

Design and Realization of Practical FIR Filter Using Hybrid Window and CSD Algorithm

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Abstract: Multiplier-less FIR filter design is the major requirement in VLSI signal processing. In this work the practical FIR low pass filter is designed using hybrid window for various mathematical operations like addition, average, exclusive-or and multiplication. Their frequency responses are obtained by using Matlab. Multiplication based design of FIR Low Pass Filter (LPF) is realized in direct form structure and implemented in VLSI. This structure consists of adders, multipliers and delay elements. In VLSI multipliers consume more power so that for multiplication of input with coefficients, the Canonical Signed Digit (CSD) algorithm is applied for multiplication process to reduce the power consumption. Apart from this the simulation, synthesis, delay and power reports are analyzed by using Xilinx 13.1 ISE and XPower Estimator 11.1

Index terms: CSD algorithm, Double precision format, FIR filter, Hybrid window, Q format.

I. Introduction

A FIR filter is a Linear Time Invariant (LTI) system, which exhibits linear phase and inherent stability has been seen in many applications such as equalization of Communication channels, signal detection in radar, sonar, pulse shaping etc.,^{1,2,3,4}. To avoid the effect of Gibbs phenomenon, windowing method is used to design FIR filters. Rectangular, Bartlett, Hamming, Hanning, Blackman and Kaiser are the different windowing techniques. Out of all these Blackman window is superior in side lobe level reduction and Kaiser window is best in less transition width^{1,3,4,9}. So, the pros of these two windows are taken to design a new hybrid window¹ in this paper. Hybrid window is a new concept which is obtained by the combination of two different windows with different mathematical operations. The characteristics of FIR low pass filter, the windowing technique and the required equations for the hybrid windows are explained further. We used MATLAB tool for implementation of the filters. The output for the frequency response of FIR low pass filter using hybrid window are shown further. Tabular forms for Relative side lobe attenuation are shown further. The VLSI implementation of a linear phase Finite impulse response (FIR) filter with fixed coefficients, large amount of power are required for multiplications when compared to additions and subtractions involved. The number of additions/subtractions used for coefficient multiplications increases the complexity of FIR filters². The CSD codes minimize the number of adders/subtractors required in each coefficient multiplications and thus reduces the complexity of FIR filters and power required^{5 to 8, 10, 11}.

II. Background

2.1 FIR filter Design Using Hybrid Window Functions

The hybrid window is formed by combining two window functions by some mathematical operations like addition, multiplication, averaging, Exclusive-or[1].

The windows which taken here are Blackman window and Kaiser Window

- **Blackman window:**

$$\begin{aligned}wb(n) &= 0.42 + 0.5 \cos(2 * \pi * n / N - 1) + 0.08 \cos(4 * \pi * n / N - 1) \\ \text{for } |n| &\leq (N-1)/2 \\ &= 0 \text{ otherwise}\end{aligned}\quad (1)$$

- **Kaiser window:**

$$\begin{aligned}wk(n) &= I_0[\alpha(1 - (2 * n / N - 1)^2)^{0.5}] / I_0[\alpha] \\ \text{for } |n| &\leq (N-1)/2 \\ &= 0 \text{ otherwise}\end{aligned}\quad (2)$$

Where, alpha is adjustable parameter

$I_0(x)$ = zeroth-order Bessel function

• **Addition:**

$$w_n(n) = w_b(n) + w_k(n) = 0.42 + 0.5\cos(2\pi n/N - 1) + 0.08\cos(4\pi n/N - 1) + I_0[\alpha(1 - (2n/N - 1)^2)^{0.5}] / I_0[\alpha] \quad (3)$$

• **Multiplication:**

$$w_n(n) = w_b(n) * w_k(n) = 0.42 + 0.5\cos(2\pi n/N - 1) + 0.08\cos(4\pi n/N - 1) * I_0[\alpha(1 - (2n/N - 1)^2)^{0.5}] / I_0[\alpha] \quad (4)$$

