IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 19, Issue 1 Ser. II (Jan. – Feb. 2022), PP 01-16
www.iosrjournals.org

Increasing the Accuracy of the Range Corrections for the Air Virtual Temperature

Vladimir Cech[1], Ondrej Rozehnal[2]
[1](*Department of Weapons and Ammunition, University of Defence, Kounicova 65, 662 10 Brno, Czech Republic*)
[2](*Department of Weapons and Ammunition, University of Defence, Kounicova 65, 662 10 Brno, Czech Republic*)

Abstract: *If for any reason it is not possible to use an automated fire control system, then especially in combat conditions it is necessary to use an alternative method of calculating the mean point of impact position. To do this, tabular firing tables (TFT) are used together with the corresponding type of artillery meteorological message. If TFTs created according to STANAG 4119 are used, only the met message METB3Q must be used for surface-to-surface fire. The calculation of corrections for deviations in virtual temperature is very problematic. We have been dealing with the problem since 2013. In this article, we propose a solution that increases the accuracy of calculations and is applicable in combat conditions. However, the main advantage of the proposed method is the fact that met data can be obtained from any available met message (METB3Q, Meteo - 11 (meteo - average), METCMQ, METGM and GRIB).*

Keywords*: Exterior ballistic; sensitivity analysis; non-standard (perturbed) projectile trajectory; Green's function; weighting factor function; effect function; norm effect; corrections for virtual temperature deviations; FIR filter*

Date of Submission: 01-01-2022 Date of Acceptance: 12-01-2022

I. Introduction

1.1 *Motivation*

We have been studying the same theme since 2013 during which our motivation has not change: It follows from the analysis of artillery fire errors, e.g. [1], that approximately two-thirds of the inaccuracy of indirect artillery fire is caused by inaccuracies in the determination of meteo parameters included in the error budget model. Consequently, it is always important to pay close attention to the problems of including the actual meteo parameters in ballistic calculations. The following *meteo parameters* μ are primarily utilized: Wind vector $\boldsymbol{w}$, air virtual temperature T_v, air pressure p, air density ρ and speed of sound a.

This paper only deals with problems relating to *unguided projectiles (including Base Burn projectiles) without propulsion system* for the sake of clarity of the solved problems. We analyze the problem of only one meteorological parameter, namely the virtual air temperature T_v..

1.2 *References*

We have gradually published a summary of the necessary theory in materials [2 - 13], using all the available literature.

In the article we focus on the issue, which is defined by definitions and requirements for creating tabular firing tables - TFTs according to STANAG 4119 [14]. The basis of this STANAG are procedures for creating US TFTs. This type of TFTs was introduced in the United States in the 1930s.

The procedure for obtaining experimental inputs for the creation of these TFTs is specified primarily in STANAG 4144 [15]. The required procedure for calculating projectile trajectories is defined in STANAG 4355 [16]. Only [17] the ICAO Standard Atmosphere [18, 19] must be used in the calculations.

The measured met data must be sent and used only in the form of a meteorological (met) message METBKQ [20] for "surface-to-surface" fire (K = 3).

In this article we are still working with other met messages: METCMQ [21] and METGM [22]. In 2019, STANAGs [17, 20 - 22] were canceled and their original content was embodied in the new edition 5 of STANAG 6015 [23]. We also refer to the met message of the World Meteorological Organization GRIB [24].

Met message Meteo - 11 (meteo - average) [25, 26] was introduced in the former Soviet Union in 1956. It was and continues to be used in the countries of the former Soviet bloc and in countries to which artillery weapons from the Soviet bloc were supplied, especially China, India, Vietnam and many other Asian, American and African countries. The use of this met message is therefore still important and current.

In Europe, another type of TFTs (Old European TFTs) was used, which was gradually replaced by TFTs according to STANAG 4119 [14]. In the countries of the former Soviet bloc, Soviet TFTs were and are used. They are modified Old European TFTs [25, 26] and since 1956 they have been supplemented by the reference height of trajectory Y_R [2, 7, 25, 26]. The values of reference height of trajectory (RHT) are required to read met data from the Meteo -11 met message.

1.3. *The main objectives of the contribution*

Three basic groups of methods are used to determine the artillery fire-control solution (Quadrant elevation, bearing (azimuth) and fuze setting):
- basic procedures,
- spare (alternative, substitute) procedures and
- emergency procedures.

Basic procedures (methods) have been based on the direct numerical integration of differential equations that describe the trajectory of a projectile since the 1960s. Calculations are also performed in field conditions on a digital ballistic computer, which forms the core of the automated Fire Control System - FCS. Met data for calculation are supplied in the form of suitable met messages, for example METCMQ [21, 23], METGM [22, 23] and Meteo-11 [25, 26].

If the ballistic computer is not available, which is typical of underdeveloped countries, or is damaged, which is common in combat conditions, it is necessary to use *spare (alternative) procedures*. It is therefore clear that the use of these methods is very common even today.

The basis of these methods is the use of TFTs together with the corresponding met message. Currently, there are the two most common combinations:

- TFTs defined using STANAG 4119 together with the METB3Q met message,
- Soviet format of TFTs together with met message Meteo - 11 (meteo - average).

In the case of both groups of procedures mentioned above, the corresponding met messages are required not to be too outdated, so, their staleness is acceptable.

If the staleness of met messages is unacceptable, one of the methods of the third group of procedures (*emergency procedures*) must be used. These methods are always based on experimental firing designed to obtain replacement met data.

In the article we will deal with one method of the second group, namely the combination of TFTs according to STANAG 4119 and the METB3Q met message.

Based on analyzes performed in previous years, we found that the method of data preparation for the METB3Q assembly is very inappropriate.

We first published [2 – 4] an analysis of the problem for calculating the range and cross ballistic winds. We have shown that it is far more advantageous to use a procedure based on the use of a reference height of trajectory (RHT) together with a met message Meteo-11 or METCMQ [2, 3, 7, 8].

Further analysis showed that a similar situation applies to the calculation of ballistic air density ρ_B, more precisely its relative deviation $\delta\rho_B$ from the standard course of air density ρ_{STD}.

However, the most problematic is the calculation of the ballistic virtual temperature $T_{v,B}$, more precisely its relative deviation $\delta T_{v,B}$ from the standard course of the virtual temperature $T_{v,STD}$. The problem is further complicated by the common occurrence of the so-called "norm effect". The norm effect has been known for about 100 years [27]. The proposal for its compensation published so far [28] has not been applied in practice.

In order to successfully solve the problem, we had to create the necessary theoretical foundations. Gradually, we published an improved theory of the generalized meteo-ballistic weighting factor functions [5, 6, 9, 10] and an improved theory of the projectile trajectory reference height Y_{CR} [7 - 10].

In this paper, we use our improved theories to design a way

- to suppress the effects of the norm effect on the mean point of impact (MPI) correction accuracy and
- *to replace data from a completely inappropriate met message METB3Q by data from any other met message*. In particular, the data reported in METCMQ and Meteo-11, as well as data from METGM and GRIB, are suitable.

The results of the analysis are formulated into two alternative proposals for the content of annexes for current TFTs according to STANAG 4119. The second proposal is more advantageous in terms of its practical use.

The proposed modifications of TFTs can be easily modified not only for Rocket Assisted Projectiles and, unguided rockets but also for illuminating and cargo projectiles and for the passive section of projectile trajectory with terminal guidance.

1.4 *Overview of basic formulae*

The area of mathematical theory we use is based on a combination of perturbation theory [29] and Green's functions theory [30].

This area of mathematical theory is commonly used in automatic control [31 - 33] and in many of its applications. An application in the theory of aerospace vehicle control is important to us [34]. In addition, in the theory of automatic control, this theory is combined with differential sensitivity analyses [35], for example [31, 32]. The following names are most often used for this theory: *differential sensitivity analysis of the control system* or *sensitivity of a system to parameter variations*.

In exterior ballistics, this theory began to be used during the second half of the First World War, i.e. from about 1916. However, the results were only published after the end of the war, for example [36]. This theory is traditionally referred to as the *theory of trajectory (differential) corrections*.

The theory was commonly used until about the first half of the 1960s [28, 37 - 40]. With the advent of digital computers, it was completely illogically abandoned, unlike most technical fields, in particular the theory of flight control of aircraft and missiles. *At present, it is a completely forgotten theory for most exterior ballisticians*. Exceptions are those who also deal with the theory of missile flight control. A more detailed analysis of the causes of the situation is given in [5].

Gradually, other terminology was introduced in mathematics, automatic control and exterior ballistics. Table 1 compares the names for the most important terms.

