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Comparative Study on Image Denoising and Inpainting Techniques

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Abstract: *Observed image signals are often corrupted by acquisition channel or artificial editing. The goal of image restoration techniques is to restore the original image from a noisy observation of it. Image denoising and inpainting are common image restoration problems that are both useful by themselves and important preprocessing steps of many other applications. Digital images are represented as matrices of equally spaced pixels, each containing a photon count. Due to the nature of light and environment it's natural that all images are noisy. Ever since digital images have existed, numerical methods have been proposed to improve the quality by reducing noise interference. These 'denoising' and inpainting methods require a noise model and an image model. It is relatively easy to obtain a noise model. And in case of inpainting, case by case method could be applied to find the best possible result. Since last decade the use of partial differential equations (pdes) have opened new frontier for image restoration techniques. We tried to look at an image the simplest way possible which is a 2D array. Using Laplacian and the method of calculating neighborhood values by Taylor expansion and iterative numerical approach we find that it's a possible way for both denoising and inpainting purpose.*

Index Terms: *Denoising, Filtering, Inpainting, Restoration, Partial Differential Equations (PDEs), Gaussian noise, Salt and pepper noise*

I. Introduction

The purpose of image restoration techniques is to restore the original image from a noisy observation of it. This is because humans have historically relied on their vision for tasks ranging from basic instinctive survival skills to detailed and elaborate analysis of works of art. Our ability to guide our actions and engage our cognitive abilities based on visual input is a remarkable trait of the human species, and much of how exactly we do what we do and seem to do it so well remains to be discovered. The ability to see and recognize things has limited functions for human eyes. As a result the sometimes the image itself is needed to be enhanced to make it clearer and take it to the level of human eye. In order to do this the field of image processing has been introduced to the modern world.

Image denoising and inpainting are the fundamental works and challenges in the field of image processing due to image signal corruption in acquisition process or artificial editing. Denoising an image is to correct defects acquired during the acquisition process of any scene i.e. photos, images and its reproduction on any display. It is one of the fundamental tasks for engineers in digital communication and hence image processing. In modern digital image processing data denoising is a well-known problem and it is the concern of diverse application areas. From the very beginning engineers and scientists have been trying to implement techniques to get rid of the noise and interference that corrupts the message signal.

On the other hand the modification of images in a way that is nodetectable for an observer is called inpainting[1].

A very large portion of digital image processing is devoted to image restoration. For example as part of the first demonstration of laser communication with a satellite at the moon, NASA Goddard scientists transmitted an image of the Mona Lisa from Earth to the Lunar Reconnaissance Orbiter at the moon by laser pulses that routinely track the spacecraft[2].

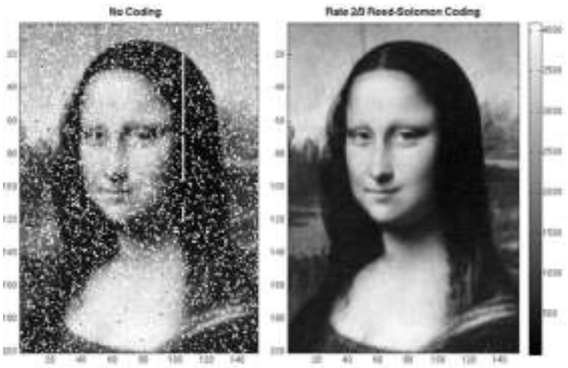


Fig. 1. Photo of Mona Lisa sent to the moon. Left one is before and the right one is after reconstruction, Image courtesy: Xiaoli Sun, NASA Goddard.

This depicts the importance of image reconstruction hence denising and inpainting in modern communication technology. Not only this but also the field of image processing has grown considerably during the past decade with the increased utilization of imagery in myriad applications coupled with improvements in the size, speed and cost effectiveness of digital computers and related signal processing technologies. Fields which traditionally used analog imaging are now switching to digital systems, for their flexibility and affordability. Important examples are medical image and video production, photography, remote sensing, and security monitoring. These and other sources produce huge volumes of digital image data every day, more than could ever be imagined. For all these purposes the noise free and non degraded images is the need of the time.

