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Dynamic Analysis Of Laminated Composite Plates Using Higher Order Theory Of 18 Degree Of Freedom Adopting Finite Element Approach

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Abstract: *Laminated composite plates (LCP) are extensively used to solve special problems in engineering applications so static, dynamic and stability behaviors are important to the designers. The project aims at dynamic analysis of these plates with higher order shear deformation theory (HSDT18). The application of higher-order theory that accounts for the realistic variation of in-plane and transverse displacements through the thickness for the dynamic response analysis of thick multi-layered composite plates also shall be studied. Finite element formulation for 18 degree of freedom which was be extended to examine the static responses of the laminated composite plates.*

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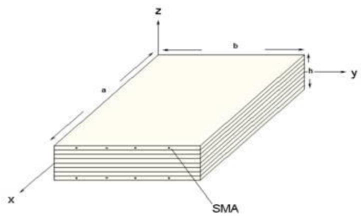
Background*:* A plate is a thin and flat structural element. The word "thin" suggests that the plate is transverse. When compared to its other two dimensions, the laminated composite plate's thickness is quite modest. i.e. the dimensions of its length and width.



Where,
h- Thickness of the plate,
a/b - Length or width dimension.
Composite plates have been widely used in a variety of fields, including structural, marine & aerospace engineering, due to their high strength to weight ratio and corrosion resistance.

Method*:* Mathematical formulation to obtain the equations of equilibrium. Governing differential equations by using HSDT18. Stress-strain relationship by Simple Deformation Theory. Formulation of the solution of above shear differential equation using finite element method. Finite element formulation and writing codes for above formulations to obtain frequency of laminated composite plate. To validate obtained results with available literature.

Results: For comparison, the dimensionless natural frequencies of Laminated Composite Plate (LCP) with simply supported are taken into account. To verify the effectiveness and accuracy of the current work, the results are compared with those from a number of other previously published works. The frequency is obtained for the problem one and problem two for 2 layers (0/90),3 layer (0/90/0) and 4 layers (0/90/90/0) for different a/h ratio which was carried out.

I. Introduction

With creation of higher-order terms in Taylor's expansions of the displacement in the thickness coordinate, this theory overcame the limitation of First Order Shear Deformation Theory. Laminated composite plates are analysed using HSDT for redirection, stresses, natural frequencies, and buckling loads. With this approach, transverse shear stresses have a parabolic distribution and no shear correction coefficients are necessary. According to these theories, an additional dependent unknown is added to the equation for each power of the thickness coordinate. Finite element formulation and writing codes using MATLAB programming to obtain frequency of laminated composite plate.

II. Method

Based on the assumptions of displacement model, a higher order shear deformation theory (HSDT) is developed to analyze the stresses. The displacement model with EIGHTEEN degrees of freedom is in the following form:

The displacement model for unsymmetrical laminates,

$$u(x,y,z)=u_o(x,y)+z\Theta_x(x,y)+z^2u_o^*(x,y)+z^3\Theta_x^*(x,y)+z^4u_o^{**}(x,y)+z^5\Theta_x^{**}(x,y)$$

$$v(x,y,z)=v_o(x,y)+z\Theta_y(x,y)+z^2v_o^*(x,y)+z^3\Theta_y^*(x,y)+z^4v_o^{**}(x,y)+z^5\Theta_y^{**}(x,y)$$

$$w(x,y,z)=w_o(x,y)+z\Theta_z(x,y)+z^2w_o^*(x,y)+z^3\Theta_z^*(x,y)+z^4w_o^{**}(x,y)$$

Stress-Strain Relationship

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \Upsilon_{12} \\ \Upsilon_{23} \\ \Upsilon_{13} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\gamma_{21}/E_2 & \gamma_{31}/E_2 & 0 & 0 & 0 \\ 1/E_{216} & -\gamma_{32}/E_3 & -\gamma_{12}/E_2 & 0 & 0 & 0 \\ 1/E_3 & -\gamma_{13}/E_1 & -\gamma_{23}/E_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{13} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \\ \tau_{23} \\ \tau_{13} \end{Bmatrix}$$

