carried out over two days in consideration of the geometric layout of satellites and the atmospheric conditions. In addition, in order to determine the elevation with an accuracy of 3cm, the observation times of the first and second days were performed at different times of geometric arrangement. Ellipsoidal heights based on observation data were reviewed to determine whether they were within the allowable range. Then, orthometric heights of triangulation points in mountainous areas were recomputed using GNSS/leveling, and the applicability of this indirect leveling was evaluated. Survey results showed the validity of recomputing the heights of triangulation points through GNSS/leveling, and they also indicate that accurate height determination can be achieved by indirect leveling in mountainous areas.

Keywords: GNSS/leveling; Geometric arrangement; Orthometric height; Geoid model.

Date of Submission: 02-12-2020

Date of Acceptance: 17-12-2020

I. Introduction

In order to determine the height of a point in a mountainous area by GNSS/leveling, verification and consideration of various factors, including mean sea level and geoid and ellipsoidal height, are required. Traditionally, the deflection of a vertical component is determined from astro-geodesy(Lachapelle G. et al., 1984, Hirt C. and Seeber G., 2008, Wang Y. M. et al., 2017), gravimetry(Jekeli C., 1999, Hirt C., 2012), GNSS/leveling(Tse C. M. and Baki Iz. H., 2006, Ceylan A., 2010), etc., from which it is easier to calculate the deflection of the vertical with an Earth gravity field model and a geoid model(Xingfu Z. et al., 2018). The most widely used geoid refinement method is the remove–compute–restore (RCR) technique(Schwarz K. et al., 1990), which uses the global gravity field model as a long-wavelength term, and performs Stokes integration on the residual gravity anomaly to obtain high-frequency signals that are missing from the gravity field model(Qiong W. et al., 2020). The RCR method has been widely applied to the refinement of geoids in countries like Turkey(Ayhan M. E., 1993), Canada(Fotopoulos G. et al., 2000), Australia(Zhang K. et al., 1998), the United States(Smith D. A. and Milbert D. G., 1999), Ghana(Caleb I. Y. et al., 2017), and Japan(Odera P. A. et al., 2012). In fact, the indirect effect (δN) for a topographic height of 3 km is less than 50cm(Caleb I. Y. et al., 2017, Odera P. A. et al., 2012, Hofmann-Wellenhof B. and Moritz H., 2006). As studies of the literature have shown, the most iconic and globally recognized peaks, such as Mount Everest(Ward M., 1995, Junyong C. et al., 2005, J. De Graaff-Hunter., 1955), Mont Blanc(De Beer G., 1956), Aconcagua(Poretti G. G., 1999) and Kilimanjaro(Saburi J. et al., 2000, TeamKILI2008., 2009), have already been thoroughly tested using the same GNSS technique(Krystian K. and Kamil M., 2020) that used in this case study. Hui evaluated global mean sea surface height through cyclone-GNSS reflectometry(Hui Q. and Shuanggen J., 2020), and Hayden et al. estimated vertical datum offsets using GNSS/leveling benchmark information of Canada and GOCE global geopotential models(Hayden T. et al., 2012). In addition, based on the integration of gravity data and GNSS/leveling data(Albertella A. et al., 2008, Chen Y. Q. and Luo Z., 2004), Alessandra et al. conducted

research on a geoid model with centimeter-level precision for application to the Jeddah region of Saudi Arabia (Alessandra B. et al., 2020). Furthermore, Dinh et al. improved the accuracy of GNSS/leveling through a study on the quasigeoid-derived transformation model for height system unification in Vietnam (Dinh T. V. et al., 2020).

Similarly, in South Korea, geoid models have been steadily updated for GNSS/leveling, and related research has been carried out. Lee et al. updated the geoid model for South Korea using the latest gravity data (Lee J. S. et al., 2011), and verified the precision of the constructed geoid model (Lee J. S. et al., 2012). Kim et al. evaluated the accuracy of geoid height data on national control points using a high-degree geopotential model (Kim K. B. et al., 2020). Lee and Kwon further developed this model, and evaluated the precision of the global geopotential model based on GNSS/leveling data for unified control points (Lee J. S. et al., 2020). Based on the results of research in a flatland area, Lee determined the precise geoid heights of the areas in and around Jirisan (Lee S. B., 2018), and Jung et al. analyzed the accuracy of vertical heights of open leveling loop areas (Jung S. C. et al., 2018). In addition, Shin G. S et al., 2014 and Jung S. C et al., 2018 conducted studies on the determination and accuracy of orthometric heights in mountainous areas.

A triangulation point is a reference point in surveying used to accurately determine the shape, boundary, area, etc., of the land, or to determine the positions of objects related to the design and construction of various facilities. Triangulation points in South Korea are classified into four types, from first-order to fourth-order. First-order triangulation points are control points of the highest order installed at intervals of about 20km, on average, and the second-order, third-order, and fourth-order triangulation points are placed at 10km, 5km, and 2.5km intervals, respectively. The survey results for triangulation points are provided through a web service of the National Geospatial Information Service (NGIS). These survey results were first determined in 1975 through triangulation with the trilateration method using electronic distance measurement (EDM). With the introduction of GNSS surveying technology, horizontal positioning of triangulation points has been determined through GNSS surveying since 1997. In addition, gravity surveying was conducted on triangulation points from 2011 to 2016 to improve the precision of the national geoid model.

The traditional control point system is divided into horizontal control points (determined through triangulation based on the horizontal origin point), and vertical control points (determined through leveling based on the vertical origin point). However, since 1997, it has become possible to determine not only latitude and longitude, but also ellipsoidal height belonging to height information at a single point through GNSS surveying. In order to increase the efficiency of surveying, control points installed thereafter have been converted into those that have both horizontal and vertical survey results. In the case of triangulation points for which the GNSS/leveling network adjustment has been made, their elevations were determined by performing GNSS adjustments after fixing the signal targets directly or indirectly connected and measured from benchmarks by using the results of leveling network adjustment announced in 2007, and new results obtained from re-surveying data of some benchmarks. In other words, their heights were obtained through network adjustments calculating the heights of triangulation points after fixing the latitude and longitude of Continuously Operating Reference Stations (CORS) regarding horizontal positions, and fixing the heights of the signal targets.

II. Material And Methods

2.1 Selection of triangulation points

To perform GNSS/leveling in mountainous areas, known and unknown points can be defined as follows. Known points refer to the unified control points that will be used to recompute the height of a triangulation point. A unified control point is a multifunctional national survey reference point where the horizontal position, height, and gravity value have already been determined. In addition, an unknown point is a triangulation point that is a national control point where the position has already been identified through triangulation (National Geographic Information Institute : Suwon, Korea., 2019).

The specific procedure of this study consisted of 1) verification of known points, 2) height determination of unknown points through GNSS/leveling, and 3) determination of survey results. In order to determine the elevation of a triangulation point (an unknown point) through GNSS/leveling, it is necessary to make GNSS observations at the triangulation point. Unified control points, or benchmarks that can be used as known points, must be present around the unknown point. To determine the orthometric heights of triangulation points, known and unknown points for height determination through GNSS/leveling were finally selected by applying the criteria presented in Figure 1. The selection criteria can be briefly described as follows. Initially, triangulation points were selected where official height values differed by 50cm or more in absolute terms from the elevation computed based on KNGeoid18. Then, 12 points in the areas in and around Jirisan were selected as unknown points, and their heights were determined through GNSS/leveling.