Table 1

generally	δT_v	ballistics	automatic control	mathematics
Q_p	ΔX	effect function (EF)	step response function (SRF)	no name
G	G	no name or GsF	impulse response function (IRF)	Green's function (GsF)
r	r	weighting factor function (WFF)	normed SRF	no name
g	g	no name or NGsF	normed IRF	normed GsF (NGsF)

We will clarify the used procedures and formulae using a specific numerical example in order to achieve the greatest possible clarity.

We chose the following parameters of the ballistic system: caliber d = 155 mm, mass of the projectile m_p = 43.2 kg, ballistic coefficient c = 0.563 m^2/kg (Soviet drag model of 1943, Fig. 1), initial (muzzle) velocity v_0 = 850 m/s. The shape of the projectile is very similar to the shape of the M107 projectile. The altitude of the gun muzzle is h_G = 0 m. The range X and summit (vertex) height y_S versus departure angles Θ_0 is shown in Fig. 2, 3.

There are explicit and implicit calculation algorithms [5]. Green's function $G(t_p, t_{PI})$ is calculated directly using explicit algorithms [28, 33, 36, 37, 39, 40]. These algorithms are illustrative, but less suitable for use on a digital computer [5,6].

The effect function $Q_p(t_p, t_{PI})$ is calculated directly using implicit algorithms [38]. These algorithms are very convenient for use on a digital computer, so we use them. A detailed analysis of this issue is given in [5, 6].

All functions $Q_p(t_p, t_{PI})$, $G(t_p, t_{PI})$, $r(t_p, t_{PI})$, and $g(t_p, t_{PI})$ (Table 1) have two independent variables t_{PI} and t_p [5, 6, 25, 28, 36 - 40].

Time t_{PI} is the flight time to the point of impact PI for a non-perturbed (standard) trajectory. For surface-to-surface fire, PI is the same as the point of fall ($y_F = 0$). Therefore, $t_{PI} = t_F$. The value of t_F is given in TFTs.

The time $t_p \in \langle 0, t_{PI} \rangle$ is the moment of the beginning of the perturbation of the projectile trajectory. In the time interval $t \in \langle 0, t_p \rangle$ the trajectory is non-perturbed (standard) and in the time interval $t \in \langle t_p, t_{PI} \rangle$ the trajectory is perturbed. The vertical coordinate of the trajectory y = 0 only occurs for the perturbed trajectory at time $t = t_{end}$ [5, 9].

In our case, the perturbation parameter is the relative deviation of the virtual temperature δT_v. We chose $\delta T_{v,P}$ = + 0.1 = const (10%) for the calculations.

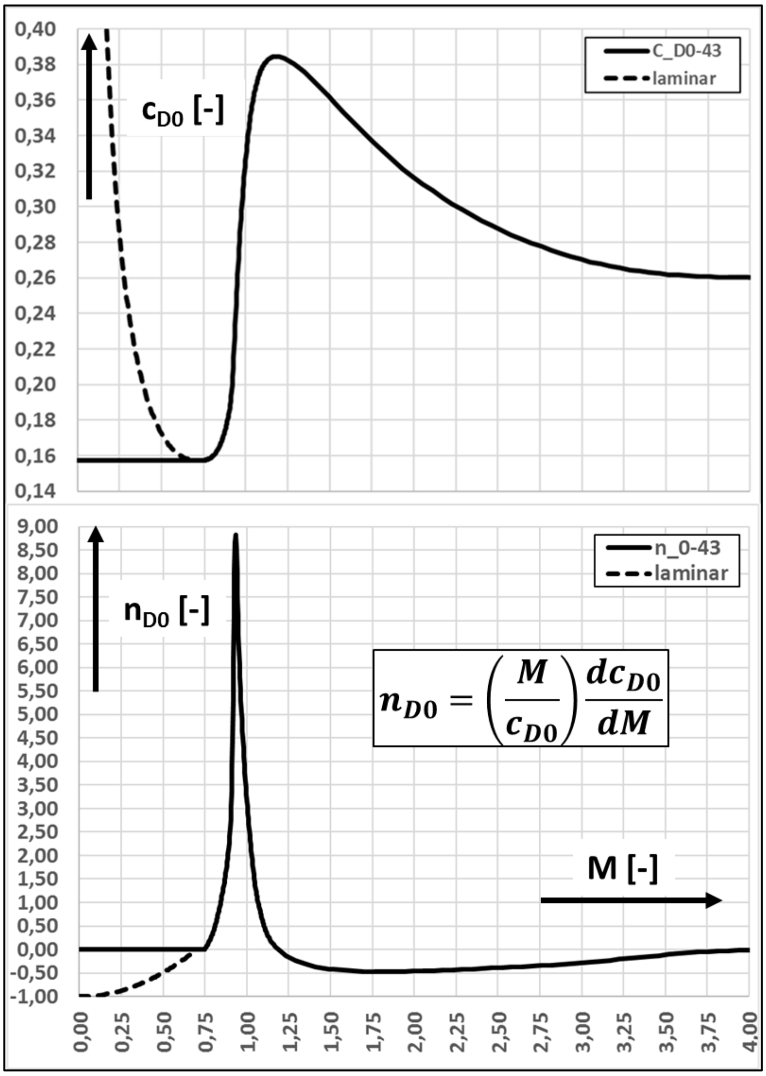

Fig. 1 Soviet drag model of 1943 and its fully normed sensitivity coefficient n_{D0} [26]

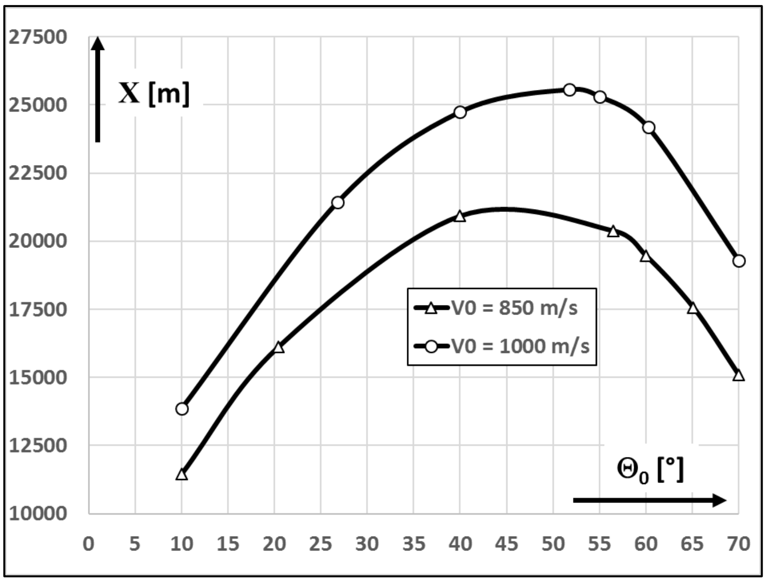

Fig. 2 Range X versus angle of departure Θ_0.

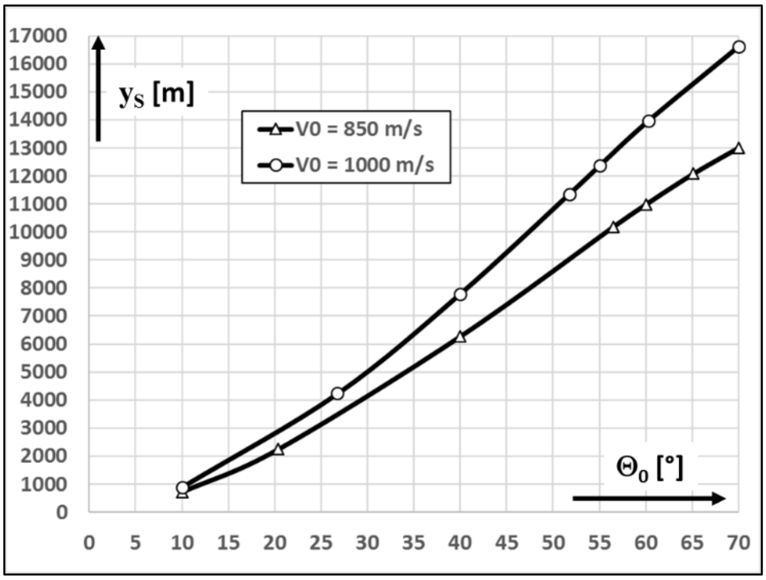

Fig. 3 Summit (vertex) height y_S versus angle of departure Θ_0.

The effect function Q_p is the perturbation of the range $\Delta X(t_p, t_{PI})$ in this case, where

$$\Delta X(t_p, t_{PI}) = X(t_p, t_{PI}) - X_{STD},$$

- X_{STD} is the range for the non-perturbed (standard) trajectory – Fig. 2 and
- $X(t_p, t_{PI})$ for the perturbed trajectory.

