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Thermodynamic Concepts in Civil Engineering

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Abstract
Thermodynamics is not always done well or usefully brought to bear in civil engineering. This paper addresses historical aspects of misunderstandings of thermodynamic concepts; applies the Second Law of Thermodynamics to applications including shock waves in compressible fluid flow, the tidal bore, spillway flow, and junction flow. Additional applications of thermodynamics in civil engineering are discussed. These include deriving hydraulic transient wave celerities for waterhammer analyses; the First Law of Thermodynamics for a closed system and for a control volume; one-dimensional flow, energy loss due to friction; parallel incompressible flow; application of the control volume to a pressure conduit; the modified Bernoulli equation; tees with small inflow and outflow branches; isentropic flow of a perfect gas and its application to flow metering, determination of choked flow conditions, and determination of the shaft power required to drive a blower.
Keywords: *Thermodynamics, Compressible flow, Tidal bore, Spillway flow, Energy dissipation*

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I. Introduction

In 2001, the author submitted a paper entitled "Concepts of Spatially Varied Flow" to the ASCE Journal of Hydraulic Engineering. The article was rejected based largely on a reviewer's assertion that "the author is under a 'common misconception' in believing that Newton's Second Law and the First Law of Thermodynamics are independent relationships of fluid mechanics." The reviewer claimed that a "number of papers have been published on this topic", but rather than cite them simply says "most of them are cumbersome to follow". The author asked that the reviewer cite those "papers", but there was no response. The one reference that was cited by the reviewer in this context was Jain [1] as had also been cited by the author. That reviewer had a very basic misunderstanding of the First Law of Thermodynamics as it relates to fluid mechanics. Jain [1] in fact used the First Law of Thermodynamics in all but name.

The author subsequently expanded substantially on this topic in the context of spatially varied flow [2]. Presented therein is a history of the erroneous notion that the First Law of Thermodynamics derives from Newton's Second Law and disbelief in the concept of entropy, both dating back to the year 1877. The confusion and errors have persisted about the separate roles of the First Law of Thermodynamics and Newton's Second Law of Motion, as noted in Graber's [2] discussion of various references by Eisenlohr [3], Kalinske [4], Yen and Wenzel [5], Yen [6], Contractor [7], Yen [8], and Jain [1].

Graber [9] simplified the derivation of the differential energy equation for fluid flow by expressing the energy in terms of an observer moving with the fluid element. When the derivation is carried out in that manner, only the First Law of Thermodynamics is utilized and it is not necessary to bring Newton's Second Law into the derivation as is done in other derivations. The dissipation function thus derived represents the energy expended per unit volume, caused by fluid stresses in distorting elements of fluid, that is converted to internal energy, heat, and sound waves which are exchanged with the environment. By then expressing the stress terms using Stoke's stress-strain relationships, the resulting relationship shows that energy is ultimately dissipated at the microscopic scale through the action of viscosity.

Graber [2] proceeded to present relationships for spatially varied flow with combined increasing and decreasing discharge, related those relationships to prior work, and provided an exposition of spatially varied flow concepts. The applicability and usefulness of energy (First Law of Thermodynamics) and momentum (Newton's Second Law) principles to spatially varied flows are addressed. Although both energy and momentum principles are applicable to spatially varied flows, they differ in their usefulness, particularly for spatially increasing flow. Momentum and energy principles are equally suitable for spatially decreasing flows if the principles are properly applied. The momentum equation provides a complete solution for spatially decreasing as well as spatially increasing flows, and provides fundamental analytical insight. Both principles are usefully applied to spatially decreasing flow, but careful selection of control volumes and interpretation are required. It is shown that those who have claimed to have avoided use of the First Law of Thermodynamics in deriving spatially varied free surface flow equations have, in fact, used that Law in all but name.

An important point that is occasionally overlooked in practice for spatially increasing flows is that even if the boundary friction is negligible an appreciable energy loss may still be incurred. That is due to non-conservative energy exchange between the inflow and main flow accompanying their exchange of momentum, which causes an energy loss in addition to that due to wall friction alone. For spatially increasing flow the First Law of Thermodynamics is incapable of independently evaluating the energy exchange between the inflow and the main flow (just as the energy principle alone is incapable of relating the conditions upstream and downstream of a hydraulic jump, sudden pipe expansion, or normal shock wave, but does usefully provide the energy loss once the problem has been solved by the momentum equation). This limitation of the energy principle was discussed by Hinds [10] in connection with side channel spillways, but has remained a source of confusion. The momentum and energy principles are derived from Newton's Second Law and the First Law of Thermodynamics, respectively, totally independent fundamental laws of nature each having its own inherent value in fluid flow problems.

Graber [9] further utilized the First Law of Thermodynamics and the concepts of energy and work to properly derive and overturn the concept of “G-value” (rms velocity gradient) which had, since the 1940s, been widely used (and often misused) by environmental engineers. In conjunction with experimental observations, the author conducted a critical assessment of the various processes to which G-value theory has been applied. The author demonstrated the fallacy of G-value and presented proper methods for the analysis and design of induced circulation in channels, basic reactor concepts (short-circuiting), mixing, flocculation and floc breakup, and filtration and fluidized beds. Graber [11,12] provided related criticisms of references applying G-value to a flocculating baffled channel and coagulation kinetics. Correct notions for backwashing based on conceptual reasoning and experimentation are given in Camp, Graber, and Conklin [13,14] and Graber [9].

The hydraulics of discrete flow takeoffs forms an important basis for the proper understanding of wastewater pressure-distribution systems. Pertinent aspects of the hydraulics of discrete tee takeoffs and discrete orifice takeoffs are discussed in Graber [15]. Some of the present conceptual and practical deficiencies in wastewater pressure-distribution system design are demonstrated in Graber [15] and include an erroneous energy balance (misapplication of the First Law of Thermodynamics) and other errors at junctions by Otis [16,17] and others including federal and state agencies that have used the Otis method (Graber [15]. Graber [18] dividing-flow manifolds with the emphasis on pressure distribution systems. Graber [19] identifies an erroneous application of the First Law of Thermodynamics by U.S. federal agencies in connection with the hydraulic design of geosynthetic and aggregate subsurface drains.

Some references in the civil engineering literature that incorporate mention of entropy are purposely omitted here because the author finds them to be lacking in fundamental background, necessity (i.e., alternative assumptions would be at least as useful), or practical implications. One apparent exception is the interesting work on estuarine mixing by Di Toro [20,21].

