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Attitude Estimation of A Rocking Ship with The Angle of Arrival Measurements Using Beacons

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Abstract: *To improve the accuracy of navigation solutions such as position, velocity and attitude, various navigation techniques have been developed. For example, the global positioning system (GPS) and inertial navigation system (INS) have been used in general navigation systems. Nowadays, to provide more accurate navigation solution, extended Kalman filter (EKF) based GPS/INS coupled systems, which calibrate the navigation solution of INS by using the GPS, are also widely used for military systems. However, with the ordinary GPS/INS coupled systems the attitude cannot be calibrated, since the GPS is able to calibrate position and velocity only, and this feature can be vulnerable to certain applications where attitude information is needed. To overcome this drawback, an angle-based attitude estimation technique using beacons is proposed in this paper. The proposed technique uses the angle of arrival (AOA) measurements of the received signals using an array antenna, and estimates the own-ship attitude. The proposed technique is also coupled with INS and the performance of the proposed technique is demonstrated through simulation.*
Keywords *- attitude estimation, beacon, extended Kalman filter (EKF), INS/GPS coupled system*

I. Introduction

The inertial navigation system (INS) and global positioning system (GPS) are widely used independently as general position estimation techniques in navigation. However, coupled navigation systems such as GPS/INS coupled systems have been also used to compensate the drawback of each; INS diverges markedly as time goes on and GPS only cannot improve the precision over certain level [1], [2]. While GPS is able to determine the position and velocity of the own-ship, attitude cannot be determined directly using GPS. This limitation cannot be a problem when the position and velocity estimation is our final need. However, if we need subsequent control actions such as firing thruster or weapons, accurate own-ship attitude information is essential. If we rely only on gyroscopes and accelerometers, then the resulting attitude estimate would be unreliable especially for rocking small ships.

In this paper, a beacon-based position and attitude algorithm is proposed. We assume that the array antenna is installed at the sea carriage and using this array antenna, Angle of Arrival (AOA) of the signals transmitted by the several beacons and distance between array antenna and several beacons can be measured [3]. Using the distance measurement, the position estimate can be obtained through the triangulation method and in turn the attitude becomes calculated by using AOA of the signals based on the estimated position. The technique is described in 3D, in addition, the INS/beacon coupled system based on extended Kalman filter (EKF) is also suggested. To evaluate the performance of the proposed technique, Monte-Carlo simulation is performed in terms of accuracy of AOA information, the number of beacons, distance between sea carriage and beacons and so on. Simulation results show the proposed technique estimates the position and attitude well. Also, the performance of the INS/beacon coupled system is also compared with conventional techniques.

The rest of this paper has four sections. In section II, an AOA-based attitude estimation technique using beacons is proposed in 3D. An INS/beacon coupled system is described in section III. To evaluate the performance of the proposed technique, several simulation results are given in section IV. Finally, section V concludes this paper.

II. Angle-Based Attitude Estimation

To define the own-ship attitude, definitions of coordinate frames should be preceded. Table 1 describes four coordinate frames that are widely used in navigation applications [4].

Table 1: Coordinate Frames

Frame	Axis			
	origin	x-axis	y-axis	z-axis
Inertial frame	Earth center	Vernal equinox	Right-hand rule	Axis of earth rotation

(i-frame)				
Earth frame (e-frame)	Earth center	Intersection between equatorial plane and Greenwich meridian plane	Right-hand rule	Axis of earth rotation
Navigation frame (n-frame)	user	Northward direction	Eastward direction	Down direction
Body frame (b-frame)	Center of gravity of the user	Forward direction	Transverse direction	Right-hand rule

In this paper, we assume that the origin of the n-frame and that of the b-frame are the same and define the angles roll (ϕ), pitch (θ) and yaw (ψ) which are the Euler angles with respect to the x-axis, y-axis and z-axis from the n-frame to the b-frame as shown Fig. 1.