II. Different Types of Noises in Digital Images

Noise can be defined as any undesired artifact that contaminates an image. The presence of noise in an image can be due to several sources, resulting in different types of noise, from thermal noise in acquisition devices to periodic noise in the communication channel used to transmit an image from a remote sensing location to a base station, among many others. In this section, we present an overview of the main types of noise

A. Image acquisition and Noise Modeling

Noise is defined as a process M, which affects the acquired image G and is not part of the original image F. For example a model with additive noise can be written as

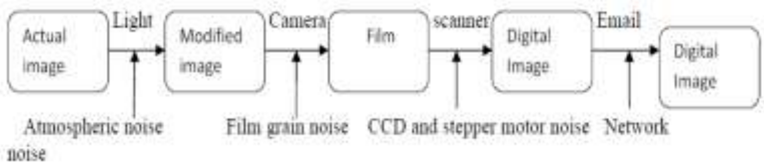


Fig. 2. Noise introduced at each step in the acquisition process.

$$F(i,j)=G(i,j)+M(i,j) \qquad (1)$$

Image acquisition is the process of obtaining a digitized image from a real world source. Each step in the acquisition process may introduce random changes into the values of pixels in the image. These changes are called *noise*.

The example illustrated the manner in which an image may be affected by noise during the acquisition process. The properties of the noise introduced at each capture step are likely to vary. However, there are three standard noise models which give the idea about types of noise encountered in most images: additive, multiplicative, and impulse noise. Noise (M) may be modeled either by histogram or probability density function (PDF) which is superimposed on probability density function of the original image (G) [3]

In the following the most common noises are discussed in brief.

B. Uniform noise

The uniform noise cause by quantizing the pixels of image to a number of distinct levels is known as quantization noise. It has approximately uniform distribution. In the uniform noise the level of the gray values of the noise are uniformly distributed across a specified range. Uniform noise can be used to generate any different type of noise distribution. This noise is often used to degrade images for the evaluation of image restoration algorithms. This noise provides the most neutral or unbiased noise.

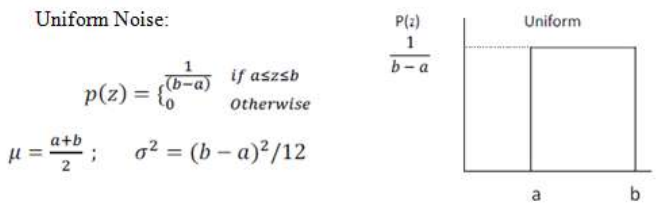


Fig. 4. PDF, mean, variance of uniform noise.

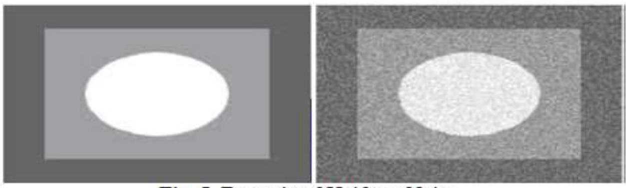


Fig. 5. Example of Uniform Noise.

C. Gaussian Noise or Amplifier Noise

This noise has a probability density function [pdf] of the normal distribution. It is also known as Gaussian distribution. It is a major part of the read noise of an image sensor that is of the constant level of noise in the dark areas of the image. Gaussian noise is evenly distributed over signal. This means that each pixel in the noisy image is the sum of the true pixel value and a random Gaussian distributed noise value.

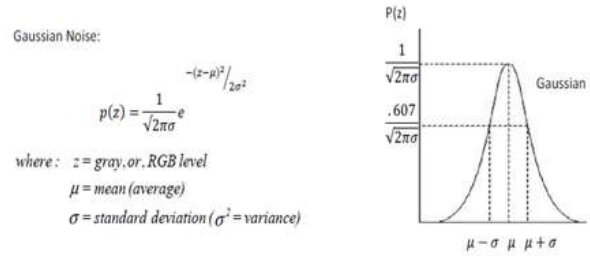


Fig. 6. PDF, mean, variance of Gaussian Noise.