In principle, the observation network for height determination by GNSS/leveling should be constructed so that an unknown point is surrounded by nearby known points in a triangular shape. In order to determine the precise position, target areas for height determination were selected according to criteria such as a relatively

large height difference between known and unknown points and the presence of control points that can be additionally fixed around the area. However, since triangulation points are situated on the tops of mountains, and unified control points and benchmarks are distributed along roads (and cannot be located at locations higher than the triangulation points), a method of fixing and utilizing the highest unified control point or benchmark located near the unknown point was used.

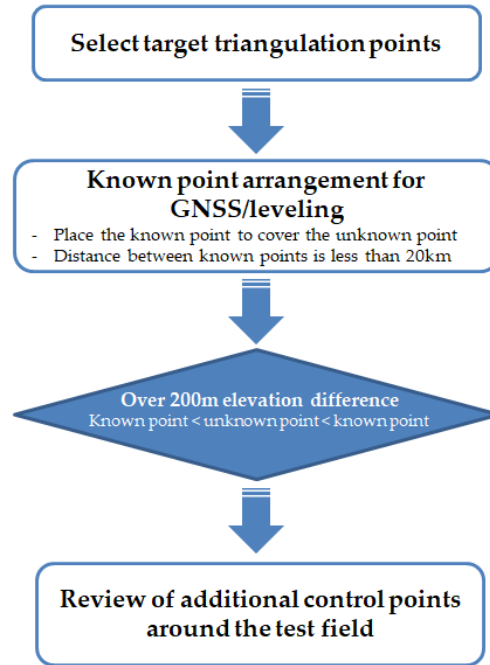


Figure 1. Triangulation point selection standard for GNSS/leveling

Table 1. Triangulation points with a difference of 50cm or more between official height and calculated height from KNGeoid18

| Triangulation points | Official height | | | | Calculated height from KNGeoid18 (m) | | |
|----------------------|-----------------|--------------|------------------------|------------------------|--------------------------------------|-------------------|----------------------|
| | Lat. (deg.) | Long. (deg.) | Ellipsoidal height (m) | Orthometric height (m) | KN- Geoid18 height | Calculated height | Difference in height |
| | | | a | b | | | |
| Unbong 25 | 35.3***** | 127.5***** | 1332.234 | 1305.379 | 27.462 | 1304.772 | 0.607 |
| Unbong 26 | 35.3***** | 127.6***** | 1610.646 | 1583.399 | 28.138 | 1582.326 | 1.073 |
| Unbong 305 | 35.3***** | 127.6***** | 952.199 | 924.904 | 28.013 | 924.186 | 0.718 |
| Unbong 12 | 35.2***** | 127.5***** | 1529.759 | 1502.894 | 27.711 | 1502.048 | 0.846 |
| Unbong 434 | 35.2***** | 127.5***** | 934.845 | 907.772 | 27.895 | 906.950 | 0.822 |
| Hadong 21 | 35.2***** | 127.6***** | 970.870 | 943.734 | 27.886 | 942.984 | 0.750 |
| Hadong 22 | 35.1***** | 127.6***** | 1143.520 | 1116.157 | 28.062 | 1115.458 | 0.699 |
| Hadong 304 | 35.2***** | 127.7***** | 933.697 | 906.240 | 28.191 | 905.506 | 0.734 |
| Hadong 408 | 35.2***** | 127.6***** | 664.998 | 637.697 | 28.020 | 636.978 | 0.719 |
| Sanchung 444 | 35.2***** | 127.7***** | 717.800 | 690.155 | 28.357 | 689.443 | 0.712 |
| Sanchung 448 | 35.3***** | 127.8***** | 555.790 | 528.123 | 28.292 | 527.498 | 0.625 |
| Hadong 409 | 35.2***** | 127.7***** | 1100.675 | 1073.264 | 28.337 | 1072.338 | 0.926 |

2.2 Instrumentation and observation conditions

The GNSS equipment used in this study included a dual-frequency GNSS receiver and a wooden tripod combined with a GNSS fixing pole, as shown in Table 2. The same type of antenna was used for the known and unknown points where observations were simultaneously carried out. Figure 2 shows the GNSS device installed at a unified control point (U0402), which is a known point, and a triangulation point (Unbong 12), which is an unknown point. In this study, height determination by GNSS/leveling was carried out based on an accuracy level within 3cm by applying the observation criteria shown in Table 3. Data obtained at intervals of 15 seconds were used to ensure the reliability of survey results obtained at the triangulation points (National Geographic Information Institute Available online., 2019). In addition, considering the geometric layout of GNSS satellites and atmospheric conditions, survey results were computed using the observation data received for four hours per day over two days, and then cross-calculations were performed. Also, the ‘3cm’ accuracy standard was selected according to guidelines of the National Geographic Information Institute.

Table 2. Standards for the survey equipment in GNSS/leveling

| Device | Standards |
|----------|---|
| Receiver | - Frequency : more than L1 and L2 - Performance : $\pm 5\text{mm} + 1\text{ppm} \times D$ (km) |
| Antenna | - Known and unknown points use the same type of antenna - Using an antenna with absolute correction for phase center variation |
| Tripod | - Accuracy level (3 cm) : wooden tripod and GNSS dedicated pole - Accuracy level (5 cm) : wooden tripod |



(a)



(b)

Figure 2. GNSS equipment used for GNSS/leveling: (a) GNSS equipment installed at the known point (unified control point U0402) and (b) GNSS equipment installed at the unknown point (Unbong 12)

Table 3. GNSS/leveling standard by accuracy

| Item | Accuracy level | |
|----------------------------------|--|-------------------------|
| | 3cm | 5cm |
| Section | 2 days | 1 day |
| Observation time | 4 hours or more per day | 2 hours or more per day |
| Epoch | 30 seconds or less | 15 seconds or less |
| Antenna height | Average value before and after observation | |
| Available satellites | GPS, GLONASS | |
| Number of observation satellites | 5 or more GNSS satellites | |
| Mask angle | 15 degrees or more | |

2.3 GNSS/leveling

2.3.1 Surveying for verification of known points

The guidelines of the National Geographic Information Institute for GNSS/leveling specify that performance verification should be done for the known points selected for GNSS/leveling, as shown in Figure 3.