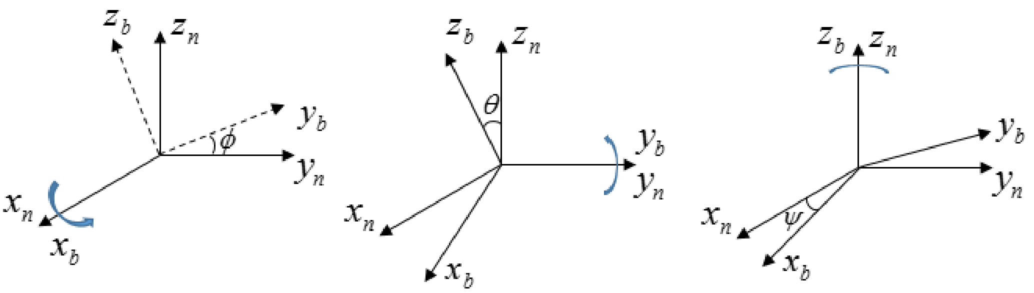

Fig.1 Definitions of roll, pitch and yaw.

The direction cosine matrix (DCM) C_n^b which transforms n-frame to b-frame can be written in terms of 3-2-1 Euler angles:

$$C_n^b = C_x(\phi)C_y(\theta)C_z(\psi)$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(1)

where $C_a(b)$ is the rotation matrix which rotates counterclockwise by angle b with respect to the a-axis.

As described above, our attitude parameters are related to the n-frame and b-frame. Therefore, the localization of the own-ship is essential to obtain the attitude parameters, since the n-frame and the b-frame can be decided after positioning. In this regards, we may calculate the own-ship position by trilateration [5], using the range measurements between the own-ship and beacons. Such trilateration technique is well-known and is omitted in this paper.

After the position of the own-ship is determined, the attitude estimation may be carried out. Note that the AOAs of the beacons are measured in the b-frame. Therefore, in the sequel i, $\mathbf{X}_{bi}$, $\mathbf{X}_u$, $\theta_{El,\text{i}}$ and $\theta_{Az,i}$ will denote respectively, the index of beacon ($i = 1, 2, ..., N$), the position of the *i*th beacon, the own-ship (user) position, the elevation angle of the *i*th beacon and the azimuth angle of the *i*th beacon. Since the own-ship is located at the origin of the b-frame, the coordinate of the own-ship is set to $(0, 0, 0)^T$. If we denote the distance between *i*th beacon and the own-ship r_i, the position of *i*th beacon can be expressed as (2).

$$\mathbf{X}_{bi} = \left(r_i \cos\theta_{El.i} \cos\theta_{Az.i},\ r_i \cos\theta_{El.i} \sin\theta_{Az.i},\ r_i \sin\theta_{El.i}\right)^T \quad (2)$$

Since the n-frame is fixed when the own-ship position is fixed, the coordinate of the *i*th beacon in the n-frame is also fixed. Then, using the DCM C_n^b, the relationship between $\mathbf{X}_{bi}$ and $\mathbf{X}_{ni}$, which is the coordinate of the *i*th beacon in the n-frame can be expressed as follows:

$$\mathbf{X}_{bi} = C_n^b \mathbf{X}_{ni} \quad (3)$$

In equation (3), we want to determine the Euler angles ϕ, θ and ψ using measurement $\mathbf{X}_{bi}$ ($\mathbf{X}_{ni}$ is a-priori information). Substituting (1) for C_n^b in (3), we can rewrite (3) as (4).

$$\mathbf{X}_{bi}=\begin{bmatrix}x_{bi}\\y_{bi}\\z_{bi}\end{bmatrix}=C_n^b\mathbf{X}_{ni}=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}\begin{bmatrix}x_{ni}\\y_{ni}\\z_{ni}\end{bmatrix}$$
$$=\begin{bmatrix}C_{11}x_{ni}+C_{12}y_{ni}+C_{13}z_{ni}\\C_{21}x_{ni}+C_{22}y_{ni}+C_{23}z_{ni}\\C_{31}x_{ni}+C_{32}y_{ni}+C_{33}z_{ni}\end{bmatrix}=\mathbf{g}_i\left(\phi,\,\theta,\,\psi\right) \tag{4}$$

where

$$\begin{aligned}C_{11}&=\cos\theta\cos\psi\\C_{12}&=\cos\theta\sin\psi\\C_{13}&=-\sin\theta\\C_{21}&=\sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi\\C_{22}&=\sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi\\C_{23}&=\sin\phi\cos\theta\\C_{31}&=\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi\\C_{32}&=\cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi\\C_{33}&=\cos\phi\cos\theta\end{aligned}\;. \tag{5}$$