The Green's function $G(t_p, t_{PI})$ is obtained by numerical derivation of the corresponding effect function $Q_p(t_p, t_{PI})$ [5, 9, 31 - 33]

$$G\left(t_p, t_{PI}\right) = -\frac{dQ_p(t_p,t_{PI})}{t_p} = -\frac{d\Delta X(t_p,t_{PI})}{t_p} . \tag{1}$$

Examples of calculated effect functions $\Delta X(t_p, t_{PI}) = \Delta X$ are shown in Fig. 4 (time t – domain).

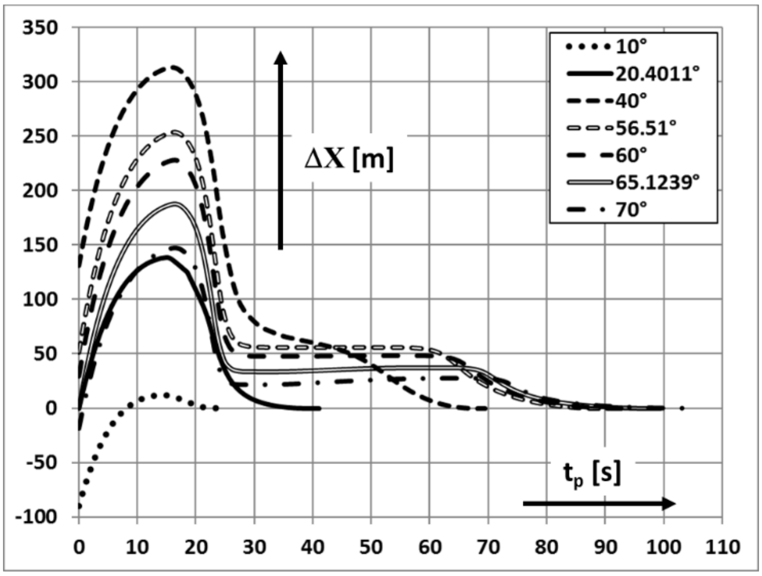

Fig. 4 Examples of effect functions $\Delta X(t_p, t_{PI}) = \Delta X$ for 7 values angle of departure Θ_0 and $\delta T_{v,P} = 0.1$
(t – domain)

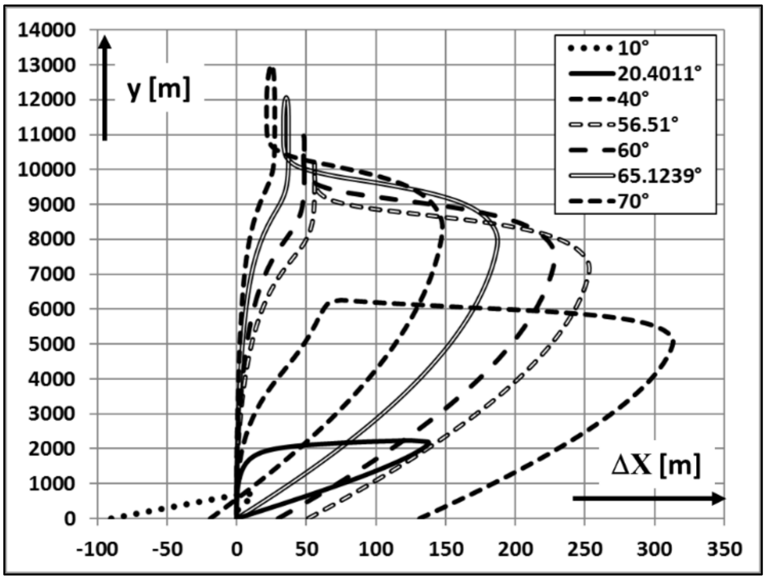

Fig. 5 Two branched effect functions (y – domain)

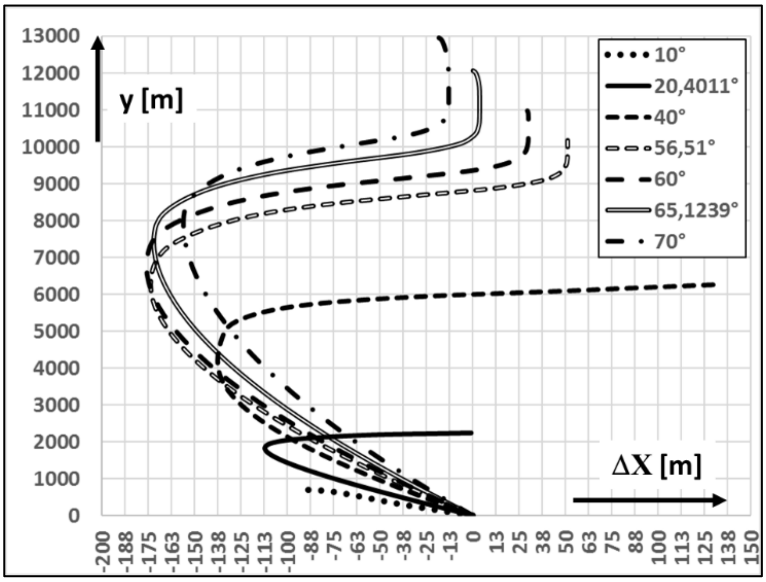

Fig. 6 Effect functions in y – domain (Bliss' notation)

In practice, we work with effect functions ΔX, which are expressed as the vertical coordinate of trajectory y functions (y - coordinate domain). To calculate them, it is first necessary to calculate two branched effect functions [4, 25, 28, 37, 38, 40] - Fig. 5.

Effect functions in the y - domain can be calculated in two ways, which we call Garnier's notation [4, 38] and Bliss' notation [1 – 13, 25 – 28, 36, 37, 40]. In practice, the use of Bliss' notation prevails - Fig. 6. In this case, a formula similar to formula (1) applies, but with a +1 sign [5, 9].

Normalized function curves are used for calculations: r - weighting factor functions - WFFs and g - normed Green's functions.

We use relative time

$$\tau = \frac{t_p}{t_{PI}} \quad (2)$$

and relative vertical coordinates

$$\eta = \frac{h - h_{min}}{h_{max} - h_{min}} = \frac{y - y_{min}}{y_{max} - y_{min}} \ , \quad (3)$$

where

$h = h_G + y$ - instantaneous altitude of the projectile [m]

h_G - altitude of the muzzle of the gun barrel (beginning of the ballistic coordinate system, $x = 0, y = 0, z = 0$)
y - instantaneous height of the projectile trajectory
$h_{min} = \min(h)$ - the minimum altitude of the projectile between the beginning of the trajectory and the point of impact/burst (h_{PI})
$h_{max} = \max(h)$ - the maximum altitude of the projectile between the beginning of the trajectory and the point of impact/burst (h_{PI})

In the following, we will assume that the point of impact PI is the same as the point of fall ($y_F = 0$ m), then it holds that

$$\eta = \frac{y}{y_S}\,, \tag{4}$$

$y_S = Y_S$ - vertical coordinate of the summit (vertex) of the trajectory.

We use the following relationships to calculate WFFs ($s = \tau$ or η)

$$r(s) = \frac{Q_p(s)}{Q_{p,N,s}} = \frac{\Delta X(s)}{\Delta X_{N,s}}\,, \tag{5}$$

and normed Green's functions

$$g(s) = \frac{G(s)}{Q_{p,N,s}} = \frac{G(s)}{\Delta X_{N,s}}\,, \tag{6}$$

where

$Q_{p,N,s} = \Delta X_{N,s} = \sigma_{N,s} \cdot N_s$ - generalized norm (standardization value, scale value)
$\sigma_{N,s} = +/- 1$ - a sign chosen to hold $r(\tau) > 0$ for $\tau = 0$ ($r(0) > 0$) or $r(\eta) > 0$ for $\eta = 1$ ($r(1) > 0$). In the case of the exact norm effect, ie if $r(\tau) = 0$ for $\tau = 0$ or $r(\eta) = 0$ for $\eta = 1$ applies, then the choice of the sign is a contractual matter [5, 9].
N_s – norm defined according to mathematical rules ($N_s > 0$) [5, 7, 9]

$$N_s = \max_s(Q_p(s)) - \min_s\left(Q_p(s)\right) = \max_s(\Delta X(s)) - \min_s(\Delta X(s))\,.$$
(7)

A (unit) correction factor is published in TFTs ($r(1) = r(\eta)$ for $\eta = 1$) [5, 7, 9, 14]

$$\Delta X_{T,1} = -\Delta X_{N,\eta} \cdot \left(\frac{\delta T_{v,N}}{\delta T_{v,p}}\right) \cdot r(1), \tag{8}$$

where

$\delta T_{v,p}$ - the value δT_v with which $\Delta X_{N,\eta}$ was calculated. In our example $\delta T_{v,p} = 0.1$.
$\delta T_{v,N}$ - the value δT_v defined in the TFTs [14]. In our case $\delta T_{v,N} = 0.01$ (1 %).