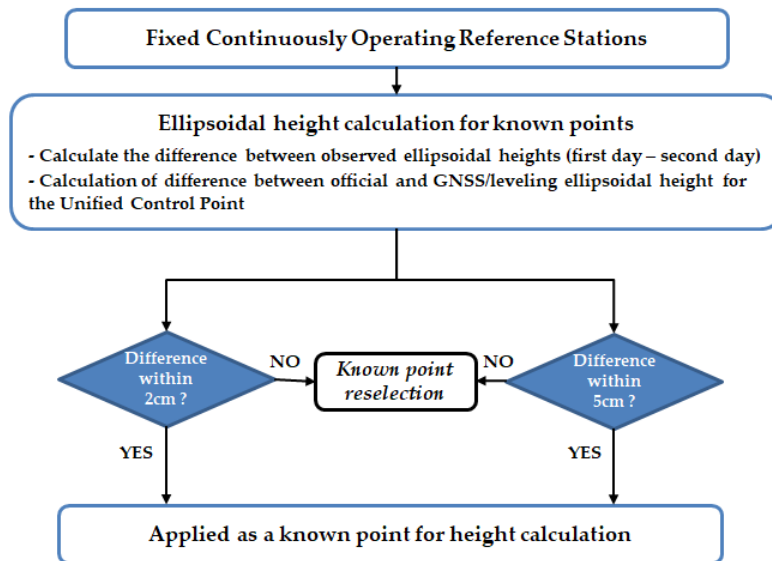


Figure 3. Known point verification method in GNSS/leveling

To comply with the guidelines, after performing a baseline analysis and a network adjustment, during which the CORS are fixed, the following two conditions must be satisfied. The first condition is that the difference between the ellipsoidal heights determined through GNSS observations for four hours per session on Day 1 and Day 2 must not exceed 2cm. This condition is intended to examine the reliability of new ellipsoidal height values themselves by examining the consistency in the new survey results. The second condition is that the mean ellipsoidal height determined by taking into account the observations of both Day 1 and Day 2 must not differ by more than 5cm from the official ellipsoidal height value. This condition is intended to check whether there are any errors in the official data.

The guidelines for GNSS/leveling require two sessions of observations to be carried out over two days, and they should be performed during different periods of time when the geometric layout of satellites is different. In other words, as shown in Table 4, when Case 1 is selected, if the observation is performed from 09:00 to 13:00 on Day 1, it should be conducted between 16:00 and 22:00 on Day 2. In addition, when Case 2 is selected, if the observation is performed from 13:00 to 17:00 on Day 1, it should be performed between 08:00 and 14:00 on Day 2. In this study, GNSS/leveling was performed by safely selecting Case 2 in consideration of earlier sunsets in mountainous areas. To carry out data processing, precise ephemeris provided by the International GNSS Service (IGS) was used, and the IGS variation model was used to adjust the antenna for phase center variation.

In order to determine the heights of triangulation points through GNSS/leveling, the unified control points to be used as known points were first examined in order to determine loss and observation conditions prior to the observations, and when it was considered difficult to conduct observations due to obstruction by trees, either shrubs were removed or the known points were changed. Table 5 shows that the triangulation points selected for this study were divided into four sites considering the distances. A list of the known points that could include them is also shown in Table 5. For Site A, an adequate observation network was built with only three known points, but for sites B, C, and D with weak network strength, four known points were selected in consideration of distance and height in order to include all unknown points. The positions of unknown and known points in each site are shown in Figure 4.

Table 4. Observation times for each day

| Case | Day 1 observation time | Day 2 observation time | Notes |
|--------|------------------------|------------------------|-----------|
| Case 1 | 09:00 ~ 13:00 | 16:00 ~ 22:00 | |
| Case 2 | 13:00 ~ 17:00 | 08:00 ~ 14:00 | selection |

Table 5. List of known points to be used for observation of unknown points by site

| Triangulation point (unknown point) | | Investigated known points |
|-------------------------------------|--|---|
| Site | Point name | |
| Site A | Unbong 25, Unbong 26, Unbong 305 | U Unbong 30, U Unbong 41, U Unbong 87 |
| Site B | Unbong 12, Unbong 434, Hadong 21 | U Unbong 71, U Unbong 87, U Hadong 23, U Hadong 42 |
| Site C | Hadong 22, Hadong 304, Hadong 408 | U Gonyang 11, U Unbong 87, U Hadong 23, U Hadong 29 |
| Site D | Sancheong 444, Sancheong 448, Hadong 409 | U Gonyang 11, USancheong 55, U Unbong 87, U0882 |

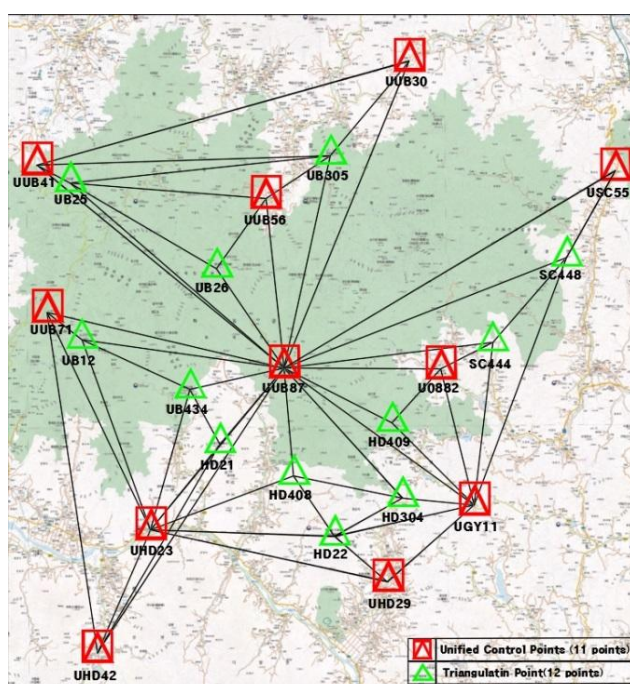


Figure 4. Network of unknown and known points by site

Site A: In Site A, known points that required verification for GNSS/leveling were three unified control points (U Unbong 30, U Unbong 41, and U Unbong 87), and ellipsoidal heights of known points were computed by performing GNSS data processing and network adjustments after fixing four Continuously Operating Reference Stations: Geochang (GOCH), Jinju (JINJ), Namwon (NAMW), and Suncheon (SONC). Figure 5 shows the layout of the CORS and the known points located in Site A.

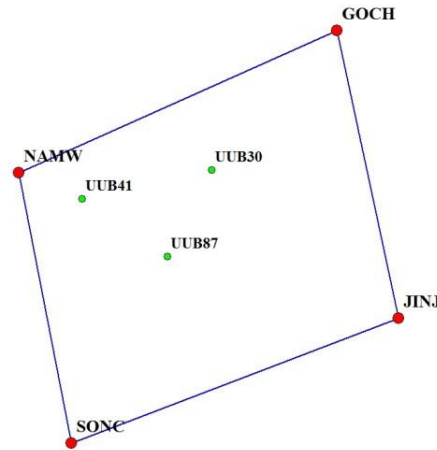


Figure 5. Network of Continuously Operating Reference Stations and known points at Site A

As to the observation times for Site A, data were received for four hours per session from 13:00 to 17:00 on Day 1 and from 08:00 to 12:00 on Day 2. As shown in Table 6, the difference between the ellipsoidal heights determined based on observation data on Day 1 and Day 2 was less than 2cm. In addition, at the three known points, the difference between the mean ellipsoidal height obtained through GNSS observations and the official ellipsoidal height value was less than 5cm, which is the inspection standard. Thus, the known points were determined to have an adequate level of reliability. The verification results of the known points in Site A are summarized in Table 6.