For N beacons, equation (4) can be extended simply as:

$$\begin{bmatrix}\mathbf{X}_{b1}\\\vdots\\\mathbf{X}_{bN}\end{bmatrix}=\begin{bmatrix}\mathbf{g}_1\left(\phi,\,\theta,\,\psi\right)\\\vdots\\\mathbf{g}_N\left(\phi,\,\theta,\,\psi\right)\end{bmatrix} \tag{6}$$

where $\mathbf{g}_i\left(\phi,\,\theta,\,\psi\right)$ is nonlinear function with respect to ϕ, θ and ψ.

To solve the above nonlinear equation using the Newton-Raphson method, we may rewrite (6) as a zero-problem (ZP):

$$\begin{bmatrix}\mathbf{f}_1\left(\phi,\,\theta,\,\psi\right)\\\vdots\\\mathbf{f}_N\left(\phi,\,\theta,\,\psi\right)\end{bmatrix}=\begin{bmatrix}\mathbf{X}_{b1}\\\vdots\\\mathbf{X}_{bN}\end{bmatrix}-\begin{bmatrix}\mathbf{g}_1\left(\phi,\,\theta,\,\psi\right)\\\vdots\\\mathbf{g}_N\left(\phi,\,\theta,\,\psi\right)\end{bmatrix}=\mathbf{0} \tag{7}$$

In (7), the solution of $\begin{bmatrix}\mathbf{f}_1\left(\phi,\,\theta,\,\psi\right)\\\vdots\\\mathbf{f}_N\left(\phi,\,\theta,\,\psi\right)\end{bmatrix}=\mathbf{0}$ becomes the estimated attitude. If we set the initial value of ϕ, θ and ψ, which are used in Newton-Raphson method as $\left[\phi_{ini},\,\theta_{ini},\,\psi_{ini}\right]^T$, respectively, the linearized (7) becomes (8)

$$\mathbf{0}=\begin{bmatrix}\mathbf{f}_1\left(\phi_{ini},\,\theta_{ini},\,\psi_{ini}\right)\\\vdots\\\mathbf{f}_N\left(\phi_{ini},\,\theta_{ini},\,\psi_{ini}\right)\end{bmatrix}$$
$$+\left.\begin{bmatrix}\frac{\partial}{\partial\phi}\mathbf{f}_1\left(\phi,\,\theta,\,\psi\right)&\frac{\partial}{\partial\theta}\mathbf{f}_1\left(\phi,\,\theta,\,\psi\right)&\frac{\partial}{\partial\psi}\mathbf{f}_1\left(\phi,\,\theta,\,\psi\right)\\\vdots&\vdots&\vdots\\\frac{\partial}{\partial\phi}\mathbf{f}_N\left(\phi,\,\theta,\,\psi\right)&\frac{\partial}{\partial\phi}\mathbf{f}_N\left(\phi,\,\theta,\,\psi\right)&\frac{\partial}{\partial\phi}\mathbf{f}_N\left(\phi,\,\theta,\,\psi\right)\end{bmatrix}\right|_{\phi=\phi_{ini},\,\theta=\theta_{ini},\,\psi=\psi_{ini}}\begin{bmatrix}\phi-\phi_{ini}\\\theta-\theta_{ini}\\\psi-\psi_{ini}\end{bmatrix} \tag{8}$$
$$=\begin{bmatrix}\mathbf{f}_1\left(\phi_{ini},\,\theta_{ini},\,\psi_{ini}\right)\\\vdots\\\mathbf{f}_N\left(\phi_{ini},\,\theta_{ini},\,\psi_{ini}\right)\end{bmatrix}+A_{ini}\begin{bmatrix}\phi-\phi_{ini}\\\theta-\theta_{ini}\\\psi-\psi_{ini}\end{bmatrix}$$