• **Averaging:**

$$w_n(n) = (w_b(n) + w_k(n)) / 2 = 0.21 + 0.5\cos(2\pi n/N - 1) + 0.04\cos(4\pi n/N - 1) + 0.5I_0[\alpha(1 - (2n/N - 1)^2)^{0.5}] / I_0[\alpha] \quad (5)$$

• **Exclusive-or:**

$$w_n(n) = k * w_b(n) + (1 - k) * w_k(n) \quad [3.21]$$

$$= k * (0.42 + 0.5\cos(2\pi n/N - 1) + 0.08\cos(4\pi n/N - 1)) + (1 - k) * (I_0[\alpha(1 - (2n/N - 1)^2)^{0.5}] / I_0[\alpha])$$

Where $k=0$ to 1 . (6)

To improve the frequency response of the filter, hybrid window technique can be used. The combination of windows reduces the ripples in filter response, decrease the stop band attenuation and increases the accuracy compared to filter response using single window.

2.1.1 FIR filter design steps

1. Choose desired frequency response of the filter $H_d(\omega)$
 2. Take inverse Fourier transform of $H_d(\omega)$ to obtain the desired response $h_d(n)$.
- By definition of inverse Fourier transform

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega n} H_d(\omega) \quad (7)$$

3. Choose window function $w(n)$ and determine the product of $h_d(n)$ and $w(n)$.

Let product this given by

$$h(n) = h_d(n) * w(n) \quad (8)$$

4. The transfer function $H(z)$ of the filter is obtained by taking z-transform of $h(n)$. Realize the filter by a suitable structure.

2.1.2 Specifications

The table 1 lists the specifications used in this work to design a FIR low pass filter.

Order (N)	15
Cut-off frequency (ω_c)	0.25π rad/sec
Window used	Hybrid
α	7
β	11
eps	0.01

2.2 Q_{n.m} Format for Fixed-point Arithmetic

Most signal processing and communication systems are first implemented in double precision floating point arithmetic using tools like MATLAB. While implementing these algorithms the main focus of the developer is to correctly assimilate the functionality of the algorithm. Q_{n.m} is a fixed positional number system for representing floating point numbers. The Q_{n.m} format of an N bit number sets n bits to the left and m bits to the right of the binary point. In cases of signed numbers, the MSB is used for the sign and has negative weight. A two's complement fixed point number in Q_{n.m} format is equivalent to $b = b_{n-1} b_{n-2} \dots b_1 b_0 b_{-1} b_{-2} \dots b_m$, with equivalent floating point value:

$$-b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \dots + b_12^1 + b_0 + b_{-1}2^{-1} + b_{-2}2^{-2} + \dots + b_m2^{-m} \quad (9)$$

For example to compute the floating point equivalent of 0 1101 1010 0000000 in signed Q_{1.15} format. The fields of the bits and their equivalent weights are shown in Figure 1.

2^0	.	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}	2^{-9}	2^{-10}	2^{-11}	2^{-12}	2^{-13}	2^{-14}	2^{-15}
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Assigning values to bit locations gives the equivalent floating point value of the Q format fixed point number:
 0 1101 1010 0000 000 = $1/2 + 1/4 + 1/16 + 1/32 + 1/128 = 0.85156$

2.3 Canonical Signed Digit Representation

CSD is a radix 2 signed digit coding. It codes constant using signed digits 1, 0 and -1. An N bit constant C is represented as:

$$C = \sum_{i=0}^{N-1} s_i 2^i \text{ for } s_i \text{ belongsto } \{-1, 0, 1\} \tag{10}$$

The expression implies that the constant is coded using signed digits 1, 0 or -1, where each digit s_i contributes a weight of 2^i to the constant value. The CSD representation has the following properties:

- No two consecutive bits in CSD representation of a number are non zero.
- The CSD representation of a number uses a minimum number of non zero digits.
- The CSD representation of a number is unique.