Table 6. Results from verification of ellipsoidal heights of known points at Site A

| Point name | Official ellipsoidal height | Height by GNSS/leveling | | | | Difference in ellipsoidal height |
|-------------|-----------------------------|-------------------------|------------|----------------------|---------------|----------------------------------|
| | | First day | Second day | Difference in height | Mean | |
| | a | b | c | $d= b -c $ | $e=(b + c)/2$ | $f= a - e $ |
| U Unbong 30 | 283.8676 | 283.841 | 283.823 | 0.018 | 283.832 | 0.0356 |
| U Unbong 41 | 707.4651 | 707.417 | 707.432 | 0.015 | 707.4245 | 0.0406 |
| U Unbong 87 | 304.131 | 304.136 | 304.12 | 0.016 | 304.128 | 0.003 |

Site B:In Site B, the known points that required verification for GNSS/leveling were four unified control points (U Unbong 71, U Unbong 87, U Hadong 23, and U Hadong 42), and ellipsoidal heights of the known points were determined by performing GNSS data processing and network adjustment after fixing the Continuously Operating Reference Stations: Gochang (GOCH), Jinju (JINJ), Namwon (NAMW), and Suncheon (SONC). Figure 6 shows the layout of the CORS and known points in the area of Site B.

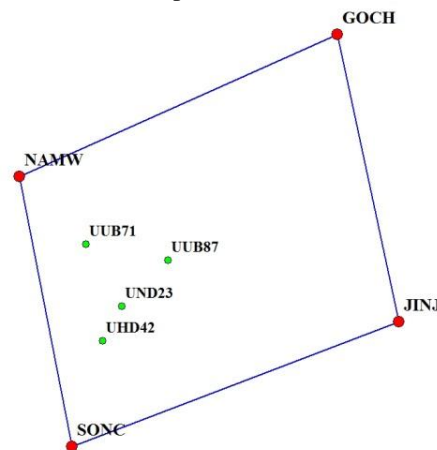


Figure 6. Network of Continuously Operating Reference Stations and known points at Site B

Regarding the observation times for Site B, as with Site A, data were received for four hours per day from 13:00 to 17:00 on Day 1 and from 08:00 to 12:00 on Day 2, and the difference between the ellipsoidal heights derived from GNSS observations on Day 1 and Day 2 was less than 2cm, as shown in Table 6. In addition, at the four known points, the difference between the mean ellipsoidal height obtained through surveying and the official ellipsoidal height value was found to be less than 5cm, which is the inspection standard, so the known points were considered to have an adequate level of reliability. The verification results for the known points of Site B are summarized in Table 7.

Table 7. Results from verification of ellipsoidal heights of known points at Site B

| Point name | Official ellipsoidal height | Height by GNSS/leveling | | | | Difference in ellipsoidal height |
|-------------|-----------------------------|-------------------------|------------|----------------------|---------------|----------------------------------|
| | | First day | Second day | Difference in height | Mean | |
| | a | b | c | $d= b - c $ | $e=(b + c)/2$ | $f= a - e $ |
| U Unbong 71 | 1118.1717 | 1118.16 | 1118.179 | 0.019 | 1118.1695 | 0.0022 |
| U Unbong 87 | 304.131 | 304.142 | 304.123 | 0.019 | 304.133 | 0.002 |
| U Hadong 42 | 131.935 | 131.898 | 131.907 | 0.009 | 131.9025 | 0.0325 |
| U Hadong 23 | 52.0582 | 52.084 | 52.065 | 0.019 | 52.0745 | 0.0163 |

Site C: The known points that required verification for GNSS/leveling in Site C were four unified control points (U Gonyang 11, U Unbong 87, U Hadong 23, and U Hadong 29), and ellipsoidal heights of the known points were computed by GNSS data processing and network adjustment after fixing four Continuously Operating Reference Stations: Geochang (GOCH), Jinju (JINJ), Suncheon (SONC), and Namwon (NAMW). Figure 7 shows the layout of the CORS and the known points in the area of Site C.

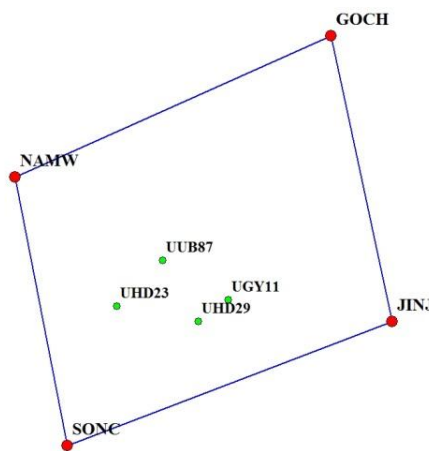


Figure 7. Network of Continuously Operating Reference Stations and known points at Site C

In Site C, observation data were received for four hours per session from 13:00 to 17:00 on Day 1 and from 08:00 to 12:00 on Day 2, and as with Sites A and B, and the difference between the ellipsoidal heights determined based on observation data on Day 1 and Day 2 was less than 2cm, as shown in Table 6. In addition, at the four known points, the difference between the mean ellipsoidal height obtained through surveying and the official ellipsoidal height value was less than 5cm, which is the inspection standard, so the known points were determined to have an adequate level of reliability. The verification results for the known points in Site C are summarized in Table 8

Table 8. Results from verification of ellipsoidal height of known points at Site C

| Point name | Official ellipsoidal height | Height by GNSS/leveling | | | | Difference in ellipsoidal height |
|--------------|-----------------------------|-------------------------|------------|----------------------|---------------|----------------------------------|
| | | First day | Second day | Difference in height | Mean | |
| | a | b | c | $d= b -c $ | $e=(b + c)/2$ | $f= a - e $ |
| U Gonyang 11 | 287.048 | 287.021 | 287.028 | 0.007 | 287.0245 | 0.0235 |
| U Unbong 87 | 304.131 | 304.139 | 304.12 | 0.019 | 304.1295 | 0.0015 |
| U Hadong 23 | 52.0582 | 52.086 | 52.091 | 0.005 | 52.0885 | 0.0303 |
| U Hadong 29 | 94.3611 | 94.316 | 94.315 | 0.001 | 94.3155 | 0.0456 |

Site D:The known points that required verification for GNSS/leveling in Site D were four unified control points (U 0882, U Gonyang 11, U Sancheong 55, and U Unbong 87). The ellipsoidal heights of known points were examined by conducting GNSS data processing and network adjustment after fixing the CORS: Geochang (GOCH), Jinju (JINJ), Suncheon (SONC), and Namwon (NAMW). Figure 8 shows the layout of the Continuously Operation Reference Stations and known points located in Site D.

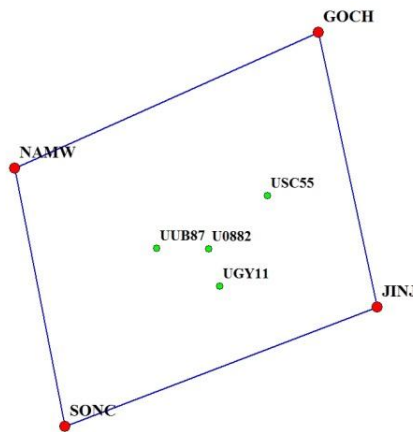


Figure 8. Network of Continuously Operating Reference Stations and known points at Site D

In Site D, data were received for four hours per session from 13:00 to 17:00 on Day 1 and from 08:00 to 12:00 on Day 2, as was done in the other three sites. The difference between the ellipsoidal heights based on observation data from Day 1 and Day 2 was less than 2cm, as shown in Table 9. In addition, at the four known points, the difference between the mean ellipsoidal height and the official data was less than 5cm, which is the inspection standard, so the known points are thought to have an adequate level of reliability.