Fig. 7. Original Image (Left) Example of Gaussian Noise (Right).

D. Salt and Pepper noise

Salt and Pepper noise is an impulse type of noise. It is actually the intensity spikes. This type of noise is coming due to errors in data transmission. This noise arises in the image because of sharp and sudden changes of image signal. Dust particles in the image acquisition source or over heated faulty components can cause this type of noise. Salt pepper noise contains two values, a and b. The probability of each is typically less than 0.1. If the numbers are greater than this numbers the noise will swamp out image. For 8-bit image the typical value for 255 for salt-noise and pepper noise is 0. The corrupted pixels are set alternatively to minimum or to maximum value, giving the image an appearance of “salt and pepper”.

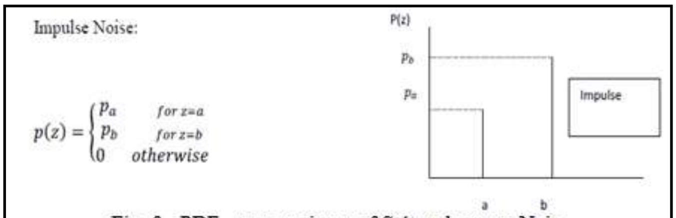


Fig. 8. PDF, mean variance of Salt and pepper Noise.

Fig. 9. Original Image (Left) Example of Salt and Pepper Noise(Right).

There are many other noises which may affect digital images. Some of the noises by which images gets most commonly distorted are

A. Gamma noise
B. Rayleigh noise
C. Photon Noise
D. Brownian noise

III. Image De-Noising and Filters

Image de-noising is very important task in image processing for the analysis of images. Ample image de-noising algorithms are available, but the best one should remove the noise completely from the image, while preserving the details. De-noising methods can be linear as well as non-linear. Where linear methods are fast enough, but they do not preserve the details of the images, whereas the non- linear methods preserve the details of the images. Broadly speaking, De-noising filters can be categorized in the following categories:

- Averaging filter
- Order Statistics filter
- Adaptive filter

g(x,y) ⟹ Filter ⟹ f(x,y)

Let $w(x)$ be the input signal subjected to filtering, and $z(x)$ be the filtered output. If the filter satisfies certain conditions such as linearity and shift invariance, then the output filter can be expressed mathematically in simple form as

$$z(x) = \int w(t)h(x-t)dt \qquad (1)$$

Basic Idea is to replace each pixel intensity value with n new value taken over a neighborhood of fixed size.

Linear filters are used to remove certain type of noise. Gaussian or Averaging filters are suitable for this purpose. These filters also tend to blur the sharp edges, destroy the lines and other fine details of image, and perform badly in the presence of signal dependent noise [4]. Also in recent years, a variety of non-linear median type filters such as rank conditioned, weighted median, relaxed median, rank selection have been developed to overcome the shortcoming of linear filter.

E. Different Type of Linear and Non-Linear Filters

1) Mean Filter:

The mean filter is a simple spatial filter. It is an averaging linear filter [5]. It is a sliding-window filter that replaces the center value in the window. It replaces with the average mean of all the pixel values in the kernel or window. The window is usually square but it can be of any shape.

Mean filter takes the average of intensity values in a m x n region of each pixel (usually m = n) and takes the average as the new pixel value

$$h(i,j) = \frac{1}{mn}\sum_{k\in m}\sum_{l\in n} f(k,l) \qquad (2)$$

It is implemented with a convolution mask, which provides a result that is a weighted sum of the values of a pixel and its neighbors. It is also called a linear filter. The mask or kernel is a square.

$$A = \begin{bmatrix} 50 & 55 & 70 \\ 66 & 230 & 61 \\ 59 & 63 & 58 \end{bmatrix}, \qquad g(i,j) = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

For example using the convolution mask g(i,j) we can have the value of A.