The strain corresponding to displacement model can be written as,

$$\epsilon_x = \epsilon_{xo} + zk_x + z^2\epsilon_{xo}^* + z^3k_x^* + z^4\epsilon_{xo}^{**} + z^5k_x^{**}$$

$$\epsilon_y = \epsilon_{yo} + zk_y + z^2\epsilon_{yo}^* + z^3k_y^* + z^4\epsilon_{yo}^{**} + z^5k_y^{**}$$

$$\epsilon_z = \epsilon_{zo} + zk_z^* + z^2\epsilon_{zo}^* + z^3k_z^{**} + z^4\epsilon_{zo}^{**}$$

$$\gamma_{xy} = \epsilon_{xyo} + zk_{xy} + z^2\epsilon_{xyo}^* + z^3k_{xy}^* + z^4\epsilon_{xyo}^{**} + z^5k_{xy}^{**}$$

$$\gamma_{yz} = \phi_y + zk_{yz} + z^2\phi_y^* + z^3k_{yz}^* + z^4\phi_y^{**} + z^5k_{yz}^{**}$$

$$\gamma_{xz} = \phi_x + zk_{xz} + z^2\phi_x^* + z^3k_{xz}^* + z^4\phi_x^{**} + z^5k_{xz}^{**}$$

where,

$$(\epsilon_{xo}, \epsilon_{yo}, \epsilon_{xyo}) = \left[\frac{\partial u_o}{\partial x}, \frac{\partial v_o}{\partial y}, \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x}\right]$$

$$(\epsilon_{xo}^*, \epsilon_{yo}^*, \epsilon_{xyo}^*) = \left[\frac{\partial u_o^*}{\partial x}, \frac{\partial v_o^*}{\partial y}, \frac{\partial u_o^*}{\partial y} + \frac{\partial v_o^*}{\partial x}\right]$$

$$(\epsilon_{xo}^{**}, \epsilon_{yo}^{**}, \epsilon_{xyo}^{**}) = \left[\frac{\partial u_o^{**}}{\partial x}, \frac{\partial v_o^{**}}{\partial y}, \frac{\partial u_o^{**}}{\partial y} + \frac{\partial v_o^{**}}{\partial x}\right]$$

$$(\epsilon_{zo}, \epsilon_{zo}^*, \epsilon_{zo}^{**}) = (\Theta_z, 3\Theta_z^*, 5\Theta_z^{**})$$

$$(k_x, k_y, k_z^*, k_z^{**}, k_{xy}) = \left[\frac{\partial \theta_x}{\partial x}, \frac{\partial \theta_y}{\partial y}, 2w_o^*, 4w_o^*, \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right]$$

$$(k_x^*, k_y^*, k_{xy}^*) = \left[\frac{\partial \theta_x^*}{\partial x}, \frac{\partial \theta_y^*}{\partial y}, \frac{\partial \theta_x^*}{\partial y} + \frac{\partial \theta_y^*}{\partial x}\right]$$

$$(k_x^{**}, k_y^{**}, k_{xy}^{**}) = \left[\frac{\partial \theta_x^{**}}{\partial x}, \frac{\partial \theta_y^{**}}{\partial y}, \frac{\partial \theta_x^{**}}{\partial y} + \frac{\partial \theta_y^{**}}{\partial x}\right]$$

$$(k_{xz}, k_{yz}) = \left[2u_0^* + \frac{\partial \theta_z}{\partial y}, \ 2v_0^* + \frac{\partial \theta_z}{\partial y}\right]$$

$$(k_{xz}^*, k_{yz}^*) = \left[4u_0^{**} + \frac{\partial\theta_z^*}{\partial x},\ 4v_0^{**} + \frac{\partial\theta_z^*}{\partial y}\right]$$

$$(\phi_x, \phi_x^*, \phi_x^{**}, \phi_y, \phi_y^*, \phi_y^{**}) = \left[\theta_x + \frac{\partial w_o}{\partial x}, 3\theta_x^* + \frac{\partial w_o^*}{\partial x}, 5\theta_x^{**} + \frac{\partial w_o^{**}}{\partial x}, \theta_y + \frac{\partial w_o}{\partial y}, 3\theta_y^* + \frac{\partial w_o^*}{\partial y}, 5\theta_y^{**} + \frac{\partial w_o^{**}}{\partial y}\right]$$