The correction for the virtual temperature δT_v is calculated from the relationship [2, 5, 28, 37]

$$\Delta X_T = \Delta X_{T,1} \cdot \left(\frac{\delta T_{v,B}}{\delta T_{v,N}}\right)\,, \tag{9}$$

where

$\delta T_{v,B}$ - is the ballistic value of the relative deviation of the virtual temperature δT_v [2, 5, 7, 9, 25, 26, 28, 37, 38, 40]

$$\delta T_{v,B} = \int_0^1 \delta T_v(\eta) \cdot g_\eta(\eta) \cdot d\eta = \int_0^1 \delta T_v(\tau) \cdot g_\tau(\tau) \cdot d\tau. \tag{10}$$

This Duhamel's convolutional integral has three possible interpretations:

- in a purely mathematical sense, it is a solution of a given differential equation at a given time $t = t_{PI}$,
- it is usually interpreted as a formula for calculating the weighted average. To calculate the arithmetic average, $g = 1$ is chosen.
- formula describing the operation of a *Low-pass digital Finite Impulse Response (FIR) filter* [11]. It is therefore an application of signal processing procedures in exterior ballistics. This interpretation makes it possible to correctly understand the true nature of the so-called *ballistic values of individual met parameters μ*. These are therefore the values at the output of the assigned digital FIR filter. Green's function is then referred to as the *impulse response function of the FIR filter*.

In fact, we work with discretized functions [2, 5, 25, 37, 38]

$$T_{v,B} \approx \sum_{j=1}^{n} \delta T_{v,j} \cdot q_j(n), \tag{11}$$

where
$q_j(n) = r_j - r_{j-1} = r(\eta_j) - r(\eta_{j-1})$, $j = 1, 2, \ldots, n$, $r_0 = 0$, $r_n = r(1) \in \langle 0, 1\rangle$ - weighting factors -WFs
$\delta T_{v,j}$ - the average value on the interval $\eta \in \langle \eta_{j-1}, \eta_j\rangle$, $\eta_0 = 0$, $\eta_n = 1$.

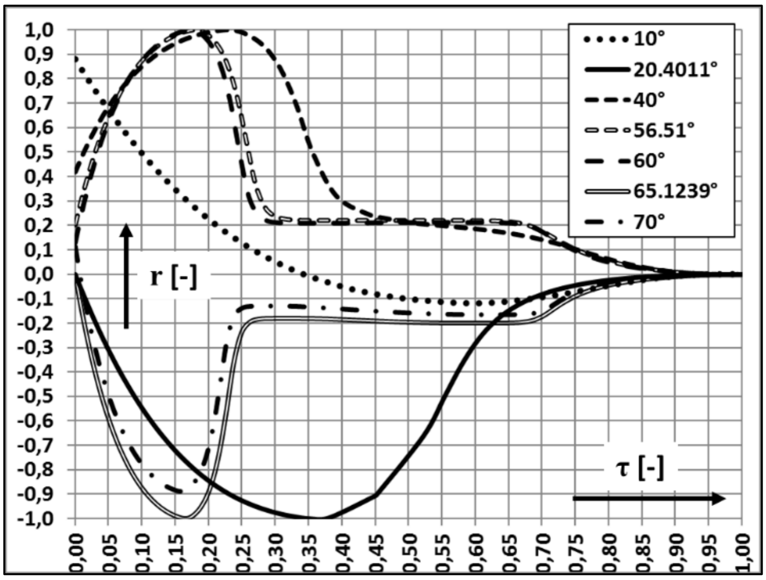

Fig. 7 Weighting factor functions - WFF for t - domain

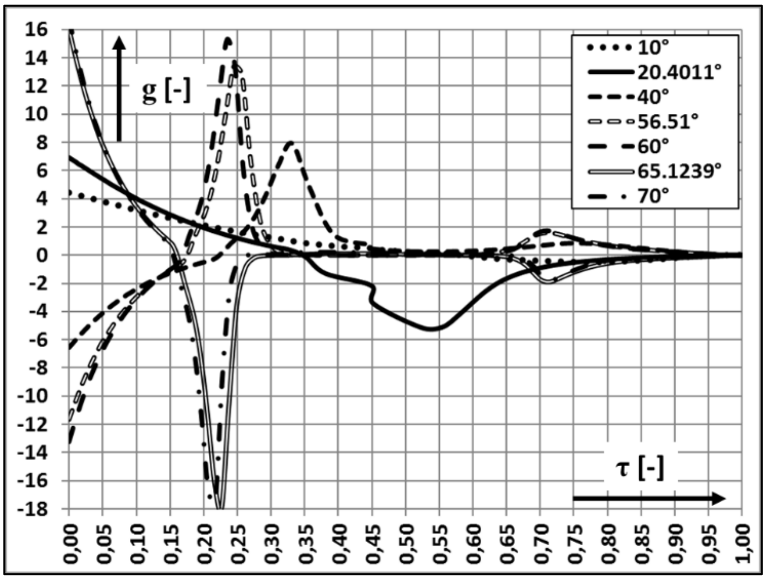

Fig. 8 Normed Green's functions for t - domain

For example, the values of weighting factors $q_j(n)$ are tabulated in STANAG 4061 [20] and 6015 [23]. The values of $\delta T_{v,B}$ are given in METBKQ for K = 2, 3

The courses of WFFs and normed Green's functions are shown in Figs. 7 – 10.

From the analysis of normed Green's functions courses in the time domain Fig. 8 it is clear that they are strongly influenced by the course of the fully normed sensitivity coefficient n_{D0} Fig. 1. This conclusion is confirmed by a theoretical analysis [40].

It follows that not only the zero - drag coefficient c_{D0} but also n_{D0} must be accurately approximated, otherwise the Green's function estimates will be very inaccurate. In general, this applies to approximations of all aerodynamic coefficients $c_j(M)$ and their derivatives. Most published approximations of the relevant data do not meet this requirement at all. This is one of the obstacles to using models with 4 or more degrees of freedom.

Also note that in the past, instead of n_{D0}, the coefficient $n = n_{D0} + 2$ was used, which is referred to as "Mayevski n" [28] or "degree of air resistance" [38, 40, 41].

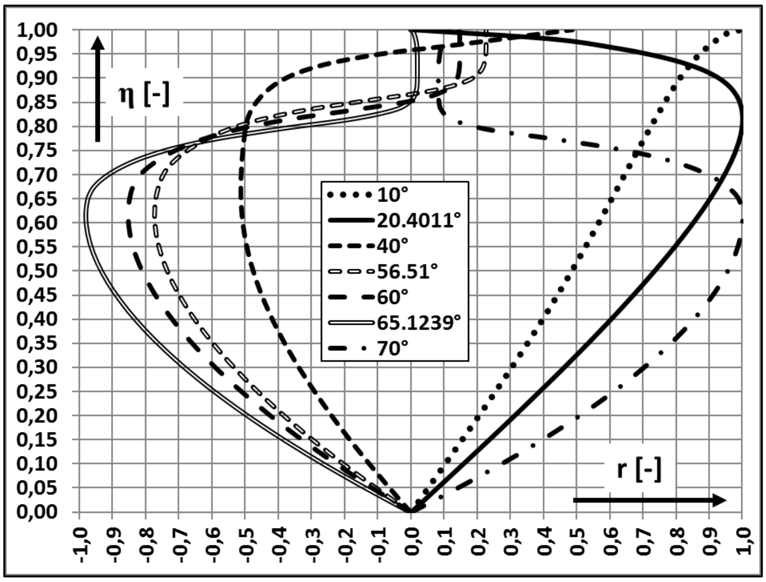

Fig. 9 Weighting factor functions - WFF for y – domain

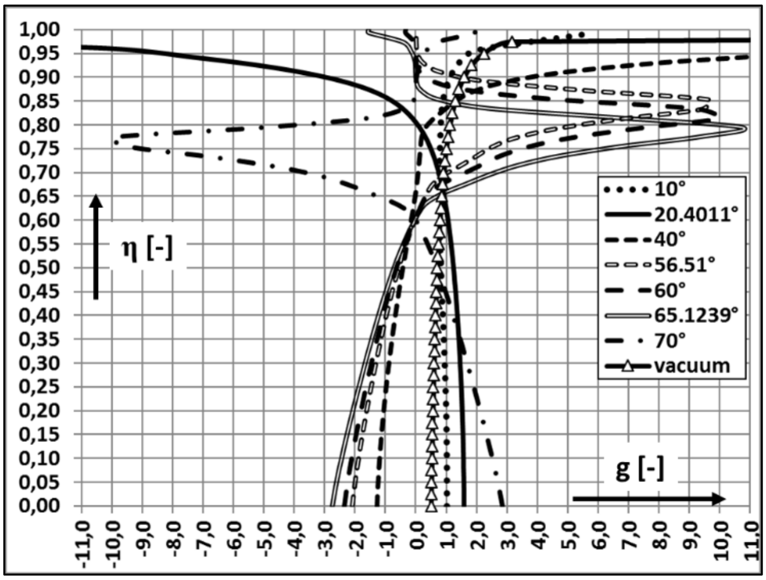

Fig. 10 Normed Green's functions for y - domain

For the WFF approximation, which was already known in the period before the First World War, the following relation applies [2, 5, 25 -28, 37, 38, 40]

$$r(\eta) \approx 1 - \sqrt{1-\eta} \quad . \tag{12}$$

This WFF approximation is commonly referred to as the Vacuum function (curve) or the Time function (curve) in the literature. The course of the derivation of this function (Vacuum Green's function) is presented for comparison in Fig. 10.