Fig. 10. Image with Gaussian noise (Left) With Mean Filter (Right)

Advantage*:*
a. Easy to implement
b. Used to remove the impulse noise, Gaussian noise.
Disadvantage: It does not preserve details of image. Some details are removes of image with using the mean filter.

2) Median Filter:

Median Filter is a simple and powerful non-linear filter which is based order statistics. Here the center value of the pixel is replaced by the median of the pixel values under the filter region [6]. It is easy to implement method of smoothing images. Median filter is used for reducing the amount of intensity variation between one pixel and the other pixel. In this filter, we do not replace the pixel value of image with the mean of all neighboring pixel values, we replaces it with the median value. Then the median is calculated by first sorting all the pixel values into ascending order and then replace the pixel being calculated with the middle pixel value. The median filter gives best result when the impulse noise percentage is less than 0.1 %.

The median filter also follows the moving window principle similar to the mean filter. A 3x3, 5x5, or 7x7 kernel of pixels is scanned over pixel matrix of the entire image. The median of the pixel values in the window is computed, and the center pixel of the window is replaced with the computed median.

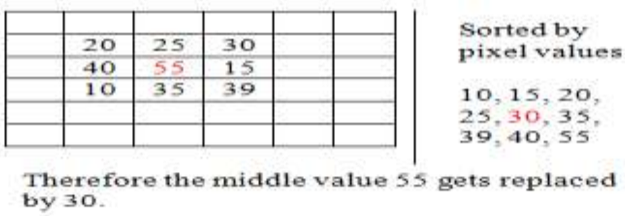


Fig 11**:** Image with impulse noise (Left) With Medean Filter(Right)

Median filter is good for salt and pepper noise. These filters are widely used as smoothers for image processing, as well as in signal processing. A major advantage of the median filter over linear filters is that the median filter can eliminate the effect of input noise values with extremely large magnitudes.

Advantage:
a. It is easy to implement.
b. Used for de-noising different types of noises.
Disadvantage**:**
a. Median Filter tends to remove image details while reducing noise such as thin lines and corners.

3) Adaptive Filter

These filters change their behavior on the basis of statistical characteristics of the image region, encompassed by the filter region. The adaptive filter does a better job of denoising images compared to the averaging filter or the mean filter. The main difference between mean filter and the adaptive filter is that in adaptive filter the weight matrix varies after every iteration on the other hand it remains constant in mean filter. Adaptive filters are capable of denoising non-stationary images, that is, images that have abrupt changes in intensity. Such filters are known for their ability in automatically tracking an unknown circumstance or when a signal is variable with little a priori knowledge about the signal to be processed. In general, an adaptive filter iteratively adjusts its parameters during scanning the image to match the image generating mechanism.

For example, BM3D is a nonlocal image modelling technique based on adaptive, high order groupwise models. Its detailed discussion can be found in [7]. Below, using the example of the denoising algorithm [8], we recall the concept of the BM3D modeling. The denoising algorithm can be split into three steps.
1) *Analysis*. Similar image blocks are collected in groups. Blocks in each group are stacked together to form 3-D data arrays, which are decorrelated using an invertible 3D transform.
2) *Processing*. The obtained 3-D group spectra are filtered by hard thresholding.
3) *Synthesis.* The filtered spectra are inverted, providing estimates for each block in the group. These blockwise estimates are returned to their original positions and the final image reconstruction is calculated as a weighted average of all the obtained block wise estimates.