CSD representation of a number can be recursively computed using the string property. The number is observed to contain any string of 1s while moving from the least significant bit (LSB) to the most significant bit (MSB). The LSB in a string of 1s is changed to $\bar{1}$ that represents -1, and all the other 1s in the string are replaced with zeros, and the 0 that marks the end of the string is changed to 1. After replacing a string by its equivalent CSD digits, the number is observed again moving from the coded digit to the MSB to contain any further string of 1s. The newly found string is again replaced by its equivalent CSD representation. The process is repeated until no string of 1s is found in the number.

Example: Converting 16'b0011 1110 1111 0111 to CSD representation involves the following recursion. Find a string while moving from LSB to MSB and replace it with its equivalent CSD representation:

0011111011110111
00111110111100 $\bar{1}$

The newly formed number is observed again for any more string of 1s to be replaced by its equivalent CSD representation:

00111110111100 $\bar{1}$
001111110000 $\bar{1}$ 00 $\bar{1}$

The process is repeated until all strings of 1s are replaced by their equivalent CSD representations:

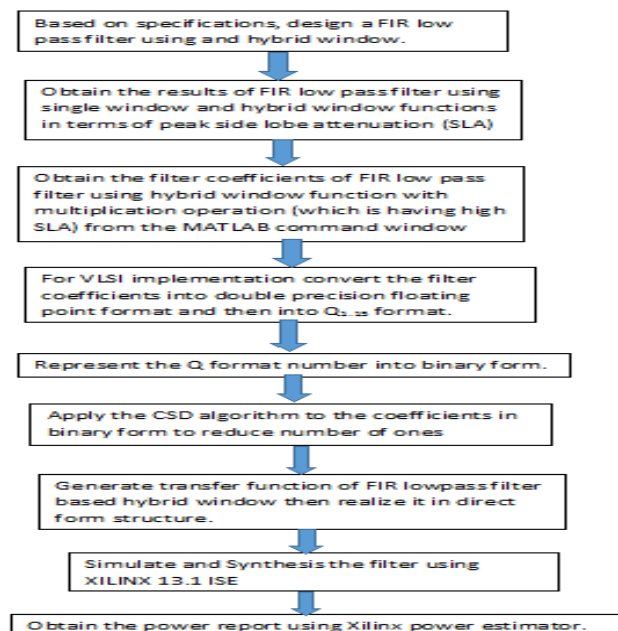
001111110000 $\bar{1}$ 00 $\bar{1}$
0100000 $\bar{1}$ 0000 $\bar{1}$ 00 $\bar{1}$

All these steps can be done simultaneously by observing isolated strings or a set of connected strings with one 0 in between. All the isolated strings with more than one 0 in between are replaced by their equivalent CSD representations, and for each connected string all the 0s connecting individual strings are changed to $\bar{1}$, and all the 1s in the strings are all changed to 0. The equivalent CSD representation computed in one step is:

0011111011110111
0100000 $\bar{1}$ 0000 $\bar{1}$ 00 $\bar{1}$

III. Proposed design

The flowchart in figure 2 explains the proposed design procedure for practical FIR low pass filter using hybrid window and CSD algorithm.



The following table 2 shows FIR filter coefficients with hybrid window (i.e., multiplication operation) of Kaiser and Black man window for N=15 in double precision floating point format, Q_{1.15} format, Binary representation, CSD representation

Table 2. FIR filter coefficients

Filter coefficients	Double Precision Floating Point Format	Q _{1.15} format	Binary representation	CSD representation
h(0)=0.2500	0.249999974297906	8192	10000000000000	10000000000000
h(1)=0.1860	0.185993627075655	6095	1011111001111	10100001010001
h(2)=0.0734	0.073385589057126	2405	100101100101	101010100101
h(3)=0.0125	0.012542059857874	411	110011011	1010100101
h(4)=-0.0000	0.000002276359643	0	0	0
h(5)=-0.0002	0.000180333843739	6	110	1010
h(6)=-0.0000	-0.000007002652330	0	0	0
h(7)=0	0	0	0	0

The following figure 3 shows the architecture of FIR low pass filter realized in direct form structure and the output expression of filter is given in equation 11.