II. Problem analysis and recommendations

In this paragraph, we will use the following identity r(1) = r(η) for η = 1.

2.1 *Meteorological message METB3Q*

In the past [2, 4], we have reconstructed the WFFs for the height zones zz = 1, 2, ..., 15, from which the weighting factors $q_j(zz)$, j = 1, 2, …,zz (formula (12)) used in the calculations of ballistic values $\delta T_{v,B}(zz)$ were calculated and which are then presented in the METB3Q met message [20, 23].

Selected WFFs are shown in Fig. 11, where y(zz) is the elevation of the upper edge of the zz zone above the Meteorological Datum Plane - MDP, whose altitude is h_{MDP}. The value y(zz) is interpreted as the summit (vertex) height y_S corresponding to the projectile trajectory. In the following, for simplicity, we assume that h_{MDP} = 0 m. The corresponding normed Green's functions are shown in Fig. 12.

If we compare the WFFs from Fig. 11 with the WFFs of our example Fig. 9 with the aid of Figs. 3, 5, it is obvious that they do not resemble each other at all, not even qualitatively. A similar comparison of the normed Green's functions shown in Fig. 12 and Fig. 10.

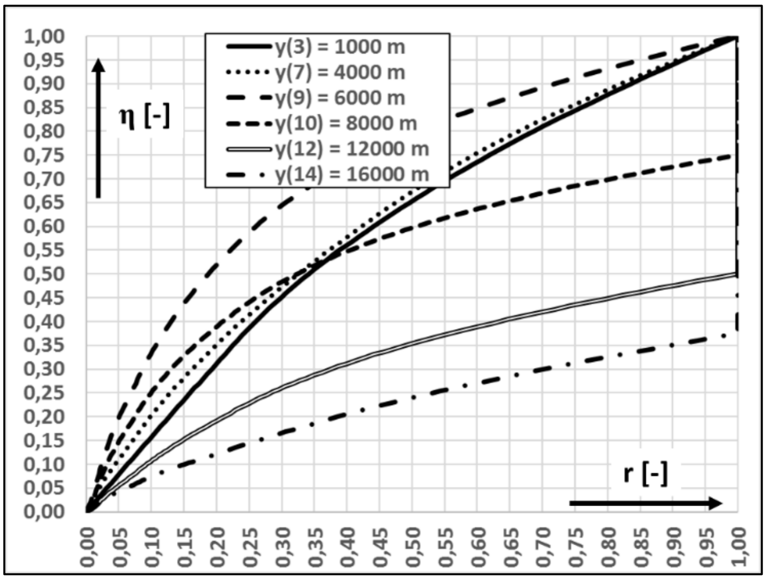

Fig. 11 Examples of WFFs from which weighting factors are calculated, which are used to calculate ballistic virtual temperature for met message METB3Q

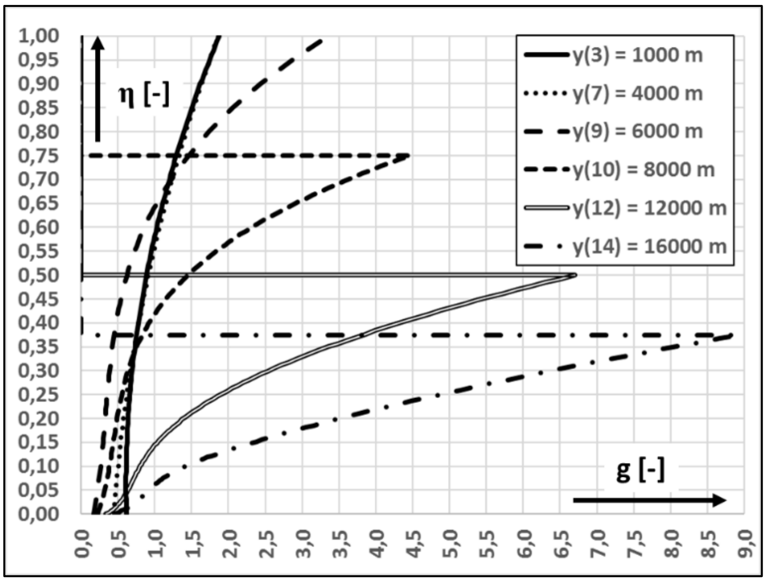

Fig. 12 Normed Green's functions, which correspond to the respective WFFs in Fig. 11.

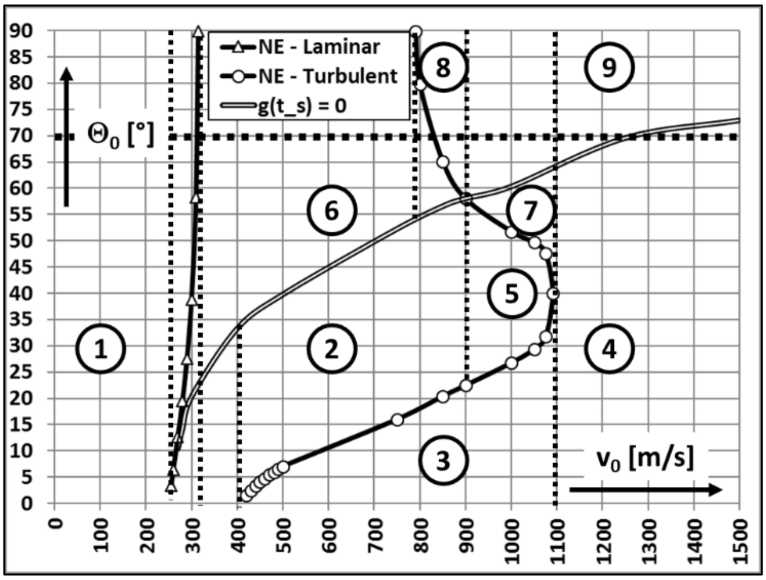

Fig. 13 Plane division (155 mm projectile): initial (muzzle) velocity v_0 - angle of departure Θ_0 to areas with similar course of WFFs and normed Green's functions

Based on the previous conclusions, we extended our example and calculated everything (Fig. 4 -10) for the range of initial (muzzle) velocities 200 to 1500 m/s. The basic findings are summarized in the graph of Fig. 13. The plane: initial (muzzle) velocity v_0 - the angle of departure Θ_0 is divided by three curves into nine basic areas. Some areas are further divided into sub-areas. In each sub-area, WFFs and normed Green's functions have a similar shape.

We also calculated analogous graphs for 82, 105 and 122 mm calibers. Their shapes are very similar to each other, so the following statement can be considered sufficiently general.

We will now proceed to the analysis of the graph in Fig. 13. The curve marked "*NE - Turbulent*" is the most important. There is an exact norm effect ($r_\tau (0) = r_\eta (1) = 0$). In our numerical example (v_0 = 850 m/s) the exact norm effect occurs twice for Θ_0 = 20.4011° and Θ_0 = 65.1239°. For example, for v_0 = 1000 m/s, Θ_0 = 26.7598° and Θ_0 = 51.7695°.

We tried to approximately model the transition of turbulent flow to laminar flow Fig.1. As a result, a second exact norm effect curve is created, marked "*NE - Laminar*" - Fig.13.