Table 9. Results from verification of ellipsoidal heights of known points at Site D

| Point name | Official ellipsoidal height | Height by GNSS/leveling | | | | Difference in ellipsoidal height |
|----------------|-----------------------------|-------------------------|------------|----------------------|---------------|----------------------------------|
| | | First day | Second day | Difference in height | Mean | |
| | a | b | c | $d= b -c $ | $e=(b + c)/2$ | $f= a - e $ |
| U Gonyang 11 | 287.048 | 287.026 | 287.035 | 0.009 | 287.0305 | 0.0175 |
| U Sancheong 55 | 418.2375 | 418.269 | 418.256 | 0.013 | 418.2625 | 0.025 |
| U Unbong 87 | 304.131 | 304.113 | 304.113 | 0 | 304.113 | 0.018 |
| U0882 | 422.9909 | 422.946 | 422.955 | 0.009 | 422.9505 | 0.0404 |

2.3.2 GNSS/leveling of unknown points

Site A

The GNSS observation network for height determination of unknown points in Site A (Unbong 25, Unbong 26, and Unbong 305) is shown in Figure 9. Regarding the distance between unified control points used as known points, the distance between U Unbong 41 and U Unbong 30 was about 21km; the distance between U Unbong 30 and U Unbong 87 was about 18km, and the distance between U Unbong 87 and U Unbong 41 was about 17km. The distance between U Unbong 41 and U Unbong 30 was 21km, exceeding the acceptable range by 1km with respect to GNSS/leveling guidelines, which require that the distance between known points be less than 20km. But that was due to the layout of the surrounding unified control points, and the guidelines were still mostly satisfied. The highest unknown point in the target area was Unbong 26, which differed in height by about 903m from U Unbong 41, which was the highest location among the known points.

As shown in Table 10, the differences between the height values obtained from observations on Day 1 and Day 2 at Unbong 25, Unbong 26, and Unbong 305 were 1.6cm, 1.5cm, and 1.8cm, respectively, which are less than 3cm, so consistency in the survey results was confirmed. The differences between the mean height values obtained through GNSS/leveling using KNGeoid18 and the official heights were 48cm, 100.3cm, and 68.2cm at Unbong 25, Unbong 26, and Unbong 305, respectively.

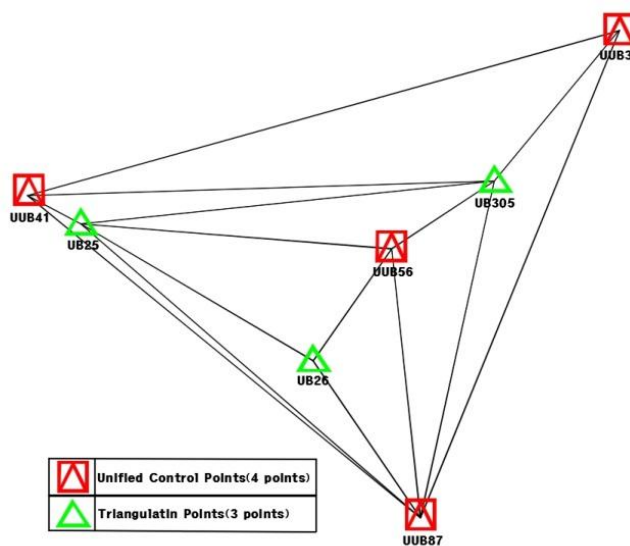


Figure 9. Network of known and unknown points in Site A

Table 10. Differences between the heights acquired by GNSS/leveling and the official heights of unknown points (Site A)

| Point name | Official height | Height by GNSS/leveling | | | | Difference in height |
|------------|-----------------|-------------------------|------------|----------------------|---------------|----------------------|
| | | First day | Second day | Difference in height | Mean | |
| | a | b | c | $d= b - c $ | $e=(b + c)/2$ | $f= a - e $ |
| Unbong 25 | 1305.379 | 1304.891 | 1304.907 | 0.016 | 1304.899 | 0.480 |
| Unbong 26 | 1583.399 | 1582.388 | 1582.403 | 0.015 | 1582.396 | 1.003 |
| Unbong 305 | 924.904 | 924.231 | 924.213 | 0.018 | 924.222 | 0.682 |

Site B

The GNSS observation network for elevation determination of unknown points in Site B (Unbong 12, Unbong 434, and Hadong 21) is shown in Figure 10. Regarding the distance between the unified control points used as known points, the distance between U Unbong 71 and U Hadong 42 was about 19km, and the distance between U Hadong 42 and U Hadong 23 was about 7km; the distance between U Hadong 42 and U Unbong 87 was about 19km, and the distance between U Unbong 87 and U Unbong 71 was about 13km. Thus, they satisfied the guidelines for GNSS/leveling, which require the distance between two known points to be less than

20km. The highest unknown point in the target area was Unbong 12, which differed in height by about 412m from U Unbong 41, which was the highest among the known points.

In Table 11, the differences between the height values obtained from observations on Day 1 and Day 2 at Unbong 12, Unbong 434, and Hadong 21 were 0.1cm, 3cm, and 2.4cm, respectively, showing consistency in the survey results. The differences between the mean height obtained through GNSS/leveling using KNGeoid18 and the official heights were 79cm, 86cm, and 74.5cm at Unbong 12, Unbong 434, and Hadong 21, respectively.

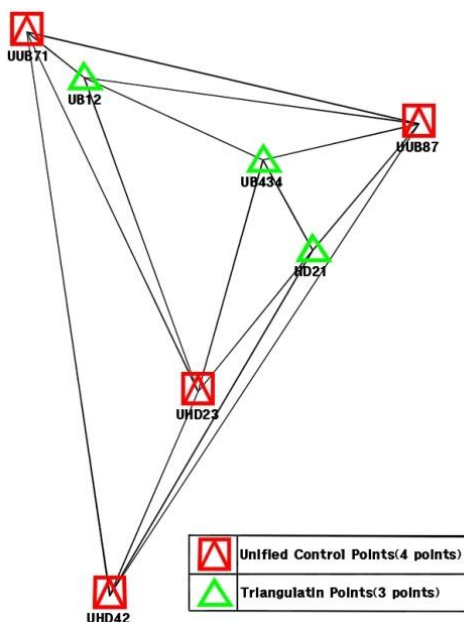


Figure 10. Network of known and unknown points in Site B

Table 11. Differences between the heights acquired by GNSS/leveling and the official heights of unknown points (Site B)

| Point name | Official height | Height by GNSS/leveling | | | | Difference in height |
|------------|-----------------|-------------------------|------------|----------------------|---------------|----------------------|
| | | First day | Second day | Difference in height | Mean | |
| | a | b | c | $d= b - c $ | $e=(b + c)/2$ | $f= a - e $ |
| Unbong 12 | 1502.894 | 1502.104 | 1502.103 | 0.001 | 1502.104 | 0.790 |
| Unbong 434 | 907.772 | 906.897 | 906.927 | 0.030 | 906.912 | 0.860 |
| Hadong 21 | 943.734 | 942.977 | 943.001 | 0.024 | 942.989 | 0.745 |

Site C

In Site C, the GNSS observation network for elevation determination of unknown points (22 Hadong, 304 Hadong, and 408 Hadong) is shown in Figure 11. As for the distance between the unified control points used as known points, the distance between U Unbong 87 and U Gonyang 11 was about 13km, and the distance between U Gonyang 11 and U Hadong 29 was about 6km; the distance between U Hadong 29 and U Hadong 23 was about 13km, and the distance between U Hadong 23 and U Unbong 87 was about 11km. Thus, they satisfied the guidelines for GNSS/leveling, which require the distance between two known points to be less than 20km. Also, among the unknown points in the target area, the highest point was Unbong 22, which differed in height by about 840m from U Unbong 87, which was the highest location among the known points in Site C.