II. Compressible Fluid Flow

The environmental engineer designs and/or specifies various systems in which gases are the fluid medium. Such gases include air, digester gas, molecular oxygen, chlorine, ozone, natural gas, and carbon dioxide. These gases are compressed, conveyed, dispersed, and diffused for a variety of purposes. Some gaseous systems, such as those providing molecular oxygenation, chlorination, and ozonation are largely manufacturer-designed, with the prime design engineer’s role being mainly that of selection and specification. On the other hand, aeration systems and digester gas systems entail the more detailed involvement of the environmental engineer. In all cases a basic understanding is desirable.

The compressibility, or density variation, of gases introduces additional considerations that must be taken into account. For compressible fluid flows, the First and Second Laws of Thermodynamics and concepts of enthalpy (the sum of internal energy and pressure energy), entropy (processes that are both reversible and adiabatic are called isentropic), specific heat, Mach Number (the ratio of the local velocity of a gas to the local velocity of sound at the same point) play major roles in compressible fluid flows in environmental engineering applications. Such applications include psychometry (involving mixtures of dry air and water vapor), pipelines, flow meters (venturi tubes, flow nozzles, orifices, positive displacement diaphragm meters, turbine-type meters, vortex shedding meters, propeller meters, and shunt meters), blowers, and compressors. Shapiro [22] is an excellent reference for such flows. It is of interest to note that shock waves may occur in the diverging sections of choked converging-diverging nozzles (e.g. venturis); this is analogous to the hydraulic jump in the diverging portion of Parshall flumes.

The mode of operation of dynamic blowers is analogous to that of dynamic pumps. As with dynamic pumps, the flow may leave the impeller in radial or axial directions, the former being termed centrifugal blowers and latter axial blowers. Centrifugal blowers are the dominant type of dynamic blowers in environmental engineering applications. A distinction has been made in mechanical engineering practice and formally approved by key engineering societies between *fans* for low pressures and *compressors* for high pressures. The demarcation is set at 7 percent increase in air density from the fan inlet to the outlet. For fans operating with less than this density increase, the assumption of incompressibility leads to substantial simplification without significant error. Shop testing is important for ascertaining compliance with specifications, and there are several categories of such testing. The category of particular concern here is that required to demonstrate compliance with a performance specification. One or more blowers of each capacity is commonly required to be tested in accordance with the ASME Power Test Code for Compressors and Exhausters (PTC 10) [23] or the ASME Performance Test Codes for Displacement Compressors, Vacuum Pumps and Blowers [24].

The specific heat ratio k is a particularly important parameter in compressible fluid processes. This ratio is defined as $k = c_p / c_v$ in which c_p is specific heat at constant pressure and c_v is specific heat at constant volume. We may, for most purposes, take k = 1.4 for air. Furthermore, we may generally assume k to be independent of temperature, pressure, and humidity, which is an important fact in blower and compressor test work. The specific heat ratio plays an important role in the discussion of shock waves which follows.

The shock wave in a compressible flow involves a transition from supersonic to subsonic flow. Of particular interest in the context of the present paper is the impossibility of a rarefaction or reverse shock, i.e. a shock wave arising in a compressible flow in which pressure increases in passing downstream across the wave, also referred to as an expansion shock wave. Shapiro [22], Kuethe and Schetzer [25], and Shames [26] provide proofs of this impossibility, using the Second Law of Thermodynamics. Shapiro's demonstration is recounted here.

Shapiro derives the following equation for the increase in entropy across a shock wave:

$$\frac{s_y - s_x}{R} = \frac{k}{k-1} \ln\left[\frac{2}{(k+1)\mathbf{M}_x^2} + \frac{k-1}{k+1}\right] + \frac{1}{k-1} \ln\left[\frac{2k}{(k+1)}\mathbf{M}_x^2 - \frac{k-1}{k+1}\right] \qquad (1)$$

in which s is entropy, R is the experimental gas constant which differs for each gas, $\mathbf{M}$ is Mach Number, and subscripts x and y respectively denote conditions upstream and downstream of the shock wave. For typical gases with $1 < k < 1.67$, the entropy change is always positive when $\mathbf{M}_x$ is greater than unity, and is always negative when $\mathbf{M}_x$ is less than unity. This can be seen by reversing the numerator on the left-hand side of the above equation, and plotting $\left(s_x - s_y\right)/R$ vs. $\mathbf{M}_x$ as in Figure 1 below. Above the x-axis shocks are possible, whereas below the x-axis shocks are impossible because entropy change must always be positive per the Second Law of Thermodynamics.

III. Tidal Bore

In most of the world's estuaries the tide rises gradually. However, in some estuaries a remarkable phenomenon known as a tidal bore occurs. Then the tide comes into the estuary abruptly as a fast-moving wall of water, in some places up to ten or more feet high. The tidal bore is a fascinating phenomenon, multiple aspects of which are discussed by Graber [27,28,29]. Estuarine tidal bores have been the subject or cause of legend, poetry, recreation, tourism, shipping disasters, and scientific interest for centuries. Graber [27,28,29] discusses tidal-bore history and legends and presents explanations of the formation, variation, maximum height, and frictional dissipation of the bore. Present-day or historic (extinct) tidal bores are discussed therein, among them the famous bore on China's Qiantang River.