Element In Cartesian Co-Ordinate

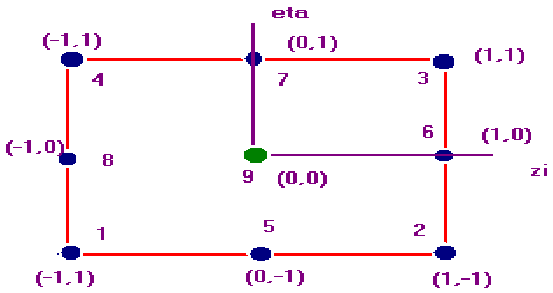


Figure. No. 02 - Element in Cartesian coordinate

Shape function for nine node -element

1. $N_1 = \frac{1}{4}(\xi^2 - \xi)(\eta^2 - \eta)$
2. $N_2 = \frac{1}{4}(\xi^2 + \xi)(\eta^2 - \eta)$
3. $N_3 = \frac{1}{4}(\xi^2 + \xi)(\eta^2 + \eta)$
4. $N_4 = \frac{1}{4}(\xi^2 - \xi)(\eta^2 + \eta)$
5. $N_5 = \frac{1}{4}(1 - \xi^2)(\eta^2 - \eta)$
6. $N_6 = \frac{1}{4}(\xi^2 + \xi)(1 - \eta^2)$
7. $N_7 = \frac{1}{4}(1 - \xi^2)(\eta^2 - \eta)$
8. $N_8 = \frac{1}{4}(\xi^2 - \xi)(1 - \eta^2)$
9. $N_9 = \frac{1}{4}(1 - \xi^2)(\eta^2 - \eta)$

The components of stress resultant vector $\bar{\sigma}$ for the laminate with NL number of layers are defined as,

$$\begin{bmatrix} N_x & N_x^* & N_x^{**} & M_x & M_x^* & M_x^{**} \\ N_y & N_y^* & N_y^{**} & M_y & M_y^* & M_y^{**} \\ N_z & N_z^* & N_z^{**} & M_z^* & N_z^{**} & 0 \\ N_{xy} & N_{xy}^* & N_{xy}^{**} & M_{xy} & M_{xy}^* & M_{xy}^{**} \end{bmatrix} = \sum_{L=1}^{NL} \int_{z_L}^{z_{L+1}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix} (1\ \ z^2 z^4 z z^3 z^5) dz$$

$$= \sum_{L=1}^{NL} \int_{z_L}^{z_{L+1}} \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{12} & Q_{22} & Q_{23} & Q_{24} \\ Q_{13} & Q_{23} & Q_{33} & Q_{34} \\ Q_{14} & Q_{24} & Q_{34} & Q_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \Upsilon_{xy} \end{Bmatrix} (1\ \ z^2 z^4\ \ z\ \ z^3 z^5) dz$$

$$\begin{bmatrix} Q_x & Q_x^* & Q_x^{**} & S_x & S_x^* & S_x^{**} \\ Q_y & Q_y^* & Q_y^{**} & S_y & S_y^* & S_y^{**} \end{bmatrix} = \sum_{L=1}^{NL} \int_{z_L}^{z_{L+1}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} (1 \;\; z^2 z^4 \;\; z \;\; z^3 z^5) dz$$

$\sigma_x = Q_{11}\epsilon_x + Q_{12}\epsilon_y + Q_{13}\epsilon_z + Q_{14}Y_{xy}$

$\sigma_y = Q_{12}\epsilon_x + Q_{22}\epsilon_y + Q_{23}\epsilon_z + Q_{24}Y_{xy}$

$\sigma_z = Q_{13}\epsilon_x + Q_{32}\epsilon_y + Q_{33}\epsilon_z + Q_{34}Y_{xy}$

$\tau_{xy} = Q_{14}\epsilon_x + Q_{42}\epsilon_y + Q_{43}\epsilon_z + Q_{44}Y_{xy}$

$Y_{xy} = \epsilon_{xyo} + zk_{xy} + z^2\epsilon_{xyo}{}^* + z^3k_{xy}{}^* + z^4\epsilon_{xyo}{}^{**} + z^5k_{xy}{}^{**}$

$Y_{yz} = \phi_y + zk_{yz} + z^2\phi_y{}^* + z^3k_{yz}{}^* + z^4\phi_y{}^{**} + z^5k_{yz}{}^{**}$

$Y_{xz} = \phi_x + zk_{xz} + z^2\phi_x{}^* + z^3k_{xz}{}^* + z^4\phi_x{}^{**} + z^5k_{xz}{}^{**}$