4) Histogram Equalization

Histogram equalization is common technique for enhancing the appearance of images. Suppose we have an image which is predominantly dark. Then its histogram would be skewed towards the lower end of the grey scale and all the image detail is compressed into the dark end of the histogram. If we could `stretch out' the grey levels at the dark end to produce a more uniformly distributed histogram then the image would become much clearer. Assume for a moment that intensity levels are continuous quantities normalized to the range [0, 1], and let *prr*() denote the probability density function (PDF) of the intensity levels in a given image, where the subscript is used for differentiating between the PDFs of the input and output images. Suppose that we perform the following transformation on the input levels to obtain output (processed) intensity levels, *s*,

$$S = T(r) = \int_0^T P_r(W) \qquad (3)$$

where w is a dummy variable of integration. It can be shown that the probability density function of the output levels is *uniform* [3]; that is,

$$P_s(S) = \begin{cases} 1, for\ 0 \le S \le 1 \\ 0, \quad otherwise \end{cases} \qquad (4)$$

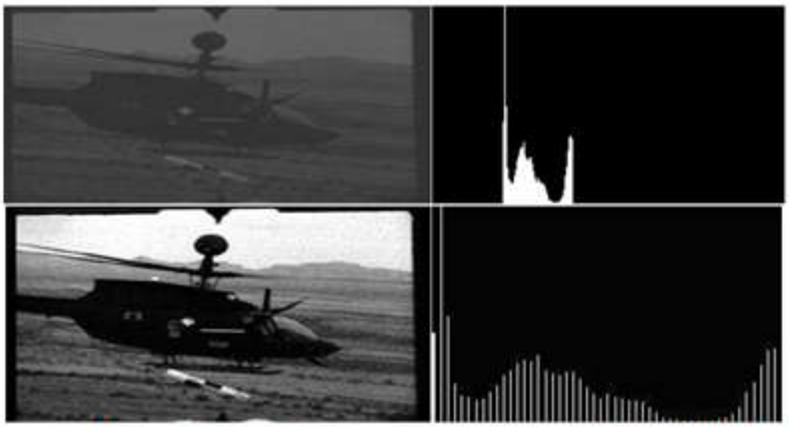


Fig12: Image with original histogram (up) With histogram eqaulization (down)

IV. Image in Painting

Image inpainting is a practice that predates the age of computers. It is a technique used by art museum craftsmen to fill in parts of a painting which have decayed or been damaged over the course of many years. It is called inpainting as literal terms for the process of painting in holes or cracks in an artwork. Digital image restoration has received attention over the last few years, because of the many applications that are has in image processing, including removal of scratches, objects, text or logos from digital images, while still retaining the consistency of the image in those areas which have been restored or retouched. The terminology for digital inpainting was first introduced by Bertalmı´o, Sapiro, Caseles and Ballester [9].

The inpainting process consists of, in general, the filling-in of missing information within a domain D or to replace this domain with a different kind of information, based upon the image information available outside of the domain D. This domain is referred to as the inpainting domain and is where the original image has been damaged due to age action or also the region that we desire to add or remove information. The filling in of missing information and the removal of noise are two very important topics in image processing, with several applications such as image coding and wireless image transmission (e.g. recovering lost blocks), special effects (e.g. removal of objects) and image restoration (e.g. fold lines, scratches and noise removal). The basic idea of inpainting algorithms is to fill-in regions with available information from their surroundings. In most cases, the available data of the original image is noisy which makes it necessary to eliminate the noise and fill-in the blank spaces (those without information). The problem in inpainting is posed as follows: given an area to be inpainted, filling in the missing areas or modifying the damaged ones in a non-detectable way for an observer not familiar with the original images [9]. Texture synthesis accepts a given sample texture and creates an output image which can have arbitrary dimensions, but which retains the texture properties derived from the original sample [10].

Both these approaches have different direct applications: where inpainting is used explicitly for filling holes in an image, texture synthesis draws from natural or artificial textures to create a textured pattern, which finds extensive applications in graphics. However, they are basically methods used to synthesize a pixel given some information about another set of pixels. We can think of the texture synthesis problem, which requires an input texture, as reducing to the inpainting problem if we assume that the sample or input texture which it attempts to replicate can be found in the same image where the region to be synthesized lies. Hence we see that, for two dimensional digital images, one can use both texture synthesis and inpainting approaches to restore the image. The two approaches can be collectively referred to as hole filling approaches, since they do try and remove unwanted features or holes in a digital image. The common requirement for hole filling algorithms is that the region to be filled has to be defined by the user. This is because the definition of a hole or distortion in the image is largely perceptual. It is impossible to mathematically define what a hole in an image is. A fully automatic hole filling tool may not just be unfeasible but also undesirable, because in some cases, it is likely to remove features of the image which are actually desired. The user should have control over the selection of the region, after which the filling is entirely automatic across the spectrum of methods to fill texture or inpaint.
There could be two schools of thought on approaching an inpainting problem: making use of information about the local neighborhood of a pixel only, and making use of global statistics to aid the local statistics.