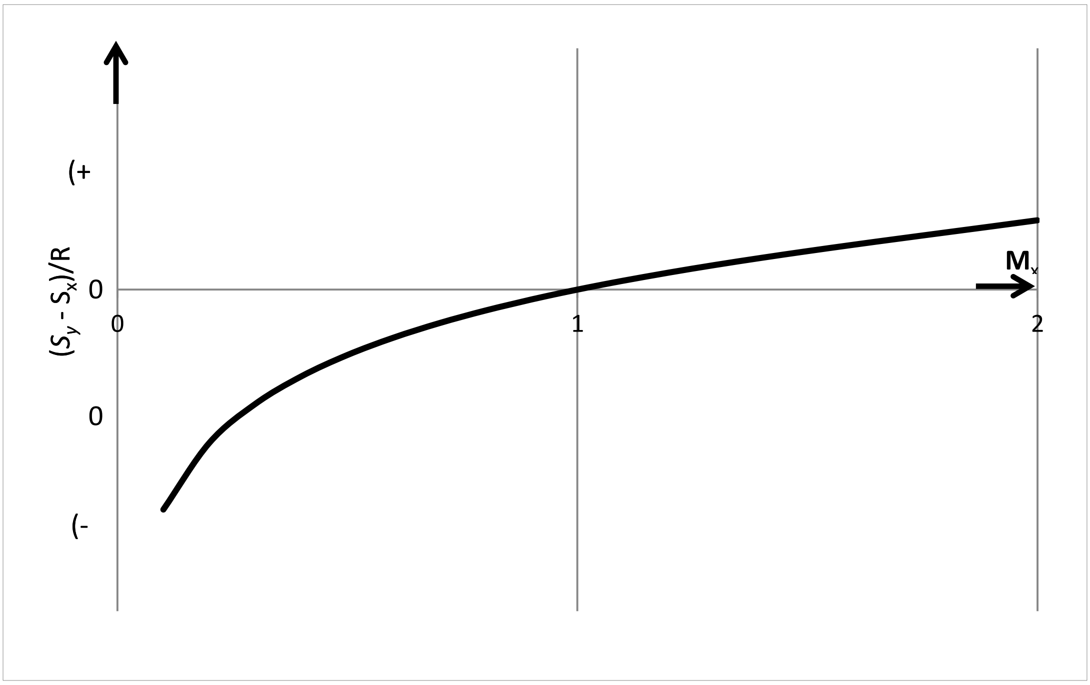

Figure 1. Nondimensional entropy difference vs. upstream Mach number.

Energy dissipation in an undular tidal bore is addressed by Chanson [30,31]. Employing the First Law of Thermodynamics and Newton's Second Law of Motion, Graber [28] demonstrated both theoretically and experimentally that Chanson's estimation of such energy dissipation was incorrect. Corrections were provided by properly using the method of Ippen and Kulin [32] actually cited by Chanson. Energy dissipation equations for the hydraulic jump given by Chow [33] and Shames [26] are incorporated in Graber's analysis. Graber (2012b) states, citing references, that channel friction retards formation of the bore and notes some controversy in that regard. Roy-Biswas and Sen [35] confirm that "in estuaries with high frictional resistance, the tidal range should be higher for the bore to form."

Although Chow [33] categorized the hydraulic drop as rapidly varied along with the hydraulic jump and tidal bore, it actually should be regarded as gradually varied by his categorizations. In an analysis of tidal ponds on Cape Cod, the authors [34] predicted a hydraulic drop at the Lighthouse Road Bridge which conformed precisely with observations. The analogous rapidly varied flow would be a reverse hydraulic jump, which leads us into a discussion of its impossibility in the context of the tidal bore.

Chow [33] also notes the dissipating role of friction, plus points out that a bore retreating to the sea (with a change of tide) will become unstable and flatten out. Schönfeld [36] and Stoker [37] also note the latter. We will address the reasons for this by first considering entropy then a more definitive development.

Analysis

Hornung *et al.* [38] addressed the entropy increase across a hydraulic jump, drawing an analogy between "the manner in which dissipative effects behave in the analogous situation of a shock wave in a compressible fluid. The square of the Mach number $\mathbf{M}^2$ corresponds to the Froude number $\mathbf{F}$ in the analogy and dissipative effects manifest themselves in the form of an entropy increase in the shock wave. This entropy change increases with the cube of $\mathbf{M}^2 - 1$, just as the manifestation of dissipative effects in the hydraulic jump (namely vorticity) increases as the cube of $\mathbf{F} - 1$ here."

Swan [39] stated: "The hydraulic jump is in many respects analogous to a shock wave arising within a compressible flow. For example, whereas the hydraulic jump provides a transition from supercritical to subcritical flow, the shock wave involves a transition from supersonic to subsonic flow. In both cases there is a critical velocity below which these transitions cannot occur, and both processes involve an increase in entropy. Indeed, in the case of a hydraulic jump the increase in entropy per unit mass is proportional to the cube of the depth change, whereas in a shock wave this increase is proportional to the cube of the pressure difference (provided this is small)."

We now address the reason for the instability of a tidal bore retreating to the sea. It will be useful in the present context to derive the equation for the hydraulic jump. We consider the basic case of a hydraulic jump in a channel of rectangular cross section with weight and drag terms neglected. The steady-flow momentum equation between upstream section "1" and downstream section "2" gives:

$$\frac{y_1}{2} b y_1 + \frac{Q^2}{g b y_1} = \frac{y_2}{2} b y_2 + \frac{Q^2}{g b y_2} \tag{2}$$

in which y is depth of flow, b is channel width, Q is volumetric rate of flow, and g is gravitational acceleration. Incorporating the upstream Froude Number $\mathbf{F}_1 = V_1 / \sqrt{g y_1}$, and continuing with the development yields:

$$\frac{y_1^2}{2} + y_1^2 \mathbf{F}_1^2 = \frac{y_2^2}{2} + \frac{y_1^3}{y_2} \mathbf{F}_1^2 \tag{3}$$

hence:

$$\left(\frac{y_2}{y_1}\right)^3 - \left(2\mathbf{F}_1^2 + 1\right)\frac{y_2}{y_1} + 2\mathbf{F}_1^2 = 0 \tag{4}$$

which can be factored to give:

$$\left[\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2\right]\left(\frac{y_2}{y_1} - 1\right) = 0 \tag{5}$$

For the non-trivial solution, we solve the following quadratic equation:

$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2 = 0 \tag{6}$$

to obtain:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8\mathbf{F}_1^2} - 1\right) \tag{7}$$

We now investigate the reverse jump by continuing to take y_1 upstream and y_2 downstream but now with $y_2 > y_1$ and $\mathbf{F}_2 = V_2 / \sqrt{g y_2} > 1$,

$$\frac{y_1}{2} b y_1 + \frac{Q^2}{g b y_1} = \frac{y_2}{2} b y_2 + \frac{Q^2}{g b y_2} \tag{8}$$

Following through as above, we obtain:

$$\frac{y_1^2}{2} + y_2^2\mathbf{F}_2^2 = \frac{y_2^2}{2} + y_2^2\mathbf{F}_2^2 \tag{9}$$

and

$$\left(\frac{y_1}{y_2}\right)^2 + 2\mathbf{F}_1^2 = 1 + 2\mathbf{F}_1^2 \tag{10}$$

leading to:

$$\left(\frac{y_1}{y_2}\right)^2 = 1 \tag{11}$$

and $y_1 / y_2 = 1$, a trivial solution.