$H_i = \frac{1}{i}\left(z_{L+1}^i \quad - z_L^i\right)$

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} D_m & D_C & 0 \\ D_C^t & D_B & 0 \\ 0 & 0 & D_S \end{bmatrix} \begin{Bmatrix} \epsilon \\ k \\ \phi \end{Bmatrix}$$

$$\sigma = D\epsilon$$

The equations of motion using **HAMILTON'S** principle can be compressed as, $M\ddot{u} + \kappa u = f$
Where,

M-System mass
ü-accelerations
κ-Stiffness matrices
f- force vector
u- Displacement
The natural frequencies and vibrational modes can be derived by solving the generalized EIGEN problem, assuming a harmonic motion,

$(\kappa - \lambda M)\, d = 0$ with $\lambda = \omega^2$

ω -Natural frequency

d- Mode of vibration

The h is the total thickness of the plate (h_1, h_2, h_3, …… are the individual thickness in the case of a layered plate). a and b is the length and width if the plate.

$$([K] - \lambda[M]) \begin{Bmatrix} u_o \\ v_o \\ w_o \\ \theta_x \\ \theta_y \\ \theta_z \\ u_o^* \\ v_o^* \\ w_o^* \\ \theta_x^* \\ \theta_y^* \\ \theta_z^* \\ u_o^{**} \\ v_o^{**} \\ w_o^{**} \\ \theta_x^{**} \\ \theta_y^{**} \\ \theta_z^{**} \end{Bmatrix} = \{0\}$$

LOAD CONDITION:

The condition considered for this project is Sinusoidal load with SS2 boundary condition where,

along x-axis, at y=0 and y=b

$v_o = w_o = v_o^* = w_o^* = v_o^{**} = w_o^{**} = \frac{\partial vo}{\partial x} = \frac{\partial wo}{\partial x} = \frac{\partial v_o^*}{\partial x} = \frac{\partial v_o^{**}}{\partial x} = \frac{\partial w_o^*}{\partial x} = \frac{\partial w_o^{**}}{\partial x} = 0$

along y-axis, at x=0 and x=a

$u_o = w_o = u_o^* = w_o^* = u_o^{**} = w_o^{**} = \frac{\partial uo}{\partial x} = \frac{\partial wo}{\partial x} = \frac{\partial u_o^*}{\partial x} = \frac{\partial u_o^{**}}{\partial x} = \frac{\partial w_o^*}{\partial x} = \frac{\partial w_o^{**}}{\partial x} = 0$

The following figure shows the same,

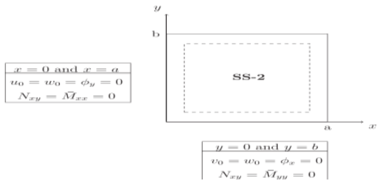


Figure. No. 03 – SS2 Condition

It is divided from the consistent matrix. The mass M in the above equation is given by

$M=\int_A^{\lambda} N^T \, m \, N dA$

Where, $N=[N_1, N_2, N_3, \ldots\ldots\ldots N_{NN},]$

$(I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} \rho \, (1, Z, Z^2, Z^3, Z^4, Z^5, Z^6, Z^7, Z^8, Z^9, Z^{10}) \, dZ$

ρ = Density

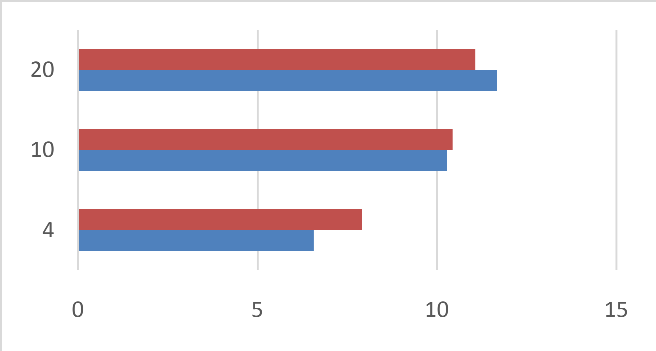
III. Result

- Problem 2: $\frac{E_1}{E_2} = 40$; $G12 = G13 = 0.6$; $\mu12 = \mu23 = \mu13 = 0.25$; $E2$ and $E3 = 1$