where A_{ini} is

$$A_{ini}=\begin{bmatrix}\frac{\partial}{\partial\phi}\mathbf{f}_1(\phi,\theta,\psi) & \frac{\partial}{\partial\theta}\mathbf{f}_1(\phi,\theta,\psi) & \frac{\partial}{\partial\psi}\mathbf{f}_1(\phi,\theta,\psi)\\ \vdots & \vdots & \vdots\\ \frac{\partial}{\partial\phi}\mathbf{f}_N(\phi,\theta,\psi) & \frac{\partial}{\partial\phi}\mathbf{f}_N(\phi,\theta,\psi) & \frac{\partial}{\partial\phi}\mathbf{f}_N(\phi,\theta,\psi)\end{bmatrix}\Bigg|_{\phi=\phi_{ini},\,\theta=\theta_{ini},\,\psi=\psi_{ini}} \tag{9}$$

Therefore, the updated ϕ, θ and ψ become

$$\begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}=-\left(A_{ini}^TA_{ini}\right)^{-1}A_{ini}^T\begin{bmatrix}\mathbf{f}_1(\phi_{ini},\theta_{ini},\psi_{ini})\\ \vdots\\ \mathbf{f}_N(\phi_{ini},\theta_{ini},\psi_{ini})\end{bmatrix}+\begin{bmatrix}\phi_{ini}\\ \theta_{ini}\\ \psi_{ini}\end{bmatrix} \tag{10}$$

Rewrite the time index ini as $n=1,\ldots,k,k+1,\ldots K$,

$$\begin{bmatrix}\phi_{k+1}\\ \theta_{k+1}\\ \psi_{k+1}\end{bmatrix}=-\left(A_k^TA_k\right)^{-1}A_k^T\begin{bmatrix}\mathbf{f}_1(\phi_k,\theta_k,\psi_k)\\ \vdots\\ \mathbf{f}_N(\phi_k,\theta_k,\psi_k)\end{bmatrix}+\begin{bmatrix}\phi_k\\ \theta_k\\ \psi_k\end{bmatrix} \tag{11}$$

As the recursion progresses, the attitude estimate becomes more accurate.

III. INS/Beacon Coupled System

In general, pure INS updates the attitude, velocity and position of the own-ship by using acceleration and angular velocity measured with accelerometers and gyroscopes, respectively [4]. However, due to characteristics of INS, navigation solution obtained from pure INS tends to diverge with the passage of time. To overcome the weakness of pure INS, EKF-based coupled systems, which calibrate navigation solution such as the INS/GPS navigation system has been used [1], [2].

In this paper, an INS/beacon coupled system is proposed. Since the difference between INS/GPS and INS/Beacon coupled system is measurement equation, we shall focus on the measurement equation only, mostly neglecting the dynamic equation.

3.1 Dynamic Equation

Typically, the dynamic equation of the EKF is based on INS error model derived from pure INS equation [6].

$$\delta\dot{x}(t)=F(t)\delta x(t)+G(t)w(t) \tag{12}$$

where $F(t)$, $\delta x(t)$, $\delta\dot{x}(t)$ and $w(t)$ are, respectively the dynamic matrix, error state variables, differentiation of error state variables and white Gaussian noise which follows $w(t)\sim N(0,Q)$ in time t. The error state variable is expressed as:

$$\delta x(t)=\begin{bmatrix}\delta Llh & \delta v_n & \delta\Psi & \nabla & \eta\end{bmatrix}^T \tag{13}$$

where δLlh, δv_n, $\delta\Psi$, ∇ and η indicate respectively, the errors of latitude, longitude, height, the velocity errors with respect to n-frame, the attitude errors, and the bias errors of accelerometer and gyroscope, with respect to the b-frame. The white Gaussian $w(t)$ models the random bias errors ∇ and η. The matrices $F(t)$ and $G(t)$ are given in [4].