Additionally, the Second Law of Thermodynamics states that entropy can be created but it cannot be destroyed. This is called the entropy balance. Therefore, the entropy change of a system is zero if the state of the system does not change during the process. The upshot of all this is that the tidal bore cannot return to the sea.

IV. Spillway Flow

Chanson [40] addressed minimum specific energy and critical flow conditions in open channels. The tangible problem on which he specifically focused is for flow over a spillway crest. For that problem he derived the following third order polynomial:

$$\left(\frac{d_c}{E_{\min}}\right)^3 - \left(\frac{d_c}{E_{\min}}\right)^2 * \frac{1}{\Lambda_{crest}} + \frac{1}{2} * \frac{\beta_{crest} * C_D^2}{\Lambda_{crest}} * \left(\frac{2}{3}\right)^3 = 0 \tag{12}$$

in which d_c = critical flow depth, i.e., flow depth at minimum specific energy; $E_{\min}$ = minimum specific energy; Λ_{crest} = pressure correction coefficient at the crest; β_{crest} = momentum correction coefficient at the crest; and C_D = dimensionless discharge coefficient. Equation (12) has one, two, or three real solutions depending on whether the discriminant, given below, is positive, zero, or negative, for which the real solutions are denoted *S*1, *S*2, and *S*3, respectively.

$$\Delta = \frac{1}{\Lambda_{crest}^6} * \frac{4}{3^6} * \beta_{crest} * C_D^2 * \Lambda_{crest}^2 * + \left(\beta_{crest} * C_D^2 * \Lambda_{crest}^2 - 1\right) \tag{13}$$

The *S*2 solution is negative and thus not physically-meaningful. The *S*1 and *S*3 solutions are positive, but the experimental data plotted by Chanson [40] only correspond to (and closely match) the *S*3 solution. [That data must be inspected closely. The pertinent data (Figure 3 in Chanson [41], Figure 2 in Chanson [42] are those represented by square blocks. The data points extending onto the S3 curve represent an undular flow in a venturi flume, which is not related to the present concern.] Chanson says it is "unclear why experimental data do not follow the solution *S*1, although it is conceivable that *S*1 might be an unstable solution." The reason, determined by the author, is given below.

The *S*1 solution is given by:

$$\frac{d_c}{E_{\min}} * \Lambda_{crest} = \frac{2}{3} * \left[\frac{1}{2} + \cos\left(\frac{\delta}{3} \right) \right] \quad (14a)$$

$$\delta = \cos^{-1}(1 - 2 * \beta_{crest} * C_D^2 * \Lambda_{crest}^2) \quad (14b)$$

And the *S*3 solution is given by:

$$\frac{d_c}{E_{\min}} * \Lambda_{crest} = \frac{2}{3} * \frac{1 - \cos(\delta/3) + \sqrt{3 * \left\{ 1 - \left[\cos(\delta/3) \right]^2 \right\}}}{2} \quad (14c)$$

with δ the same as above.

Analysis

The S1 and S3 solutions can be expressed in the following functional form, with the function differing for each solution:

$$\frac{d_c}{E_{\min}} * \Lambda_{crest} = \mathrm{fn}(\beta_{crest} * C_D^2 * \Lambda_{crest}^2) \quad (15)$$

The parameters at the crest are related to the minimum energy at the crest by $d_c + V_c^2/(2g) + h_L = E_{\min}$, in which new parameters are critical velocity at the crest, gravitational acceleration, and head loss. The Froude Number at the crest is unity, corresponding to $V_c^2/(gd_c) = 1$ so that $V_c^2/(2g) = [gd_c/(2g)] = d_c/2$. We can then write the following equation:

$$\frac{h_L}{E_{\min}} = 1 - \frac{3d_c}{2E_{\min}} \quad (16)$$

The energy loss from upstream to the crest is converted to heat which we denote by ΔQ. The entropy change [43] is $\Delta s = \Delta Q/T$ and we have:

$$\frac{\Delta Q/T}{E_{\min}} = \frac{1}{T}\left(1 - \frac{3d_c}{2E_{\min}}\right) \quad (17)$$

From this we define an Entropy Parameter equal to the variable portion of the right-hand side of Equation (17):

$$\text{Entropy Parameter} = 1 - \frac{3d_c}{2E_{\min}} \quad (18)$$

Then using the Equation (18) with the *S*1 and *S*3 solutions given above, we plot in Figure 2 the Entropy Parameter vs. the abscissa for each of those solutions as shown below. Since the Entropy Parameter is negative for the *S*1 solution, that solution is inadmissible; only the *S*3 solution is physically viable.