Table. No.01– Comparison with Kant paper

NUMBER OF LAYERS	a/h	PRESENT RESULT	KANT [1]
2	4	6.566	7.9081
	10	10.2717	10.4319

	20	11.6614	11.0663
4	4	8.538	9.287
	10	14.7913	15.1048
	20	19.1411	17.647

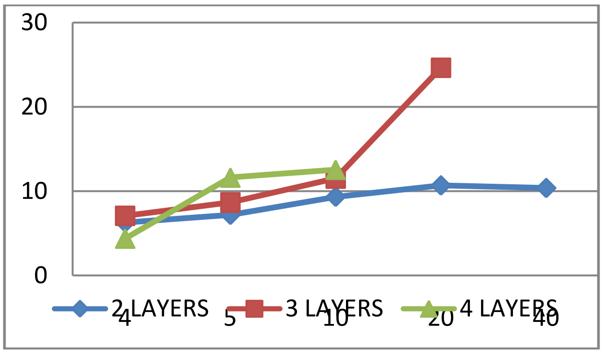


Comparison with Kant paper

- Problem 1: $\frac{E_1}{E_2}$= 25; $G12 = G13 = 0.5$; $\mu12 = \mu23 = \mu13 = 0.25$; $E2$ *and* $E3 = 1$

Table. No. 02 – Result of problem 1

NUMBER OF LAYERS	a/h	FREQUENCY
2	4	6.2768
	5	7.18
	10	9.32
	20	10.6698
	40	10.3619
3	4	7.0732
	5	8.6496
	10	11.4983
	20	44.621
4	4	4.3897
	10	11.62
	40	125.4353

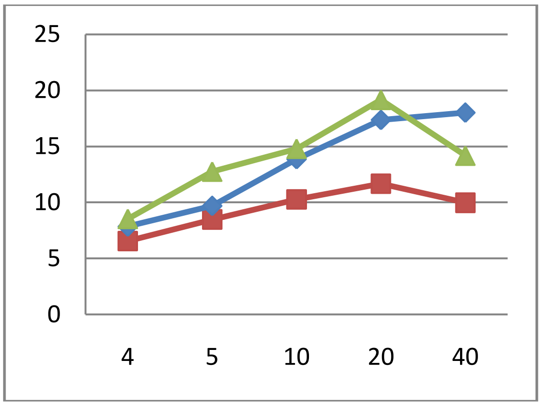


Result of problem 1

- Problem 2: $\frac{E_1}{E_2}$= 40; $G12 = G13 = 0.6$; $\mu12 = \mu23 = \mu13 = 0.25$; $E2$ *and* $E3 = 1$

Table. No. 03 - Result of problem 2

NUMBER OF LAYERS	a/h	FREQUENCY
2	4	6.566
	5	8.481
	10	10.2717
	20	11.6614
	40	9.968
3	4	7.8573
	5	9.6994
	10	13.8622
	20	17.358
	40	18.01
4	4	8.538
	5	12.7515
	10	14.7913
	20	19.1499
	40	14.1749



Result of problem 2

IV. Conclusion

Results are obtained for higher order shear deformation for 9 nodes has found out to be effective. Code worked out in MATLAB for 18 DOF gives satisfying results. The frequency increases with increase in a/h ratio consistently. Graph plotted for the comparison shows the values are almost close to result values mentioned in article of T. KANT and SWAMINATHAN.

References

[1]. T. KANT AND K. SWAMINATHAN "Analytical solutions for free vibration of laminated composite and sandwich plates based on higher- order refined theory".

[2]. V.M. SRIHARI "A detailed comparison study of first order and higher order shear deformation theories in the analysis of laminated composite plate".

[3]. T. KANT AND MALLIKARJUNA (1988) PUBLISHED AS "Dynamic of LCP With Higher Order Theory and FE Discretization**".**

[4]. NGO NHU KHOA & TRAN ICH THINH "Finite element analysis of laminated composite plates using high order shear deformation theory". SANG JIN LEE and HA RYONG KIM published as "fe analysis of laminated composite plate using a higher order shear deformation theory with assumed strains".

[5]. B. N. PANDYA & T. KANT "Finite element analysis of laminated composite plates using a higher-order displacement model".

[6]. N.D. PHAN AND J.N. REDDY (1985) published as "Analysis of laminated composite plates using a higher-order shear deformation theory".

[7]. SURESH KUMAR, T. DHARMA RAJU, K. VIJAYA KUMAR REDDY published as "vibration analysis of composite laminated plate using higher- order shear deformation theory with zig-zag function.

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