As shown in Table 12, the differences between the height values obtained through GNSS/leveling on Day 1 and Day 2 at Hadong 22, Hadong 304, and Hadong 408 were 1.5cm, 1cm, and 0.2cm, respectively, which are all less than 3cm, showing consistency in the survey results. The differences between the mean heights obtained through GNSS/leveling using KNGeoid18 and the official heights were 62.4cm, 69.6cm, and 74.7cm at Hadong 22, Hadong 304, and Hadong 408, respectively

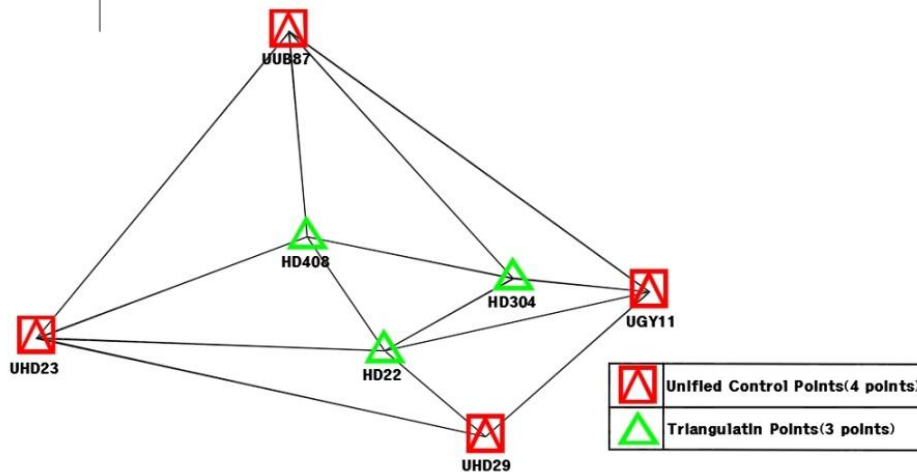


Figure 11. Network of known and unknown points in Site C

Table 12. Differences between the heights acquired by GNSS/leveling and the official heights of unknown points (Site C)

| Point name | Official height | Height by GNSS/leveling | | | | Difference in height |
|------------|-----------------|-------------------------|------------|----------------------|----------|----------------------|
| | | First day | Second day | Difference in height | Mean | |
| | | a | b | c | d= b -c | |
| Hadong 22 | 1116.157 | 1115.54 | 1115.525 | 0.015 | 1115.533 | 0.624 |
| Hadong 304 | 906.24 | 905.549 | 905.539 | 0.01 | 905.544 | 0.696 |
| Hadong 408 | 637.697 | 636.951 | 636.949 | 0.002 | 636.950 | 0.747 |

Site D

In Site D, the GNSS observation network for elevation determination of unknown points (Sancheong 444, Sancheong 448, and Hadong 409) is shown in Figure 12. Regarding the distance between the unified control points used as known points, the distance between U Unbong 87 and U Sancheong 55 was 20.9km, and the distance between U Sancheong 55 and Gonyang 11 was about 18km; the distance between U Gonyang 11 and U0882 was about 8km, and the distance between U0882 and U Unbong 87 was about 9km, while the distance between U Gonyang 11 and U Unbong 87 was about 13km. The distance between U Unbong 87 and U Sancheong 55 (20.9km) exceeds the acceptable range by 0.9km with respect to the guidelines for GNSS/leveling, but this was due to the layout of unified control points around the area, and the guidelines were still mostly satisfied. Also, the highest unknown point in the target area was Hadong 409, which differed in height by about 679m from U0082, which was the highest among the known points.

As shown in Table 13, the differences between the height values obtained through GNSS/leveling on Day 1 and Day 2 at Sancheong 444, Sancheong 448, and Hadong 409 were 1.4cm, 1.7cm, and 2.9cm, respectively, which are less than 3cm, so consistency in the survey results was confirmed. The differences between the mean heights obtained through GNSS/leveling and the official heights were 61.1cm, 55.6cm, and 88.0cm at Sancheong 444, Sancheong 448 and Hadong 409, respectively.

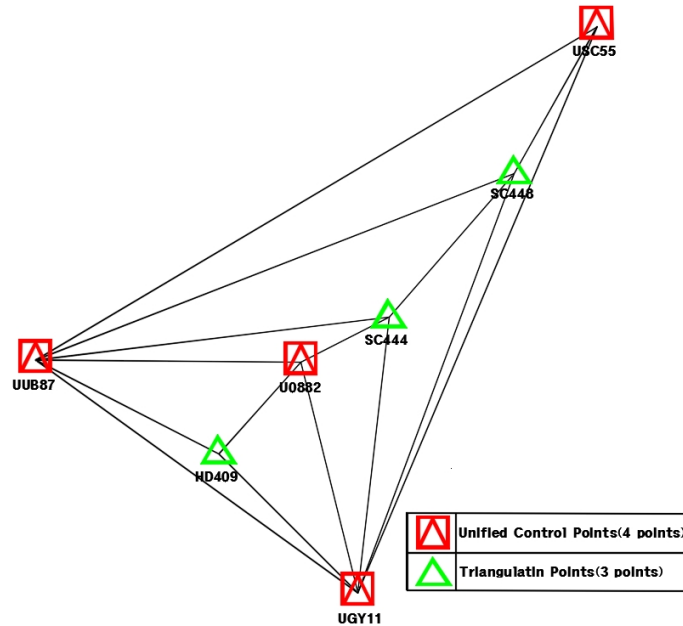


Figure 12. Network of known and unknown points in Site D

Table 13. Differences between the heights acquired by GNSS/leveling and the official heights of unknown points (Site D)

| Point name | Official height | Height by GNSS/leveling | | | | Difference in height |
|---------------|-----------------|-------------------------|------------|----------------------|---------------|----------------------|
| | | First day | Second day | Difference in height | Mean | |
| | a | b | c | $d= b -c $ | $e=(b + c)/2$ | $f= a - e $ |
| Sancheong 444 | 690.155 | 689.551 | 689.537 | 0.014 | 689.544 | 0.611 |
| Sancheong 448 | 528.123 | 527.576 | 527.559 | 0.017 | 527.568 | 0.556 |
| Hadong 409 | 1073.264 | 1072.398 | 1072.369 | 0.029 | 1072.384 | 0.880 |

III. Results

At the known points in each site, the analysis results of data received over two days from fixing the Continuously Operating Reference Stations around each area showed that the differences between the ellipsoidal heights on Day 1 and Day 2 were all less than 3cm. In addition, Table 14 shows the mean height values obtained through GNSS/leveling for the two days. Regarding ellipsoidal heights, the differences between the official ellipsoidal heights and the values obtained through GNSS observation(c) were at a modest level in most cases, but the survey results differed from the official values by about 10cm at Hadong 409 and Unbong 26. This is thought to be an acceptable level, considering that the areas are mountainous with elevations of 500m or more. As for orthometric heights, overall heights obtained through GNSS/leveling showed a large difference from the official heights. More specifically, except for Unbong 25, which showed a difference of 48cm, all height values determined through GNSS/leveling differed by 50cm or more from the official values. These results were similar to the initially analyzed differences between the elevations derived from the geoid model and the official heights, showing that the target areas were appropriately selected.