F. Local in Painting

Bertalmio et al [9] define the problem of local inpainting as, given a region to be inpainted Ω and its boundary $\partial\Omega$, to synthesize pixel values from the boundary inwards, using neighbourhood pixel information to continue the inpainting process.

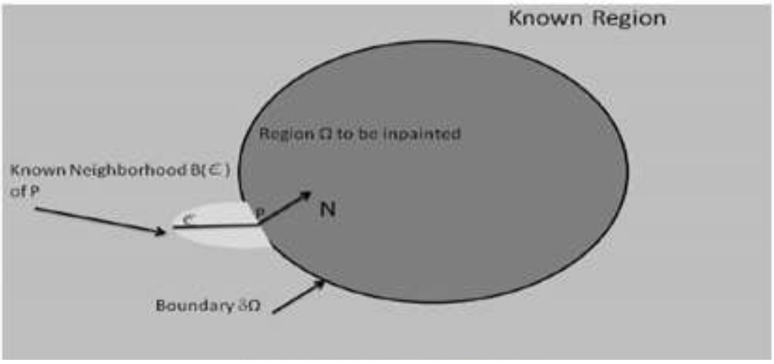


Fig 13: Local inpainting

These methods assume a prior information about the probability distribution of the relation between a pixel value and its neighborhood, which will help fill in a pixel lying on the hole boundary. Also, other properties such as high smoothness, low total variation or low curvature are assumed to create the framework on which the actual algorithm runs.
There are two significant approaches to local information based hole filling of images:

- Based on Partial Differential Equations (PDE)
- Based on convolution

1) PDE Based Methods:

PDE based models can be formally written in the general form:

$$\frac{\partial u}{\partial t}(t,x) + F\big(x, u(t,x), \nabla u(t,x), \nabla^2 u(t,x)\big) = 0 \ in\ \Omega$$
$$\frac{\partial u}{\partial N}(t,x) = 0 \ on\ \partial\Omega \ \text{(Neumann Boundary condition)}$$
$$u(0,x) = u_0(x) \quad initial\ condition \qquad (5)$$

where $u(t, x)$ is the restored version of the initial degraded image $u_0(x)$. As usual ∇u and $\nabla^2 u$ stand respectively for the gradient and the Hessian matrix of u with respect to the space variable x.

V. Proposed Method

During the previous decade or so, mathematical frameworks employing powerful tools of partial differential equations (PDEs) and functional analysis have emerged and successfully applied to various image processing (IP) tasks, particularly for image restoration [11,12]. Those PDE-based models have allowed researchers and practitioners not only to introduce effective models but also to improve traditional algorithms in image denoising. However, these PDE-based models tend to either converge to a piecewise constant image or introduce image blur (undesired dissipation), partially because the models are derived by minimizing a functional of the image gradient. As a consequence, the conventional PDE-based models may lose interesting fine structures during the denoising [13]. In order to overcome these problems researchers have studied various mathematical and numerical techniques which incorporate diffusion modulation, constraint terms, and iterative refinement [13, 14]. But more advanced models and appropriate numerical procedures are yet to be developed.

G. Proposal

We know that images are commonly represented as 2D functions of space: f(x, y).

A digital image f(m, n) may be interpreted as a discretized version of f(x, y) in a 2D array, or as a matrix. An M × N matrix has M rows and N columns; its height is M and width is N; numbering of the elements starts with (1, 1) at the top left corner and ends with (M,N) at the lower right corner of the image. This gives an idea of how an image can be represented in two dimensions.