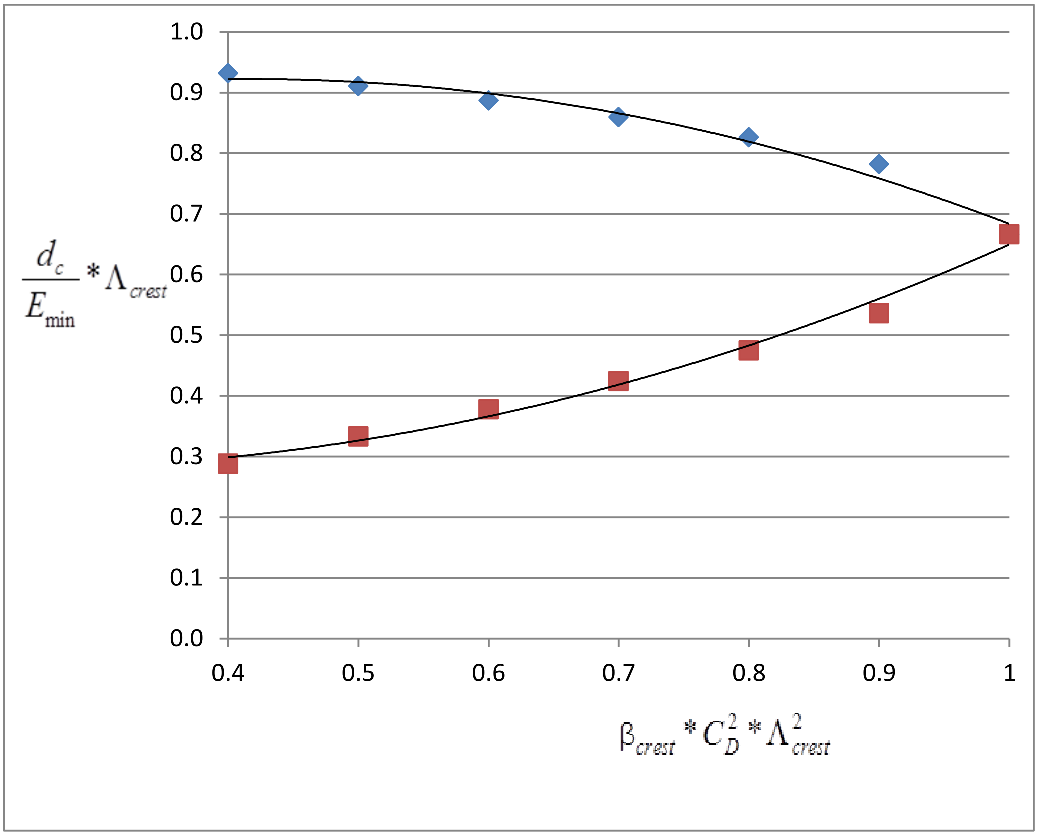

Figure 2. Entropy parameter vs. crest parameter.

V. Special Junction Flow

A complicated junction was part of a wastewater network analyzed by the author for the city of Lawrence, Massachusetts. Referred to as the Daisy Street Junction Structure, it consisted of an existing 45-inch reinforced concrete pipe (RCP) flowing into the structure, two existing 30-inch RCP lines flowing out of the structure and across the Spicket River, and a proposed 42-inch interceptor pipe flowing into the structure upstream of the river crossing. The 42-inch interceptor pipe was proposed to come into the structure at a right angle to the 45-inch RCP. The 30-inch lines flow out of the structure on approximately the same alignment as the incoming 45-inch line. Complexity was added by the existence of a drawdown to supercritical flow conditions and a hydraulic jump under certain conditions, subcritical flow throughout under other conditions, and pressure flow in the upstream section under still other conditions. Depending on the flow and depth conditions, the junction can act as either an expansion (upstream velocity > downstream velocity) or a contraction (upstream velocity < downstream velocity). It was necessary to assume one or the other and solve the corresponding momentum equation, then check the results for applicability. In addition, there are multiple roots to the governing momentum equation, and it was necessary to test the equation for the sign of the energy loss to determine which roots were permissible. A negative energy loss violates the Second Law of Thermodynamics. By disallowing impermissible roots and considering all the remaining possibilities, worst-case design conditions were derived and the suitability of the overall design established.

VI. Additional Uses Of Thermodynamics

Additional applications of thermodynamics in environmental engineering are discussed here in summary fashion.

In pressure conduits the "information" regarding the existence of a local conduit variation or disturbance is propagated through the conduit via pressure waves. The one-dimensional energy equation and concepts of enthalpy and isentropic (reversible adiabatic) flow are employed in deriving hydraulic transient wave celerities for waterhammer analyses [44,45].

Now consider the flow in a channel moving at a given velocity as it approaches a local channel variation or disturbance. The "information" regarding the disturbance is propagated along the channel via surface waves, in a way analogous to the propagation of circular ripples outward from the point at which a rock is thrown into a pond. The steady flow energy equation is used to determine the velocity of the surface waves. The surface wave celerity is directly analogous to the sonic speed or speed of sound in gases. A Froude Number of unity divides subcritical and supercritical flow in a way analogous to the Mach Number of unity dividing subsonic from supersonic gas flows.

A perfect gas is, by common definition, a gas satisfying the semi-perfect gas relationship ($p\mathcal{V} = mRT$ in which p is the absolute pressure, m is the mass of the gas in the volume $\mathcal{V}$, T is absolute temperature, and R is an experimental gas constant which differs for each gas) plus having constant values of c_v and c_p , independent of temperature. For dry air and water vapor, the specific heats at constant volume and constant pressure, c_v and c_p respectively, are actually weak functions of temperature but may be assumed constant with no significant error for our applications with temperatures below 300 deg F (760 deg K). For isentropic processes in a perfect gas, there are useful relationships between pressure, density, and temperature.

In most flow meters, the flow velocity is sufficiently high and the lengths of flow channels sufficiently short that the amount of heat transfer between the fluid and the surroundings is low enough to be considered negligible. Thus the flow can be assumed to be adiabatic without significant error. However, in order to handle the added complexity of variable flow areas (such as are encountered in certain flow metering devices) another simplifying assumption is made, namely that the flow, in addition to being adiabatic, is also reversible and thus isentropic. The short lengths of smooth flow channels justify the assumption of frictionless (reversible) flow for some well-proportioned venturi meters and flow nozzles. For other cases, particularly orifices, the frictionless assumption is less appropriate. Nevertheless, isentropic flow of a perfect gas provides a useful theoretical basis for all types of flow meters considered here, and will be addressed along with consideration of the meters themselves.

The term *closed system* is used in thermodynamics, whereas in fluid mechanics texts the term *system* alone is often used to have the same meaning. However, because of the varied usage of the word “system” in engineering work, the term closed system is less ambiguous and will be employed here. A *closed system* is defined as a prescribed mass of material, in our case a prescribed fluid mass. The concept of control volume is of vital importance in fluid mechanics. Many a problem owes its solution to the analyst’s ability to judiciously select a control volume. The *control volume* is defined as a prescribed volume fixed in space. The *control volume* is identical to the *open system* of thermodynamics. Conservation of Energy relationships can be applied to closed systems or control volumes.

Closed System

For a closed system, the First Law of Thermodynamics states that, in the absence of nuclear reactions, energy can neither be created nor destroyed. This law, also known as the law of conservation of energy, may be expressed for a system as follows: $DE/Dt = \dot{Q} - \dot{W}$ in which *DE*/*Dt* is the rate at which energy (e.g., in BTU or joules) increases in the system, $\dot{Q}$ is the rate of heat flow (e.g., in BTU/hr or watts) *into* the system, and $\dot{W}$ is the rate at which work is done *by* the system (same units as $\dot{Q}$). If the heat flows *out of* the system it is given a negative sign as is work done *on* the system. Heat and work represent the means by which energy can be transferred across the system boundaries.