The last division curve is marked "$g(t_s) = 0$". This is the value of the Green's function in the t - domain for $t_p = t_S$, where t_S is the moment at which the projectile is at the summit (vertex) of the trajectory. This curve has two properties:

- In the area above this curve, a typical loop is on the two branched effect curves Fig. 5, which then manifests itself in a typical course of WFFs for large values of y and η, Fig. 6, 9.
- If $g(t_s) = 0$, then $g(\eta) = 0$ for $\eta = 1$, Fig. 10. In addition, this curve separates the two divergence areas of Green's functions. For example, if $g(\eta) \to \infty$ for $\eta \to 1$ holds in the area below the curve, then $g(\eta) \to -\infty$ for $\eta \to 1$ must hold in the area above the curve, and vice versa, Fig. 10.

The second mentioned property of this curve leads to great numerical difficulties therefore the graph in Fig. 10 only counts for $\eta \leq 0.95 – 0.97$. A detailed analysis of the problem is given in [9].

In the numerical example (v_0 = 850 m/s) the point of this third curve has a value Θ_0 = 56.51° and therefore lies between the points corresponding to the exact norm effect. If v_0 = 1000 m/s, Θ_0 = 60.25° and this point lies above both points for the exact norm effect - Fig. 13.

Further curves (Θ_0 = 10°, 40°, 60° and 70°) are shown in Figs. 7 - 10 to illustrate courses in areas 2, 3, 6 and 8 - Fig. 13.

Finally, three more remarks on Fig. 13:

- The graph is realistic only up to an angle of departure Θ_0 of approx. 70°. For larger angles of departure, the flight of the projectile is usually unstable.
- We have simplified the graph to make it clear. There is at least one more $g(t_s) = 0$ - curve that has only the second of the properties described.
- Another dividing curve exists for large initial (muzzle) velocities v_0 (above about 1200 m/s) and very large angles of departures Θ_0. Above this curve, the two branched effect curves have not one, but two superimposed loops - Fig. 5.

Partial conclusions:

1. Based on the analysis, it can be stated that the weighting factors $q_j(zz)$, i.e. discrete values of digital FIR filters, for calculating [20, 23] the ballistic values of the relative deviation of the virtual air temperature $\delta T_{v,B}$ do not even qualitatively coincide with the values calculated by us. Qualitative agreement exists for small angles of departure ($\Theta_0 \leq$ cc. 10°). As a result, the values ($\delta T_{v,B}$ +1) given in the METB3Q met message can be expected to be extremely inaccurate under usual met situations.
2. Based on the analysis performed in [2], it can be stated that the weighting factors $q_j(zz)$, i.e. discrete values of digital FIR filters, for calculating [20, 23] the ballistic value of the wind vector are qualitatively identical with the values calculated by us, but there are significant quantitative differences. As a result, it can be expected that the values of the ballistic wind vector given in the METB3Q met massage may differ significantly from the actual values in a usual met situation.

2.2 *Consequences of norm effect for practice*

In the case of the exact norm effect, r(1) = 0 always holds, so $\Delta X_{T,1} = 0$ always holds (formula (8)). As a result, the correction for the deviation in virtual temperature ΔX_T (formula (9)) is always zero, which is a shortcoming of the theory used so far, as this correction is usually non-zero. Based on the analysis of the graph in Fig. 13, it is clear that this situation is quite common.

Now we present formulas [7, 9] without derivation, which allow to solve the given problem.

We assume that the *measured course* of the relative deviation of the virtual air temperature $\delta T_v(\eta)$ is approximated by a polynomial

$$\delta T_v(\eta) \approx \sum_{j=0}^{n} a_j \cdot \eta^j = a_0 + a_1 \cdot \eta + \cdots, \tag{13}$$

where

a_j, j = 0, 1, 2, …,n - empirical approximation coefficients - Fig. 14

$\delta T_v(\eta) = T_v(\eta)/T_{v,STD}(\eta) - 1$ [-]

$T_v(\eta)$ – measured values of the air virtual temperature [K]

$T_{v,STD}(\eta)$ – standard values of the air virtual temperature [K] [18, 19]

Substitute formula (13) into relation (10) and after rearrangement we get [7, 9]

$$\delta T_{v,B} \approx \sum_{j=0}^{n} a_j \cdot m_{WFF,j} = a_0 \cdot m_{WFF,0} + a_1 \cdot m_{WFF,1} + \cdots. \tag{14}$$

where

$m_{WFF,j}$, j = 0, 1, 2, …. - new characteristics of digital FIR filter

$$m_{WFF,j} = \int_0^1 \eta^j \cdot g_\eta(\eta) \cdot d\eta, \quad j = 0, 1, \dots \, . \tag{15}$$

Next formulae pay special [7, 9]

$$m_{WFF,0} = r(1), \tag{16}$$

$$m_{WFF,1} = r(1) - S \,, \tag{17}$$

$$S = \int_0^1 r(\eta) \cdot d\eta \, . \tag{18}$$

After substituting formulae (16) and (17) into formula (14) and adjusting, we obtain an approximate (linearized) estimate

$$\delta T_{v,B} \approx (a_0 + a_1) \cdot r(1) - a_1 \cdot S \quad , \tag{19}$$

where

r(1), S are *new characteristics of the digital FIR filter*.

In the case of exact norm effect (r(1) = 0) will apply $\delta T_{v,B} \approx - a_1 \cdot S$, Fig. 14.

Instead of formula (8) we will use the formula

$$\Delta X_{T,1} = -\Delta X_{N,\eta} \cdot \left(\frac{\delta T_{v,N}}{\delta T_{v,p}}\right). \tag{20}$$

Formulae (14) or (19), (20) and (9) make it possible to overcome the consequences of the exact norm effect described above. We will demonstrate it with an example that follows the previous example ($\delta T_{v,p} = 0.1$):

- For the first exact norm effect: Θ_0 = 20.4011°, X = 16112 m, y_S = 2242 m, $\Delta X_{N,\eta}$ = -111.9 m, and S = 0.6198, so $\Delta X_T \approx - 694 \cdot a_1$. If a_1 is a random number from the interval $\langle -0.1, 0.1 \rangle$, then the correction will be $\Delta X_T \in \langle -62, +62 \rangle$ m. Obviously, such a correction cannot generally be neglected.

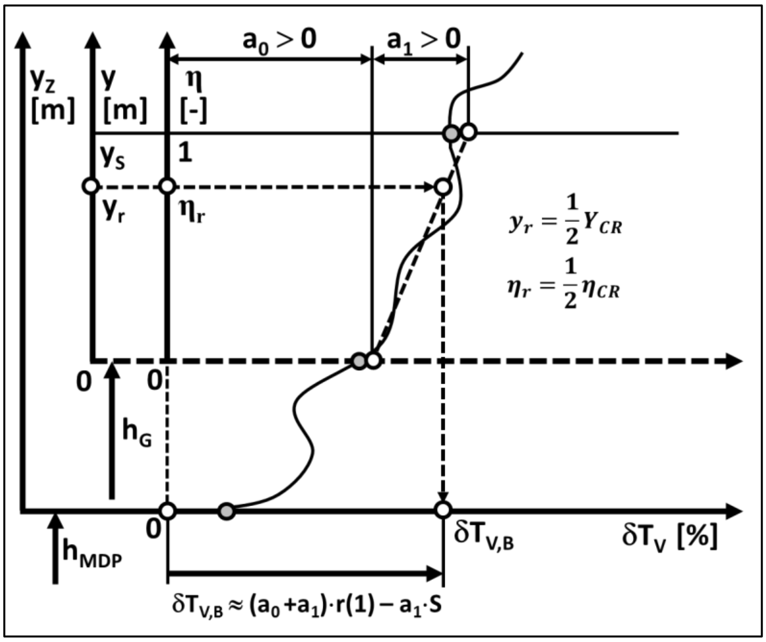

Fig. 14 Scheme to explain the algorithm for estimating the ballistic value of the relative deviation of the virtual air temperature $\delta T_{v,B}$ [-]

- For the second exact norm effect: Θ_0 = 65.1239°, X = 17569 m, y_S = 12072 m, $\Delta X_{N,\eta}$ = 175.6 m, and S = - 0.5409, so $\Delta X_T \approx$ - 950·a_1. If a_1 is a random number from the interval ⟨-0.1, 0.1⟩, then the correction will be $\Delta X_T \in$ ⟨- 95, + 95⟩ m. Obviously, such a correction cannot generally be neglected.