3.2. Measurement Equation

The measurement equation can be expressed as follows [6]:

$$z(t)=H\delta x(t)+v(t) \tag{14}$$

where $v(t)$ represents white Gaussian measurement noise distributed as $v(t)\sim N(0,R)$. In section II, to calibrate attitude, the AOA measurements from beacon signals are used. However, as deriving equations to obtain attitude, the measurement equation at time t can be written as follows.

$$z(t)=-\begin{bmatrix}\mathbf{f}_1(\phi_t,\theta_t,\psi_t)\\ \vdots\\ \mathbf{f}_N(\phi_t,\theta_t,\psi_t)\end{bmatrix}=A_t\begin{bmatrix}\delta\phi_t\\ \delta\theta_t\\ \delta\psi_t\end{bmatrix}+v(t) \tag{15}$$

If we rewrite (15) in terms of the original full error state variables, then

$$z(t) = -\begin{bmatrix} \mathbf{f}_1(\phi_t, \theta_t, \psi_t) \\ \vdots \\ \mathbf{f}_N(\phi_t, \theta_t, \psi_t) \end{bmatrix} = \begin{bmatrix} 0_{3\times3} \\ 0_{3\times3} \\ A_t^T \\ 0_{3\times3} \\ 0_{3\times3} \end{bmatrix}^T \begin{bmatrix} \delta Llh(t) \\ \delta v_n(t) \\ \delta\Psi(t) \\ \nabla(t) \\ \eta(t) \end{bmatrix} + \nu(t) \quad (16)$$

$$= H_t \delta x(t) + \nu(t)$$

Now, by carrying out the time-update and the measurement-update in (17), the errors of the pure INS can be corrected [6]:

$$\begin{aligned} \delta\hat{x}_k^- &= f\left(\delta\hat{x}_{k-1}^+\right) \\ P_k^- &= \Phi(k|k-1)P_{k-1}^+\Phi^T(k|k-1) + G(k)QG^T(k) \\ K_k &= P_k^- H_k^T\left(H_k P_k^- H_k^T + R\right)^T \\ \delta\hat{x}_k^+ &= \delta\hat{x}_k^- + K_k\left(z_k - H_k\delta\hat{x}_k^-\right) \\ P_k^+ &= \left(I - K_k H_k\right)P_k^- \end{aligned} \quad (17)$$

where $f(\square)$ is the nonlinear dynamic equation and $\Phi(k|k-1) = e^{(F(t_{k+1}-t_k))}$. Also, $\delta\hat{x}_k^-$ and P_k^- are time-updated mean and covariance matrix of the error state variables, and $\delta\hat{x}_k^+$ and P_k^+ are measurement-updated mean and covariance matrix.

IV. Simulation results

This section illustrates the performance of the AOA-based attitude estimation algorithm and the INS/beacon coupled system via Monte Carlo simulation.

4.1.AOA-based attitude estimation algorithm

The performance of the AOA-based attitude estimation algorithm is evaluated with respect to the error of measured AOA (azimuth and elevation) and the number of beacons. The distance between user and each of the beacons is set to 1000m. The errors of measured azimuth and elevation range from 0.001° to 10° and the number of beacons ranges from 2 to 10. The initial attitude (roll, pitch, yaw) used Newton Raphson method is uniformly distributed in $[0^o, 10^o]$. The simulation is carried out using 1000 trials for each AOA errors and the number of beacons.

Fig. 2. shows the standard deviation of attitude errors with respect to the AOA error when the number of beacons is two. This result indicates that attitude errors is proportional to AOA error. Also, the errors of roll, pitch and yaw have similar patterns.