The energy E may be categorized as [26]: (a) internal energy, (b) kinetic energy associated with the motion of the system, and (c) potential energy associated with the position of the system in external force fields. The specific energy or energy per unit mass e (e.g., in BTU/lbm or joule/gram) may be expressed in terms of these energy categories as follows: $e = E/m = u + v^2/2 + (\text{p.e.})$ in which *u* is the specific internal energy, $v^2/2$ is the kinetic energy with v being the velocity of the system, and (p.e.) denotes the potential energy per unit mass due to force fields. Gravity is the only external force field with which we shall be concerned; thus the potential energy per unit mass due to gravity is denoted by gz in which g is the acceleration of gravity and z is the elevation above a defined datum. The specific energy may then be written in terms of its three components as: $e = E/m = u + v^2/2 + gz$.

The work done by or on a system may be categorized as: (a) work due to pressure stresses at the boundaries of the system, (b) work done by shear stresses at the boundaries of the system, and (c) electric work. These categories are elaborated upon below.

Control Volume

The First Law of Thermodynamics may be written for a control volume as follows:

$$\dot{Q} - \dot{W}_{shear} - \dot{W}_{elec} = \iint_{c.s.} \left(e + \frac{p}{\rho} \right) \rho \bar{v} \bullet d\bar{A} + \frac{\partial}{\partial t} \iiint_{c.v.} (\rho e) d V \tag{19}$$

in which $\dot{Q}$ is the rate of heat flow *into* the control volume, and $\dot{W}_{shear}$ and $\dot{W}_{elec}$ are, respectively, the rate at which shear and electric work are done *by* the fluid in the control volume on its environment. The work done by pressure stresses at the boundary of the control volume is fully incorporated in the term

$\iint_{c.s.} (p/\rho) \rho \bar{v} \bullet d\bar{A}$. The sum *e* + *p*/*ρ* is: $e + p/\rho = u + p/\rho + v^2/2 + gz$. (20)

The shear work is of two types [22]: (a) shaft work, which is the work done *by* the part of the shaft inside the system or control volume on the part of the shaft outside the system or control volume, this work being due to the torque in the rotating shaft resulting from the shear stress in the plane cut by the boundary of the system or the control surface; and (2) the work done at the boundaries of the system or control volume on the adjacent fluid which is in motion. The latter is nonzero for a system of particles in a fluid flow, but becomes zero for a control volume in which the region outside the boundary is stationary (as when the control surface coincides with the stationary wall of a duct or casing).

Equations (19) and (20) give:

$$\dot{Q} - \dot{W}_{shear} - \dot{W}_{elec} = \iint_{c.s.} \left(u + p/\rho + v^2/2 + gz \right) \rho \bar{v} \bullet d\bar{A} \tag{21}$$

One-Dimensional Flow

One-dimensional flow is flow in which all fluid properties (u, p, ρ, v) *are assumed to vary only in the mean direction of flow*. Such an assumption is of value if property variations perpendicular to the direction of flow are much smaller than those in the direction of flow, and only the latter are of interest. Applying energy principles and continuity enables development of the *one-dimensional energy equation for a single stream*, applicable to steady flow in a control volume in which there is no electric work and all the shear work is in the form of shaft work. It is applicable to compressible as well as incompressible flow. An analogous relation for multiple flow streams may be derived. However, when dealing with pipe networks with friction it is preferable to apply the *single-stream* equation between nodes. For other multiple flow-stream problems, it is generally best to go back to the control volume formulation and proceed from there.

Energy Loss Due To Friction

The nature of the heat flow and internal energy terms merits a close look. The heat flow and internal energy terms may be grouped together as $\left[-\dot{Q}/\rho Q_1 + (u_2 - u_1) \right]$, which represents the heat transferred *out of* the control volume per unit mass of fluid plus the increase in specific internal energy of the flow. From physics or thermodynamics we recall that $\Delta u = c\Delta T$ in which *c* is the specific heat of the fluid. [More precisely, c is the specific heat in the case of a liquid and specific heat at constant volume in the case of a perfect gas.] The terms of interest may thus be written as $\left[-\dot{Q}/\rho Q_1 + c(T_2 - T_1) \right]$.

If T_1 is equal to the ambient temperature outside the control volume, then any and all heat transfer and temperature rise must be due to frictional effects within the piping and pump casing. (If T_1 differs from the ambient temperature, then additional transfers of heat and/or temperature rises are superimposed; the general reasoning, however, remains the same.) If the piping and pump were perfectly insulated, we would have $\dot{Q} = 0$ (adiabatic flow), and all the frictional heating would manifest itself as a temperature rise in the fluid. If the piping and pump freely transferred heat to the environment, we might have little or no temperature rise (isothermal flow), with all the frictional heating lost via $\dot{Q}$. The Second Law of Thermodynamics may be employed to show that such frictional heating is irreversible (meaning, roughly, irrecoverable), and thus represents a *loss* of energy. [The irreversibility of frictional heating (fluid or solid) or pressure drop due to friction may be demonstrated by methods given in basic thermodynamics texts, e.g., Mooney [44, Chapter 8).

Since, for a liquid (incompressible fluid), the other fluid properties $(p, \rho, \forall)$ are not affected by temperature, and since the temperature rise is small and usually not of concern, we find it convenient to express $\left[-\dot{Q}/\rho Q_1 + (u_2 - u_1)\right]$ for this case by the energy loss per unit mass gh_ℓ. The following equation pertains:

$$-\frac{\dot{W}_{sh}}{\rho g Q_1} = -\left[p_1/(\rho g) + V_1^2/(2g) + (z_c)_1\right] + \left[p_2/(\rho g) + V_2^2/(2g) + (z_c)_2\right] + h_\ell \qquad (22)$$

in which h_ℓ is *energy loss per unit weight*. Note that h_ℓ has units of length (e.g., feet or meters) and is termed *head loss*.

The concepts presented above may be generalized in the following specialization:

$$-\dot{Q}_f - \dot{W}_{sh} = \oiint_{c.s.} \left(p/\rho + v^2/2 + gz\right)\rho\bar{v} \bullet d\bar{A} \qquad (23)$$

Equation (23) is the steady-flow energy equation for a control volume, applicable to incompressible flow in which $\dot{W}_{elec} = 0$ and all the shear work is in the form of shaft work (see above); the term $\dot{Q}_f$ represents the rate of energy loss due to friction.