Table 14. Differences between official heights and GNSS/leveling values (ellipsoidal height and orthometric height)

| Point name | Ellipsoidal height | | | Orthometric height | | |
|---------------|--------------------|----------------------|----------------------|--------------------|----------------------|----------------------|
| | Official height | GNSS/leveling height | Difference in height | Official height | GNSS/leveling height | Difference in height |
| | a | b | c= a - b | d | e | f= d - e |
| Unbong 25 | 970.870 | 970.875 | 0.005 | 1,305.379 | 1,304.899 | 0.480 |
| Sancheong 448 | 664.998 | 664.970 | 0.029 | 528.123 | 527.568 | 0.556 |
| Sancheong 444 | 952.199 | 952.235 | 0.036 | 690.155 | 689.544 | 0.611 |
| Hadong 22 | 933.697 | 933.735 | 0.038 | 1,116.157 | 1,115.533 | 0.624 |
| Unbong 305 | 934.845 | 934.807 | 0.038 | 924.904 | 924.222 | 0.682 |
| Hadong 304 | 1,100.675 | 1,100.721 | 0.045 | 906.240 | 905.544 | 0.696 |
| Hadong 21 | 1,529.759 | 1,529.815 | 0.056 | 943.734 | 942.989 | 0.745 |
| Hadong 408 | 1,610.464 | 1,610.534 | 0.070 | 637.697 | 636.950 | 0.747 |
| Unbong 12 | 555.790 | 555.860 | 0.070 | 1,502.894 | 1502.104 | 0.790 |
| Unbong 434 | 1,143.520 | 1,143.595 | 0.075 | 907.772 | 906.912 | 0.860 |
| Hadong 409 | 717.800 | 717.901 | 0.101 | 1,073.264 | 1,072.384 | 0.880 |
| Unbong 26 | 1,332.234 | 1,332.361 | 0.127 | 1,583.399 | 1,582.396 | 1.003 |

IV. Conclusions

Traditionally, triangulation points have been used to determine the boundaries or the area of a piece of land. However, in recent years, as the number of GNSS satellites has increased, and as the functions of GNSS instruments have improved, the utilization of triangulation points has been decreasing gradually. In addition, the use of triangulation points located on the peaks of high mountains during surveying is limited by the very low efficiency of survey operations. Therefore, unified control points in urban areas or near roads are utilized. This study attempts through GNSS/leveling to recompute the heights of triangulation points located on mountain peaks in order to further increase the utility of those triangulation points and to evaluate the applicability of height determination through GNSS/leveling in mountainous areas. GNSS/leveling is a technique that computes ellipsoidal heights based on ellipsoids through GNSS surveying, and determines the elevation by subtracting the geoid height derived from a high-precision geoid model from the ellipsoidal height obtained through GNSS/leveling. When performing such surveying, determination of elevations with reference to the vertical data of known points requires computation of ellipsoidal heights by using geoid height and orthometric height under the assumption that the geoid height calculated from the geoid model and the actual geoid height are the same.

GNSS/leveling was performed for unknown points in and around Jirisan, South Korea, which was set as the target area for this study, and it was found that the height values obtained through GNSS/leveling differed by about 50cm or more from the official heights at all but one of the observation points. Examination of the ellipsoidal heights of unknown points obtained through GNSS/leveling showed that there was not a large difference between an ellipsoidal height derived from GNSS/leveling and the official ellipsoidal height, which indicated that accurate height values can be determined though GNSS/leveling in mountainous areas. In addition regarding the selection of unknown points for analysis, results showed that the selection of triangulation points where the height value derived from the national geoid model differed by 50 cm or more from the official height was appropriate. In addition, as a result of GNSS/leveling at the same point for 2 days in order to consider the geometrical arrangement of the satellites and the atmospheric condition, it was confirmed that the difference in heights was within 3cm.

In this study, islands and coastal areas were excluded from analysis because of unfavorable accessibility, although preliminary analysis showed there were a number of islands and coastal areas with a large difference between the official height and the height derived from a geoid model. Therefore, based on the findings of the present study, future plans include expanding the research on GNSS/leveling to other

triangulation points showing a large height difference in preliminary analysis. In addition, expectations are to apply and utilize the study results through research on height determination in mountainous areas using drones.

Funding: This work was supported by the Technology Innovation Program (20005066, Development of orthoimage generation technology and product standards that can process orthoimages within 8 hours with 5cm resolution for large-scale construction management) funded by the Ministry of Trade, Industry & Energy (MOTIE, Korea).

Acknowledgments: I thank NGII for providing the data used in this work.