Finally, we assess the suitability of using a generalized reference height of trajectory [7, 9]. Generalized reference height of trajectory is given by relation

$$Y_{CR} = K_{CR} \cdot \Delta h_m \,, \tag{21}$$

where
$\Delta h_m = h_{max} - h_{min} = y_{max} - y_{min}$. The next relationship applies to our assumptions $\Delta h_m = y_S$.

$$K_{CR} = \eta_{CR} = k \cdot \eta_r \tag{22}$$

k = 2 if meteo message Meteo -11 (meteo-average) is used (Soviet methodology [5, 7, 9, 25])
k = 1 if meteo message METCMQ [21] is used (newly proposed methodology [7, 9]).
In the following, we will assume the Soviet methodology, so we will assume that k = 2 – Fig. 14.

$$\eta_r = m_{WFF,1} - (1 - r(1)) \cdot \left(\frac{a_0}{a_1}\right) = (r(1) - S) - (1 - r(1)) \cdot \left(\frac{a_0}{a_1}\right), a_1 \neq 0 \quad .$$
(23)

Using the reference height of the trajectory Y_{CR} is equivalent to using relation (19), so Y_{CR} *must be considered as another characteristic of a digital FIR filter.*

If we insist on compliance with *the law of causality*, $\eta_r \in \langle 0, 1 \rangle$ must apply - Fig. 14.
From formula (23), after adjustment, we obtain an equivalent condition

$$\left(\frac{a_0}{a_1}\right) \in \langle \left(\frac{a_0}{a_1}\right)_{min}, \left(\frac{a_0}{a_1}\right)_{max} \rangle \,, \tag{24}$$

where

$$\left(\frac{a_0}{a_1}\right)_{min} = \left(\frac{a_0}{a_1}\right)_{max} - 1 \quad \text{and} \quad \left(\frac{a_0}{a_1}\right)_{max} = \frac{r(1)-S}{1-r(1)} \quad .$$

If r(1) < 1 applies, the use of the formula (21) for the generalized reference height of the trajectory Y_{CR} places a limitation on the size of the empirical coefficients a_0, a_1, therefore its use is not appropriate and *it is more appropriate to use the formula (19) directly.*

2.3 *Met data filtering*

The basic filtering of met data already takes place during the measurement and is given by the used measuring devices, their adjustment and calibration, and the used measurement methodology.

If meteorological software is used to predict changes in the measured met data, then the properties of the software that affect its output can be interpreted as the properties of a special prediction filter.

This raw data is usually distributed to users in METGM [22, 23] and GRIB [24] met messages.

We are interested in how this raw data is further processed. There are currently three methodologies [11, 12]:

- single-stage filtration,
- two-stage filtration and
- three-stage filtration.

Single-stage filtering is used separately and then its output is data given e.g. in the METCMQ met message [21, 23], or it represents the first stage in multi-stage filtering. The aim of this filtering is to reduce the volume of transmitted data in the respective met messages. It is basically a lossy data compression.

The altitude interval $\langle h_{MDP}, (h_{MDP} + y_{z,max})\rangle$, $y_{z,max}$ = 30 km is divided into layers (zones) of different heights Δh_j, j = 1, 2, …, n. The total number n of these layers is 31 for METCMQ, 21 for METBKQ and 19 for Meteo -11 met message. For each layer, the filtration takes place separately and independently.

The arithmetic mean of the Cartesian components of the wind vector is calculated (formula (10) or (11)). For the other met parameters (air virtual temperature, air pressure, air density), their relative deviations from the corresponding standard values are first calculated and then their arithmetic means are calculated. Using this average, the filtered value of the met parameters for the altitude of the mid-point of layer (zone) is calculated back. This met parameter value represents the given layer (zone) in its entirety.

In the case of *two-stage filtration*, the second stage of filtration follows using the appropriate FIR filter, which is represented by its coefficients - weighting factors. Formula (11) is used for the calculation. Filtered data is distributed, for example, using the METB3Q met message. In the previous part of the article, we showed that this procedure is very inaccurate, because *we use sets of weighting factors $q_j(zz)$ common to all artillery systems* (projectile caliber, its initial (muzzle) velocity, ballistic coefficient) *for one reason only - to reduce the number of these weighting factors* [2, 4, 5]. This problem has been known for about 100 years. During that time, several attempts have been made to solve the problem, but with questionable results – for details [2, 5].

Three-stage filtering of met data is a variant of how to overcome the difficulties associated with the two-stage variant of filtration, but at the cost of linearization - formula (19) or using the reference height of trajectory Y_{CR} - formula (21).

In the second stage of this filtration, only the arithmetic average (arithmetic FIR filter) is calculated using formula (11), so that *the results apply universally to any artillery system*. The goal is to suppress the "high frequency" components that contain the output data from the first stage of filtration [7]. The data obtained in this way are distributed using the met message Meteo-11 (meteo-average) [2, 7, 25, 26]. Met messages other than Meteo-11 can be compiled in the same way, for example from met data contained in METCMQ, or METGM or GRIB.

The third stage of this filtering method consists in reading the ballistic values of the met parameters from the Meteo - 11 met message using the reference height of trajectory Y_{CR} - in detail [7].
The reference height of trajectory Y_{CR} can be used to read ballistic values of met parameters directly from met messages METCMQ, METGM or GRIB, but at the cost of lower accuracy of read values – for details [7].

2.4 *Recommended variants of annexes to TFTs*

The problems analyzed in this article can be overcome even with the practical use of tabular firing tables - TFTs according to STANAG 4119 [14]. To do this, it is necessary to create an annex to TFTs. There are two possible variants of the content of the annex to the TFTs.

All further proposed parameters must be calculated for all ranges X used in TFTs.

The first variant is seemingly simpler, but the necessary calculations are more demanding and, in addition, the use of the annex will be more complicated in practice.

We already know [2, 11, 12] that in the case of range wind w_x, cross wind w_z and air density ρ, it always applies that r(1) = 1, so the reference height of trajectory ($Y_{CR} = Y_R$) can be used without any problems. Hence, the annex will contain reference heights $Y_{R,wx}$, $Y_{R,wz}$ and $Y_{R,\rho}$.

Alternatively, a simplified parameter [2]

$$Y_{R,w} = k_w \cdot Y_{R,wx} + (1 - k_w) \cdot Y_{R,wz} \ , \tag{25}$$

or even

$$Y_{R,AV,1} = k_\rho \cdot Y_{R,\rho} + (1 - k_\rho) \cdot Y_{R,w} \tag{26}$$

can be used to reduce the amount of data in the annex. Whereas
$k_w, k_\rho \in \langle 0, 1\rangle$ - weights. You can usually choose $k_w = 1/2$, $k_\rho = 1/3$.

To calculate corrections for the deviation of virtual air temperature $\delta T_{v,B}$ it is necessary to add:

- $\Delta X_{T,1}$ – unit correction factor - formula (20),
- $r(\eta)$ for $\eta = 1$ – formula (5) and
- S - formula (18).

This variant only allows a very complicated conversion of met data for basic TFTs to met data for Mountain TFTs [12].

The essence of *the second variant* of the annex for TFTs lies in the return to the old European TFTs in the Soviet variant.

Wind corrections are the same in both systems.

In TFTs according to STANAG 4119, corrections are made for the deviation in air density $\delta\rho_B$ (ballistic value) and the deviation in virtual air temperature $\delta T_{v,B}$ (ballistic value), which includes only its effect on the deviation in the magnitude of the Mach number M (air elasticity).

In old European TFTs, the correction for the deviation in air pressure Δp_0 for $y = 0$ compared to the table value $p(h_{FT})$ and the absolute deviation in the virtual temperature $\Delta T_{v,B}$ (ballistic value) is used, which includes both the effect of the deviation in air density and the effect of virtual temperature on deviation of the Mach number.

The advantage is that there is no exact norm effect by WFFs for the absolute deviation of the virtual temperature ΔT_v at all, so the traditional normalization can be used where $r(1) = 1$ and therefore the reference height of trajectory $Y_{R,T}$ can be used without problems.

The annex will therefore contain:

- ΔX_{p0}, ΔX_T – unit correction factors for Δp_0 and ΔT_v,
- $Y_{R,wx}$, $Y_{R,wz}$ and $Y_{R,T}$ or $Y_{R,w}$ and $Y_{R,T}$ or $Y_{R,AV,2}$, where

$$Y_{R,AV,2} = k_T \cdot Y_{R,T} + (1 - k_T) \cdot Y_{R,w} \ , \qquad (27)$$

$k_T \in \langle 0, 1\rangle$ - weight. You can usually choose $k_T = 1/3$.

According to the Soviet methodology [2, 25], $k_w = ½$ and $k_T = 1/3$ are chosen and the value of $Y_{R,AV,2}$ is rounded to the nearest hundred meters.

A modified Petrovic algorithm [7] can be used to calculate all reference heights of trajectory $Y_{R,..}$.

The second variant of the annex has three advantages:

- the necessary parameters can be calculated much faster, and
- people who are used to using the Soviet modification of the TFTs can handle the use of the annex without problems and without much training, therefore very quickly.
- very simple conversion of met data for basic TFTs to met data for Mountain TFTs [12].