We note here Hynes' interesting observation that the potential energy of the water flowing in natural streams is converted to heat generated by friction, "a fact that is well demonstrated when meltwater at 0°C carves channels in ice over which it flows" [46].

Parallel Incompressible Flow

A relation similar to the one-dimensional energy equation can be derived for incompressible flow, by making assumptions which are less restrictive and for which the significance of the assumptions are clearer than those for one-dimensional flow. This is for the case of *parallel flow*, defined here as *flow in which the fluid velocity vectors at all points in a cross section perpendicular to the mean direction of flow are assumed to be parallel to the mean direction of flow*. For flow in a conduit, the velocity components perpendicular to the direction of flow are neglected. It should be apparent that the conduit (and flow cross section in the case of open channels) must be prismatic or have only gradual variations at sections where these assumptions are to be valid. Furthermore, changes in flow direction (including those resulting from changes in slope) must be gradual so that centrifugal forces do not cause significant transverse velocity components. *As a consequence of the parallel-flow assumption, we have a hydrostatic pressure distribution over the cross section.*

Pressure Conduits

The following equation applies to the pressure-conduit control volume:

$$-\dot{Q}_f = \iint_1 \left(p/\rho + v^2/2 + gz\right)\rho\bar{v} \bullet d\bar{A} + \iint_2 \left(p/\rho + v^2/2 + gz\right)\rho\bar{v} \bullet d\bar{A} \qquad (24)$$

in which $\dot{Q}_f$ is the rate of energy loss due to friction.

We investigate the terms in the Equation (24), incorporate an energy coefficient attributable to Coriolis and analogous to the Boussinesq momentum coefficient, and employ continuity to derive the *parallel-flow energy equation for steady flow in pressure conduits with a single flow stream*. By convention, the elevations are measured above datum to the centroids of the cross sections, and the pressures are those occurring at those centroids.

Modified Bernoulli Equation

We note that the Equations (23) and (24) may be thought of as being applied between two *points* in a piping system, one point in cross section "1" and one point in cross section "2", without explicit reference to a control volume. We will extend this concept by investigating conditions on the suction side of a pump. We will apply the energy equation to a control volume bounded by the interior walls of the suction pipe, by a cross section perpendicular to the flow at the pump suction flange, and in the wet well by the portion of the control surface. The control surface in the wet well is extended out from the end of the pipe to a distance at which the velocities are very small and the kinetic energy of the flow is negligible. The pressure distribution outside of and on the surface of the wet well control volume is hydrostatic, so that $p = p_{atm} + \rho g(z_a - z)$ in which p_{atm} is the atmospheric pressure and z_a is the elevation above datum of the wet well water surface. The energy flux term at the wet well control surface is thus given by

$$\iint_1 \left(p/\rho + v^2/2 + gz \right)\rho \bar{v} \bullet d\bar{A} \cong \iint_1 \left(p/\rho + gz \right)\rho \bar{v} \bullet d\bar{A} = \iint_1 \left[p_{atm}/\rho + g(z_a - z) + gz \right]\rho \bar{v} \bullet d\bar{A}$$

$$= (p_{atm}/\rho + gz_a)\rho \iint \bar{v} \bullet d\bar{A} = -(p_{atm}/\rho + gz_a)\rho Q_1 .$$

(25)

(Similar reasoning may be applied to the discharge). The energy equation for the control volume of interest becomes:

$$\frac{\dot{Q}_f}{\rho Q_1} = -\left(p_{atm}/\rho + gz_a \right) + \left(p_s/\rho + \alpha V_s^2/2 + gz_s \right) \qquad (26)$$

in which the subscript "s" denotes the section at the pump suction flange. The above equation may be thought of as having been applied between a *point* on the water surface of the wet well and a *point* (such as the centroid) in the cross section at the pump suction flange. If gage pressures rather than absolute pressures are used, then p_{atm} may be dropped.

Tee with Small Inflow Branch

The steady-flow energy equation alone gives results of little practical value if applied to the tee with small inflow branch. However, having determined the pressure loss across the run by momentum methods (in conjunction with continuity), we can determine the change in energy per unit mass across the run. Doing so demonstrates a very important phenomenon, too often overlooked in practice: The ***exchange of momentum*** required to accelerate the branch flow (or any lateral inflow) to the velocity of the flow in the run causes a loss in pressure and energy per unit mass in the flow through the run. The pressure and energy differences between the branch and the run require experimental determination.

Tee with Small Outflow Branch

As for the inflow tee considered above, the steady-flow energy equation gives results of little value by itself. As before, however, having determined the pressure loss across the run by the momentum principle, we can determine the change in energy per unit mass across the run. Thus, in the case of the outflow tee, we have a pressure rise and head loss across the run of the tee. The pressure and energy difference between the branch and the run are amenable to approximate theoretical analysis.

Isentropic Flow Of A Perfect Gas

In dealing with isentropic flows, it becomes convenient to deal with *stagnation properties* (denoted by subscript "o"), which are the fluid properties at a reference value of zero Mach Number in relation to properties at Mach Number unity. The stagnation temperature, pressure, and density ratios (stagnation condition divided by unity Mach Number condition) can then be expressed in terms of Mach Number. Working charts and tables for the isentropic flow relationships are given by Shapiro [22] and others. It is noteworthy that p is within 1 percent of p_o for $\mathbf{M} < \mathbf{0.11}$. The corresponding values of $\mathbf{M}$ for ρ within 1 percent of ρ_o and T within 1 percent of T_o are 0.12 and 0.22, respectively. Economic pipe velocities are well within these limiting Mach

Numbers, suggesting that differences between stagnation properties and actual properties in full section pipe flow are often negligible.

The values of the temperature, pressure, and density ratios at the critical state [corresponding to minimum flow area [22] are found by setting $\mathbf{M=1}$ in the ratios mentioned in the previous paragraph.