References

- [1]. Lachapelle G., Dennler M., Lethaby J., Cannon M.E., (1984), "Special order geodetic operations for a Canadian Pacific railway tunnel in the Canadian Rockies" *Can. Surv.*, Vol. 38, pp. 163–176.
- [2]. Hirt C., Seeber G., (2008), "Accuracy analysis of vertical deflection data observed with the Hannover digital zenith camera system" *TZK2-D. J. Geod.*, Vol. 82, pp. 347–356.
- [3]. Wang Y.M., Becker C., Mader G., Martin D., Li X., Jiang T., Breidenbach S., Geoghegan C., Winester D., Guillaume S., et al., (2017), "The Geoid Slope Validation Survey 2014 & GRAV-D Airborne Gravity Enhanced Geoid Comparison Results in Iowa", *J. Geod.*, Vol. 91, pp. 1261–1276.
- [4]. Jekeli C., (1999), "An analysis of vertical deflections derived from high-degree spherical harmonic models", *J. Geod.*, Vol. 73, pp. 10–22.
- [5]. Hirt C., (2012), "Efficient and accurate high-degree spherical harmonic synthesis of gravity field functionals at the Earth's surface using the gradient approach", *J. Geod.*, Vol. 86, pp. 729–744.
- [6]. Tse C.M., Baki I. H., (2006), "Deflection of the vertical components from GPS and precise leveling measurements in Hong Kong", *J. Surv. Eng.*, Vol. 132, pp. 97–100.
- [7]. Ceylan A., (2010), "Determination of the deflection of vertical components via GPS and leveling measurement: A case study of a GPS test network in Konya, Turkey" *Sci. Res. Essays*, Vol. 4, pp. 1438–1444.
- [8]. Xingfu Z., Yongyi Z., Bofeng L., (2018), "Guangxin Q. GNSS-Based Verticality Monitoring of Super-Tall Buildings", *Appl. Sci.*, Vol. 8, pp. 991.
- [9]. Schwarz K., Sideris M., Forsberg R., (1990), "The Use of FFT Techniques in Physical Geodesy", *Geophys. J. Int.*, Vol. 100, pp. 485–514.
- [10]. Qiong W., Hongyao W., Bin W., Shengbo C., Hongqing L., (2020), "Performance Comparison of Geoid Refinement between XGM2016 and EGM2008 Based on the KTH and RCR Methods: Jilin Province, China, Remote Sens" Vol. 12, pp. 324.
- [11]. Ayhan M.E., (1993), "Geoid determination in Turkey (TG-91)", *Bull. Géodésique*, Vol. 67, pp. 10.
- [12]. Fotopoulos G., Kotsakis C., Sideris M., (2000), "A new Canadian geoid model in support of levelling by GPS", *Geomatica* Vol. 54, pp. 53–62.
- [13]. Zhang K., Featherstone W., Stewart M., Dodson A., (1998), "A new gravimetric geoid of Australia. In Proceedings of the Second Continental Workshop on the Geoid in Europe, Budapest, Hungary", Vol. 4, pp. 225–234.
- [14]. Smith D.A., Milbert D.G., (1999), "The GEOID96 high-resolution geoid height model for the United States", *J. Geod.*, Vol. 73, pp. 219–236.
- [15]. Caleb I.Y., Vagner G.F., Cosmas Y.A., (2017), "Towards the Selection of an Optimal Global Geopotential Model for the Computation of the Long-Wavelength Contribution: A Case Study", *Geosci.*, Vol. 7, pp. 113.
- [16]. Odera P.A., Fukuda Y., Kuroishi Y., (2012), "A high-resolution gravimetric geoid model for Japan from EGM2008 and local gravity data", *Earth Planets Space*, Vol. 64, pp. 361–368.
- [17]. Hofmann-Wellenhof B., Moritz H., (2006), "Physical Geodesy, 2nd ed.; Springer: Berlin, Germany", pp. 413.
- [18]. Krystian K., Kamil M., (2020), "New Heights of the Highest Peaks of Polish Mountain Ranges. Remote Sens" Vol. 12, pp. 1446.
- [19]. Ward M., (1995), "The height of Mount Everest", *Alp. J.*, Vol. 100, pp. 30–33.
- [20]. Junyong C., Yanping Z., Janli Y., Chunxi G., Peng Z., (2010), "Height Determination of Qomolangma Feng (MT. Everest) in 2005", *Surv. Rev.*, Vol. 42, pp. 122–131.
- [21]. De Graaff-Hunter J., (1955), "Various Determinations over a Century of the Height of Mount Everest", *Geogr. J.*, Vol. 121, pp. 21–26.
- [22]. De Beer G., (1956), "The history of the altimetry of Mont Blanc", *Ann. Sci.*, Vol. 12, pp. 3–29.
- [23]. Poretti G.G., (1999), "America's highest peak now measures 6962 metres!", *Report. Mag. Leica Geosystems*, Vol. 47, pp. 28–29.
- [24]. Saburi J., Angelakis N., Jaeger R., Illner M., Jackson P., Pugh K.T., (2000), "Height measurement of Kilimanjaro", *Surv. Rev.*, Vol. 35, pp. 552–562.
- [25]. Team KIL12008., (2009), "Precise Determination of the Orthometric Height of Mt Kilimanjaro", In FIG Working Week 2009; FIG: Eilat, Israel, 2009, pp. 11.
- [26]. Hui Q., Shuanggen J., (2020), "Global Mean Sea Surface Height Estimated from Spaceborne Cyclone-GNSS Reflectometry", *Remote Sens.*, Vol. 12, pp. 356.
- [27]. Hayden T., Amjadi Parvar B., Rangelova E., Sideris M.G., (2012), "Estimating Canadian vertical datum offsets using GNSS/levelling benchmark information and GOCE global geopotential models", *J. Geod.*, Vol. 10, pp. 257–269.
- [28]. Albertella A., Barzaghi R., Carrion D., Maggi A., (2008), "The joint use of gravity data and GPS/levelling undulations in geoid estimation procedures", *Bollettino di Geodesia e Scienze Affini*, Vol. 1, pp. 47–57.
- [29]. Chen Y.Q., Luo Z., (2004), "A hybrid method to determine a local geoid model-Case study", *Earth Planets Space*, Vol. 56, pp. 419–427.
- [30]. Alessandra B., Riccardo B., Omar A.B., Suhail A.M., (2020), "Centimeter Precision Geoid Model for Jeddah Region (Saudi Arabia)", *Remote Sens.*, Vol. 12, pp. 2066.
- [31]. Dinh T.V., Sean B., Sylvain B., Dominique R., Georgios S.V., (2020), "A Quasigeoid-Derived Transformation Model Accounting for Land Subsidence in the Mekong Delta towards Height System Unification in Vietnam", *Remote Sens.*, Vol. 12, pp. 817.
- [32]. Lee J.S., Kwon J.H., Keum Y.M., Moon J.Y., (2011), "The Update of Korean Geoid Model based on Newly Obtained Gravity Data", *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 29, No. 1, pp. 81–89.
- [33]. Lee J.S., Kwon J.H., Keum Y.M., Moon J.Y., (2012), "Development of Korean Geoid Model and Verification of its Precision", *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 30, No. 5, pp. 493–500.

- [34]. Kim K.B., Yun H.S., Choi H.J.,(2020), “Accuracy Evaluation of Geoid Heights in the National Control Points of South Korea Using High-Degree Geopotential Model”, *Appl. sci.*, Vol. 10, pp.1466.
- [35]. Lee J.S., Kwon J.H., (2020), “Precision Evaluation of Recent Global Geopotential Models based on GNSS/Leveling Data on Unified Control Points”, *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 38, No. 2, pp. 153-163.
- [36]. Lee S.B., (2018), “Determination of Precise Regional Geoid Heights on and around Mount Jiri, South Korea”, *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 36, No. 1, pp. 9-15.
- [37]. Jung S.C., Kwon J.H., Lee, J.S., (2018), “Accuracy Analysis of GNSS-derived Orthometric Heights on the Leveling Loop Disconnected Area.”, *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 36, No. 1, pp. 1-8.
- [38]. Shin G.S., Han J.H., Kwon, J.H., (2014), “Accuracy Analysis of Orthometric Heights Based on GNSS Static Surveying”, *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 32, No. 5, pp. 527-537.
- [39]. Jung S.C., Kwon J. H., Lee, J.S., (2018), “Accuracy Analysis of GNSS-derived Orthometric Heights in the Mountainous Area”, *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 36, No. 1, pp. 81-84.
- [40]. National Geographic Information Institute: Suwon, Korea, (2019), “NGII. Promote Upcycling and Re-observation of Height to Strengthen the Use of Triangulation Point; NGII Report; Publication”, No. 11-1613436-000215-01
- [41]. NGII. National Geographic Information Institute. Available online: http://map.ngii.go.kr/ms/mesrInfo/geoidIntro.do#tab_5 (accessed on 3 May 2019).

Kyedong Lee. “Orthometric Height Evaluation Considering the Geometrical Arrangement of Satellites and Atmospheric Condition.” *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)*, 17(6), 2020, pp. 56-71.