III. Conclusion

In the period since 2013, we have managed to theoretically process the issue of corrections of non-standard trajectories using the appropriate Green's functions or, more precisely, using digital finite impulse filters (FIR filters). In this paper, we present a proposal for adjustments of TFTs according to STANAG 4119 [14], which leads to the refinement of calculated corrections and at the same time allows *to use met data from all existing met messages (METB3Q, METCMQ, Meteo - 11 as well as METGM and GRIB) for calculations. As a result, the whole system becomes very robust, even in combat conditions.*

In the paper, we used an improved theory of the generalized reference height Y_{CR} [7, 9], which is based on traditional assumptions. We are currently working on its further improvement, which is important for ranges over about 30 km.

Acknowledgments

This work originated with the support of financing from the Research Project for the Development of the Department of Weapons and Ammunition, Faculty of Military Technology, University of Defence, Brno, DZRO VYZBROJ.

References

[1] STANAG 4635, Ed. 1 *The NATO Error Budget Model*, 2008

[2] Cech, V. Jedlicka, L. Jevicky, J. *Problem of the Reference Height of the Projectile Trajectory as a Reduced Meteo-ballistic Weighting Factor.* Defence Technology 10, Number 2 (June 2014), Issue 2: Special issue of the 28th International Symposium on Ballistics, p. 131 - 140

[3] Cech, V. Jedlicka, L. Jevicky, J. *Problem of the Reference Height of the Projectile Trajectory as a Reduced Meteo-ballistic Weighting Factor. Extended Abstract.* Proceedings of the 28th International Symposium on Ballistics, Editors: Ames, R. G., Boeka, R. D., Atlanta, September 22 - 26, 2014, p. 7 – 10.
[4] Cech, V. Jedlicka, L. Jevicky, J. *Some Problems with the Estimation of Projectile Trajectory Perturbations*. Proceedings of 20th International Conference Engineering Mechanics 2014, Book of full texts, Editor: Fuis, V., Svratka, May 12 – 15, 2014, p. 116 – 119.
[5] Cech, V. and Jevicky, J. *Improved theory of generalized meteo-ballistic weighting factor functions and their use*, Defence Technology 12 (June 2016), Issue 3: Special issue of the 29th International Symposium on Ballistics, p. 242 -254
[6] Cech, V. and Jevicky, J. *Improved theory of generalized meteo-ballistic weighting factor functions and their use, Extended abstract*. Proceedings of the 29th International Symposium on Ballistics, Volume 1, Editors: Woodley, C., Cullis,I., Edinburgh, May 9 -13, 2016, p. 92 -95.
[7] Cech, V. and Jevicky, J. *Improved theory of projectile trajectory reference heights as characteristics of meteo-ballistic sensitivity functions*, Defence Technology 13 (June 2017), Issue 3: Special issue of the 30th International Symposium on Ballistics, p. 177 – 187
[8] Cech, V. and Jevicky, J. *Improved theory of projectile trajectory reference heights as characteristics of meteo-ballistic sensitivity functions, Extended abstract*. Proceedings of the 30th International Symposium on Ballistics, Volume 1, Editors: Chocron, S., Smith, J., Long Beach, September 11 -15, 2017, p. 221 - 225.
[9] Cech V. and Jevicky J. *Some problems with numerical calculations of the meteo-ballistic sensitivity functions and their solutions*. IOSR Journal of Mechanical and Civil Engineering (IOSR – JMCE), Vol. 16, Issue 4, Ser. I (Jul. – Aug. 2019), pp. 1 – 16
[10] Cech V. and Jevicky J. *Some problems with numerical calculations of the meteo-ballistic sensitivity functions and their solutions. Extended abstract.* Proceedings of 31st International Symposium on Ballistics. Edited by V. K. Saraswat, G. Satheesh Reddy, C. Woodley, Hyderabad, India, 4–8 November 2019, p. 606 – 609.
[11] Cech V. *Problems of corrections of the mean point of impact position caused by changes of meteorological conditions (public lecture in Czech)*, University of Defence, Brno 3.6.2019
[12] Cech V. (2019d) *Standard and firing tables atmospheres (Research report in Czech).* University of Defence 2019, p.86
[13] Rozehnal O. and Cech V. *Design of suitable asymptotic courses of meteo-ballistic weighting factor functions for sensitivity analysis of perturbed projectile trajectories.* Proceedings of 26th International Conference Engineering Mechanics 2020, Editor: Fuis, V., November 24 - 25, p. 440 – 443,
[14] STANAG 4119, Ed. 2 *Adoption of a Standard Cannon Artillery Firing Table Format*. 2007
[15] STANAG 4144, Ed. A *Firing Techniques to Determine Ballistic Data for Fire Control Systems*, AOP – 65, 2018
[16] STANAG 4355 JAIS, Ed. 3 *The Modified Point Mass and Five Degrees of Freedom Trajectory Models*, 2009
[17] STANAG 4044 MET, Ed. 2, *Adoption of a Standard Atmosphere*, 1969
[18] *Manual of the ICAO standard atmosphere extended to 80 kilometers (262 500 feet).* Third edition. Doc 7488/3, International Civil Aviation Organization 1993, p. 305
[19] ISO 2533 – 1975. *Standard atmosphere*. 1978
[20] STANAG 4061 MET, Ed. 4 *Adoption of a Standard Ballistic Meteorological Message (METBKQ).* 2000
[21] STANAG 4082 MET, Ed. 2 *Adoption of a Standard Artillery Computer Meteorological Message (METCMQ)*, 1969.
[22] STANAG 6022 MET, Ed. 2 *Adoption of Standard Gridded Data Meteorological Message*, 2010
[23] STANAG 6015, Ed. 5 *NATO Meteorological and Oceanographic Codes Manual – AMETOCP – 4*, 2019
[24] Wordl Meteorological Organization – WMO, *Manual on Codes No. 306*, 2019
[25] Kovalenko V. V. and Shevkunov V. I. *Meteorological preparation of artillery fire (in Russian).* Military Artillery Academy of M. I. Kalinin, Leningrad 1975. p. 84.
[26] Lysenko L. N., Benevolskij S. V., Burlov V. V., Kazakovcev V. P. et al. *Ballistics. Textbook. (in Russian)* Artillery engineering institute of Penza. Penza 2005, p. 512
[27] Composite authors. *A Course in Exterior Ballistics. Ordnance Textbook*. Ordnance Department 1920.
[28] McShane E. J., Kelley J. L. and Reno F. V. *Exterior Ballistics.* University of Denver Press, Denver 1953, p. 834.
[29] Kato T. *Perturbation theory for linear operators*. Reprint of the 1980 edition. Springer –Verlag, Berlin, Heidelberg 1995, p. 620.
[30] Duffy, D. G. *Green's Functions with Applications*. Second Ed. Chapman & Hall/CRC, New York 2018.
[31] Raven, F. H. *Automatic Control Engineering*. Fifth Edition.McGraw – Hill, Inc., New York 1995, p. 620
[32] Kubik S., Kotek Z., Strejc V. and Stecha J. *Theory of automatic control I. Linear and nonlinear systems. (in Czech)*, SNTL – TKI, Prague 1982, p. 528.
[33] Tsien H. S. *Engineering cybernetics*. McGraw – Hill, New York 1954, p. 289
[34] Zipfel P. H. *Modeling and simulation of aerospace vehicle dynamics*. Second edition. AIAA, Education series. Reston, VA, USA 2007, p. 607.
[35] Rosenwasser Je. N. and Jusupov R. M. *Sensitivity of control systems (in Russian).* Nauka, Moscow 1981, p. 464.
[36] Bliss G. A. *Functions of lines in ballistics.* In Transactions of the American Mathematical Society. Vol. 21, No. 2 (Apr. 1920), pp. 93 – 106.
[37] Bliss G. A. *Mathematics for Exterior Ballistics*. John Wiley and Sons, Inc., London, Printed in USA 1944, p. 128.
[38] Curti P. *Introduction into Exterior Ballistics (in German).* Verlag Huber and Co., Frauenfeld 1945. p. 408.
[39] Davis L., Follin J. W. and Blitzer L. *The exterior ballistics of rockets*, D. van Nostrand Co., Princeton 1958, p. 457
[40] Molitz H. and Strobel, R. *Exterior Ballistics (in German).* Springer-Verlag, Berlin 1963, p. 610.
[41] Cranz C. *Textbook of ballistics. First volume. Exterior ballistics*. Fifth edition (in German). Springer Verlag, Berlin 1925, p. 712.).

Vladimir Cech, et. al. "Increasing the Accuracy of the Range Corrections for the Air Virtual Temperature ." *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)*, 19(1), 2022, pp. 01-16.