We will now derive a relationship for isentropic mass rate of flow which will provide a basis for subsequent relationships for flow meters. Consider a flow nozzle or venturi tube with state points 1 and 2 referring, respectively, to Section 1 immediately upstream of the nozzle or venturi and section 2 at the minimum cross section. The minimum cross section may be either an exit section or a throat section depending on whether the nozzle is converging or converging-diverging. Application of the continuity relationship and the steady flow energy equation for adiabatic flow with zero heat flow exchange between sections 1 and 2 gives:

$$w=\rho_2 A_2 \frac{\sqrt{2g(h_1-h_2)}}{\sqrt{1-(\rho_2/\rho_1)^2(A_2/A_1)^2}} \tag{27}$$

It is desired to express Equation (27) in terms of the pressures measured at or near sections 1 and 2. For isentropic flow, the density ratio is related to pressure ratio as follows:

$$\frac{\rho_2}{\rho_1}=\left(\frac{p_2}{p_1}\right)^{1/k} \tag{28}$$

For an isentropic process, the differential enthalpy is related to the differential pressure by $dh=(1/\rho)dp$ the integral of which is given by:

$$h_1-h_2=-\int_1^2 dh=-\int_1^2\frac{dp}{\rho} \tag{29}$$

Combining Equations (28) and (29) and carrying out the integration results in:

$$h_1-h_2=\frac{k}{k-1}\frac{p_1}{\rho_1}\left[1-\left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right] \tag{30}$$

Combining equations given above and adding a *discharge coefficient* C gives:

$$w=\frac{CA_2}{\sqrt{1-(p_2/p_1)^{2/k}\beta^4}}\sqrt{\frac{2gk}{k-1}p_1\rho_1\left(\frac{p_2}{p_1}\right)^{2/k}\left[1-\left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]} \tag{31}$$

in which $\beta=d_2/d_1$. This rather awkward relationship is often written in terms of the analogous incompressible flow equation:

$$w=\frac{CYA_2}{\sqrt{1-\beta^4}}\sqrt{2g\rho_1(p_1-p_2)} \tag{32}$$

in which Y is an *expansion factor* obtained by eliminating w from Equations (31) and (32), with the result given by:

$$Y=\left\{\frac{k}{k-1}\left(\frac{p_2}{p_1}\right)^{2/k}\left[\frac{1-\left(p_2/p_1\right)^{(k-1)/k}}{1-p_2/p_1}\right]\right\}^{1/2}\left[\frac{1-\beta^4}{1-\left(p_2/p_1\right)^{2/k}\beta^4}\right]^{1/2} \tag{33}$$

As the pressure drop ratio $\Delta p/p_1=(p_1-p_2)/p_1$ becomes small, p_2/p_1 approaches 1 and Y approaches 1 and the flow equations approach the relationship for incompressible flow. At the opposite extreme of large $\Delta p/p_1$, the critical pressure ratio p_2/p_o is approached. When that critical pressure ratio is reached, i.e., at:

$$\frac{p_2}{p_1}=\frac{p_2/p_o}{p_1/p_o}=\frac{p^*/p_o}{p_1/p_o}=\frac{0.5283}{p_1/p_o} \tag{34}$$

the Mach Number at the throat section becomes unity. Further increases in $(p_o-p_2)/p_o$ corresponding to further decreases in p_2/p_o are not possible (note that p_2 is at the throat or exit section) and the flow is said to be *choked*.

For choked flow, the equation for w takes on the functional form $w=CA_*\phi(p_1/p_o,\beta,k)\sqrt{gp_1\rho_1}$ where A_* denotes A_2 under choked conditions. Thus, for a choked venturi or flow nozzle, only the properties at station 1 need be measured to determine the flow. The analogy with open channel critical flow meters should be apparent. The analogy is continued by noting that the flow and conditions at station 1 in the choked nozzle are independent of downstream pressure (downstream depth in the analogous open channel) as long as the ratio p_2/p_o remains below the critical value p^*/p_o (critical submergence ratio in the open channel).

When $p_1\cong p_o$ and $p_2/p_o=p^*/p_o$ we have:

$$w=\frac{CA_*}{\sqrt{1-[2/(k+1)]^{2/(k-1)}\beta^4}}\sqrt{k\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}gp_o\rho_o} \tag{35}$$

For a relatively large entrance area, the β term becomes negligible. Then employing the perfect gas relationship $\rho=p/\left(RT\right)$ with $R=$ 53.34 ft lbf/(lbm deg R) we obtain $w=0.532CA_*p_o/\sqrt{T_o}$ in which $A_*, p_o,$ and T_o have units of ft^2, lbf/ft^2, and deg R, respectively.

The number of variables is now formidable. However, noting that the isentropic work of a blower is given by (see standard thermodynamics texts for the development of the following relationship):

$$W_s=\left(\frac{k}{k-1}\right)RT_1\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right] \tag{36}$$

Carter *et al*. [47] investigated the relation among the variables resulting from the following combination of variables:

$$\Phi\left(\frac{Q_1}{ND^3},\mu_s,\mathbf{R}_1\right)=0 \tag{37a}$$

$$\mu_s=\left(\frac{k}{k-1}\right)\frac{RT_1}{N^2D^2}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right] \tag{37b}$$

The term μ_s is dimensionless and called the *isentropic work coefficient*.

The shaft power required to drive a blower can be related to the isentropic work of compression W_s by $P_{sh} = \left(g\rho W_s \right) / \eta_{shs}$ in which η_{shs} is the isentropic efficiency. In an ASME [23,24] Class I test, the isentropic efficiency remains approximately the same at test and specified conditions. Then we have:

$$P_{sh} \propto q\rho T_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right] \tag{38}$$

With kinematic similarity and constant isentropic work coefficient, Equation (38) gives $P_{sh} \propto \rho N^3$.

We thus obtain the power conversion for a Class I test as follows:

$$(P_{sh})_{sp} = \left(\frac{N_{sp}}{N_t} \right)^3 \frac{\rho_{sp}}{\rho_t} (P_{sh})_t \tag{39}$$

VII. Conclusions

Thermodynamics is not always done well or brought usefully to bear in civil engineering. This paper addresses historical aspects of misunderstandings of thermodynamic concepts. The Second Law of Thermodynamics is applied to a variation of a previous application to shock waves in compressible fluid flow; and to new applications involving the tidal bore, spillway flow, and junction flow. Additional applications of thermodynamics in civil engineering are discussed. These include deriving hydraulic transient wave celerities for waterhammer analyses; the First Law of Thermodynamics for a closed system and for a control volume; one-dimensional flow, energy loss due to friction; parallel incompressible flow; application of the control volume to a pressure conduit; the modified Bernoulli equation; tees with small inflow and outflow branches; isentropic flow of a perfect gas and its application to flow metering, determination of choked flow conditions, and determination of the shaft power required to drive a blower.

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