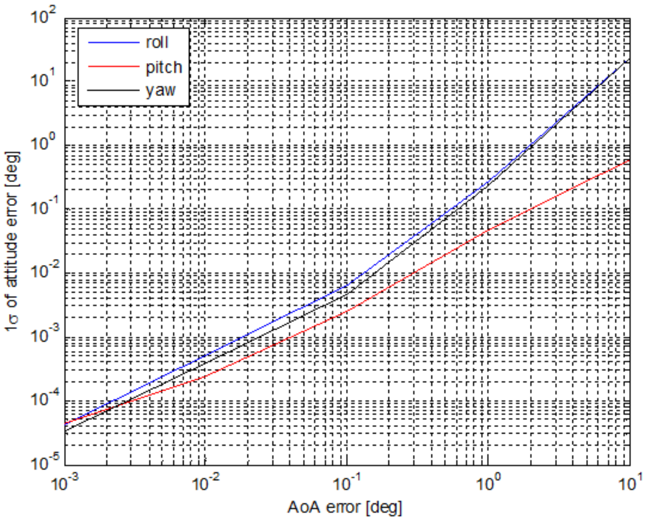

Fig. 2. Attitude error with respect to AOA error (Number of beacons = 2)

The standard deviation of the attitude errors with respect to the number of beacons is illustrated in the case of roll in Fig. 3. Note that the performance of the AOA-based attitude estimation algorithm is proportional to the number of beacons, as the measurements increase when the number of beacons increases. Also, the improvement of performance becomes converge with respect to increase of the number of beacons.

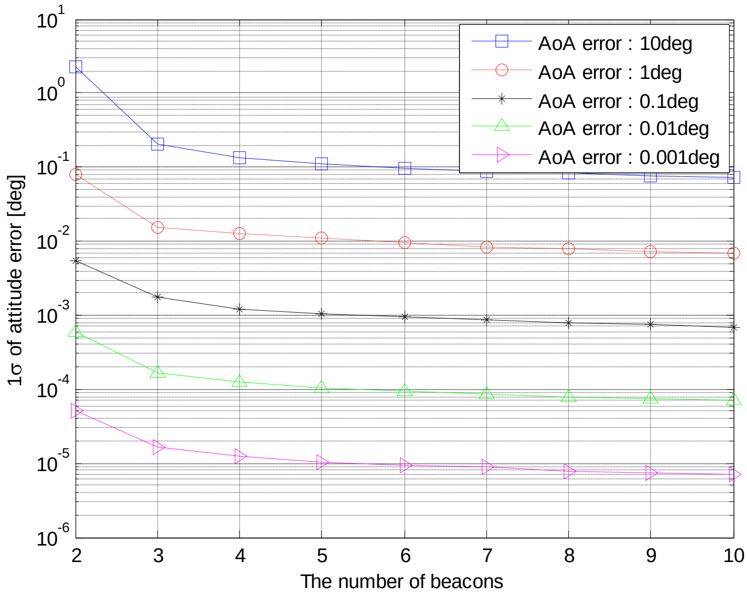

Fig. 3. Attitude errors with respect to the number of beacons.

4.2. INS/Beacon coupled system

The attitude estimation performances of the INS/GPS coupled system and the INS/Beacon coupled system are compared with a simple scenario. The standard deviations of acceleration, angular velocity measured from inertial measurement unit (IMU), GPS position error and AOA measured from beacon signals are set to $50\mu g$, $1^{\circ}/hour$, 1m and 1°, respectively. All sensors have the same data rate, 100Hz. The scenario lasted 100 seconds with no fluctuation and movement. The initial alignment which aligns initial attitude before starting navigation continues for 10 seconds, and INS/Beacon and INS/GPS coupled system are carried out for 90 seconds. The Fig. 4 shows the simulation results of both coupled systems.