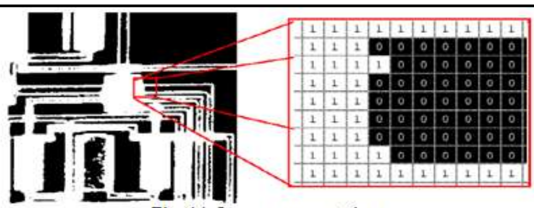
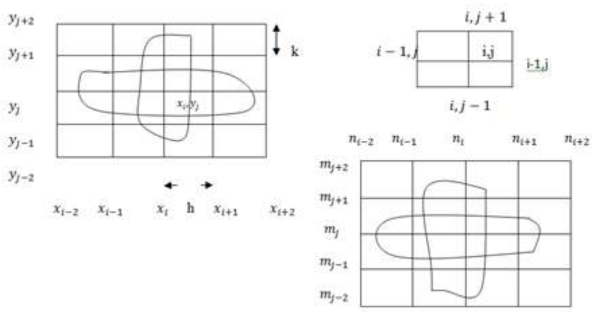


Fig. 14. Image representation



From definition for any function 'u' defined on some two-dimensional domain, the Laplacian $\Delta u\ of\ u$ is defined by

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad (6)$$

Replacing the second order derivatives with their finite difference equivalents at the points (x_i, y_j), the obtained equation is

$$\frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{h^2} + \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{k^2} = 0 \qquad (7)$$

If the step size, h=k then the equation 7 becomes

$$f_{i+1,j} - 2f_{i,j} + f_{i-1,j} + f_{i,j+1} - 2f_{i,j} + f_{i,j-1} = 0$$
$$f_{i+1,j} - 4f_{i,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1} = 0$$
$$f_{i,j} = \tfrac{1}{4}(f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1}) \qquad (8)$$

To solve the above mentioned system there could be many iterative methods. But here we would be discussing mainly Jacobi's and Gauss- Seidel method.
For Jacobi's method if we want to apply it on equation (8)

$$f_{i,j} = \frac{1}{4}(f_{i+1,j+1} + f_{i-1,j-1} + f_{i+1,j+1} + f_{i-1,j-1})$$

Let $f_{i,j}^{(n)}$ denotes the nth iterative values of $f_{i,j}$. Now the Jacobi's method suggests

$$f_{i,j}^{(n+1)} = \frac{1}{4}\left(f_{i-1,j}^{(n)} + f_{i+1,j}^{(n)} + f_{i,j-1}^{(n)} + f_{i,j+1}^{(n)}\right) \qquad (9)$$

And the Gauss-Seidel Method uses the latest iterative values available and scans the mesh points systematically from left to right along successive rows. The iterative formula is

$$f_{i,j}^{(n+1)} = \frac{1}{4}\left(f_{i-1,j}^{(n+1)} + f_{i+1,j}^{(n)} + f_{i,j-1}^{(n+1)} + f_{i,j+1}^{(n)}\right) \qquad (10)$$

Depending upon the requirement any of the methods can be used [15].

VI. Result

Here we tried to test the algorithm on an common image so that we could understand the problems that we have with the proposed method.

Fig 15: Image of Lena original (left) With Proposed method (right)

From the result, it's clear enough that this technique creates blurriness to any image although it makes an image smother. Fundamental problem with these types of algorithms is that when applied they work on the whole image. Also as it follows the iterative formula to eliminate the problems within the image, more iteration creates more blurriness. Here we tried with an iteration value of 10.

This problem could be solved if we tried this algorithm with a region of interest based technique. That way we won't be applying the algorithm on the whole image instead it would be applied on the particular area where there the problem exists.

VII. Conclusion

For decades scientists have been working with noise and denoise techniques. This window from the field of image processing will be ever growing and without any boundaries. Merging different techniques and proper knowledge will take it to near perfection. But yet there will be scope for improvement. With the increase in modern day processors more techniques could be merged together to make better and more powerful filters.

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