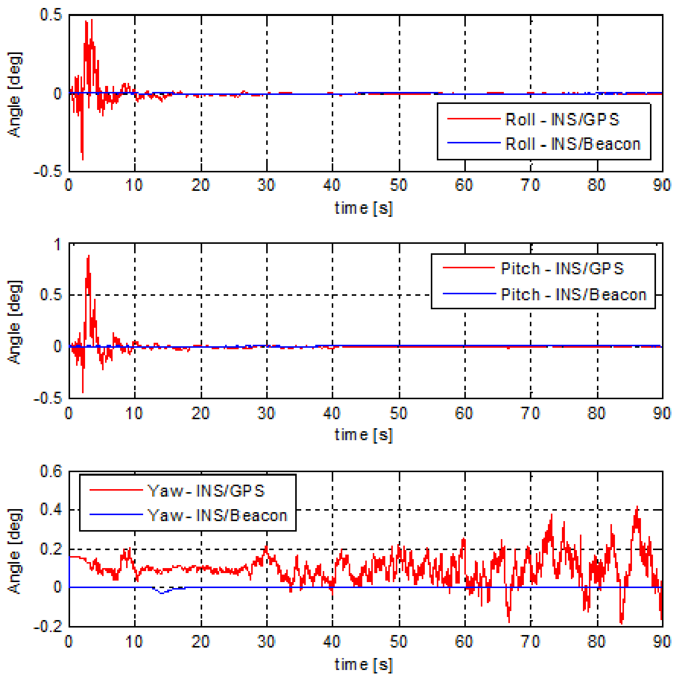

Fig. 4. Attitude estimation performance of INS/GPS and INS/beacon coupled systems

Fig. 4 illustrates the simulation results for roll, pitch and yaw. Note that the performance of the INS/beacon coupled system is superior to that of the INS/GPS coupled system in all cases.

To verify the performance of the INS/beacon coupled system in detail, Fig. 5 illustrates simulation results related to INS/beacon coupled system. As depicted in Fig. 5, estimated yaw has more errors compared to roll and pitch. Also the estimated attitude has fluctuation even if these values are extremely small. These may be caused by the measurement equation which has correlation between measurement and state variables when the measurement is obtained and could become worse as time progresses.

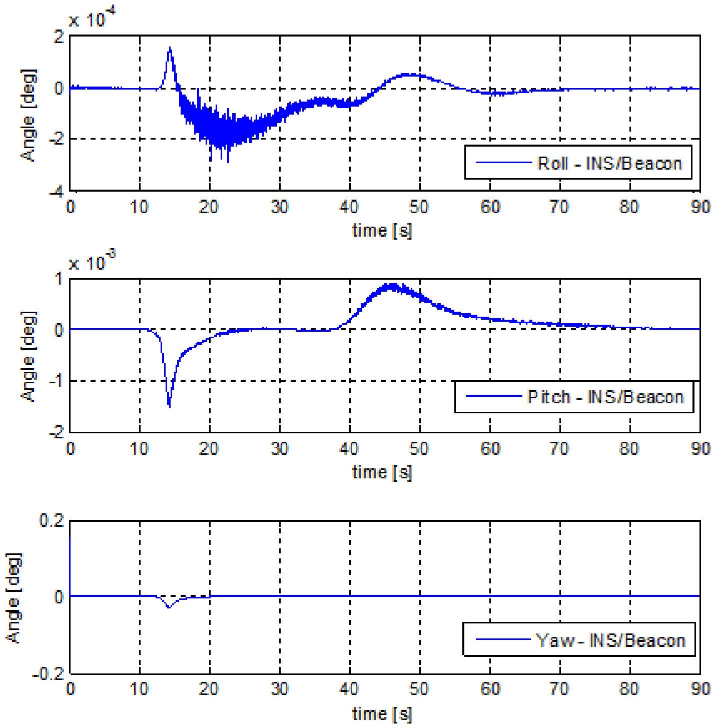

Fig. 5. Attitude estimation performance of the INS/beacon coupled system.

V. Conclusion

In this work, an AOA-based attitude estimation technique using beacon and INS/beacon coupled system are proposed to overcome the drawback of the conventional navigation systems such as INS or INS/GPS coupled system. First, an AOA-based attitude estimation technique using the Newton Raphson method is derived. Then, based on this method, an INS/beacon coupled system is proposed. The performance of the proposed techniques are assessed by Monte Carlo simulation for a simple scenario, comparing them with that of the conventional system. Although the proposed technique gives better performances compared to the conventional system, further refinement of the EKF model would be needed to increase the reliability of proposed technique.

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