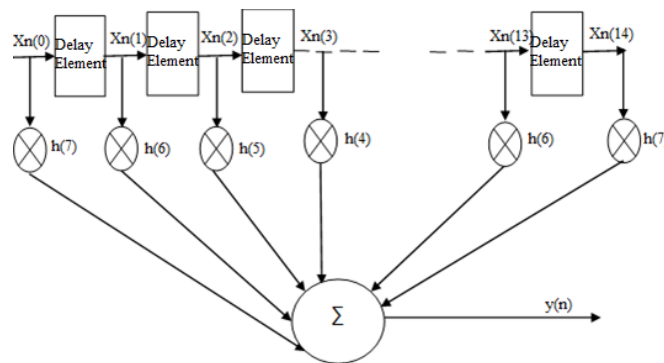


Figure 3. Architecture of FIR low pass filter

$$Y(n) = x_n(0)*h(7)+x_n(1)*h(6)+x_n(2)*h(5)+x_n(3)*h(4)+x_n(4)*h(3)+x_n(5)*h(2)+x_n(6)*h(1) + x_n(7)*h(0)+x_n(8)*h(1)+x_n(9)*h(2)+x_n(10)h(3)+x_n(11)*h(4)+x_n(12)*h(5)+x_n(13)*h(6) + x_n(14)*h(7) \quad (11)$$

IV. Results and Discussion

MATLAB Simulation Results for 15 tap FIR low pass filter using hybrid windows are shown from figures 4 to 7. Blackman window and Kaiser Window are taken here for generating new hybrid window functions using mathematical operations. Relative Side Lobe Attenuation (SLA) of FIR low pass filter is shown in table3 using different hybrid windows. From table 2, FIR filter using multiplication operator window response gives the highest attenuation compared to other hybrid windows. So, these filter coefficients are taken to realize in direct form structure. The Xilinx Integrated Software Environment (ISE) is used for performing simulation, synthesis and implementation of design. The coding was done by using Verilog language. A 15 tap FIR LPF is designed and implemented with Q_{1.15} format filter coefficients in binary and CSD representations. Simulation, power and delay reports of filter with binary and CSD representations are shown in figures 8 to 15. Table 4 shows the filter implemented using CSD consumes less power when compared to binary representation

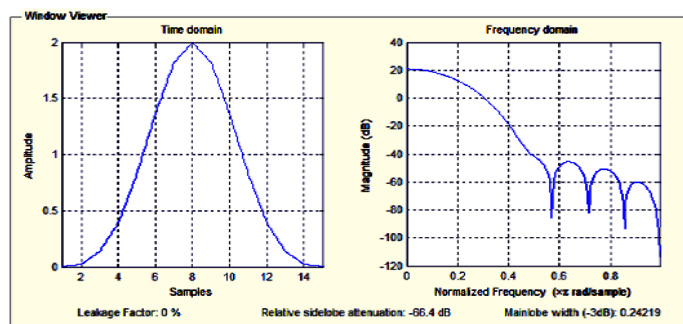


Figure 4. Response of FIR low pass filter using hybrid window with addition operator

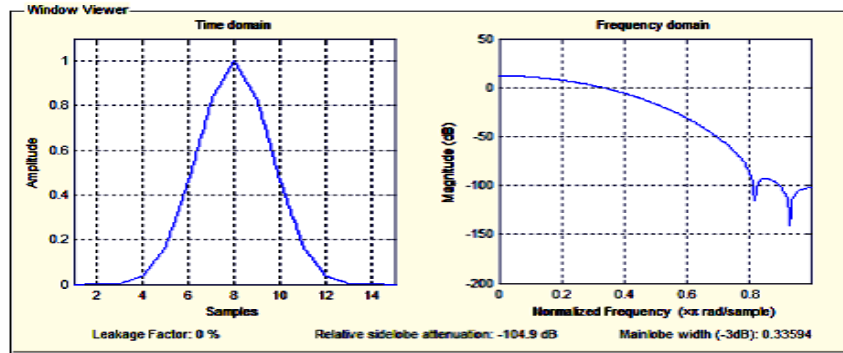


Figure 5. Response of FIR low pass filter using hybrid window with multiplication operator

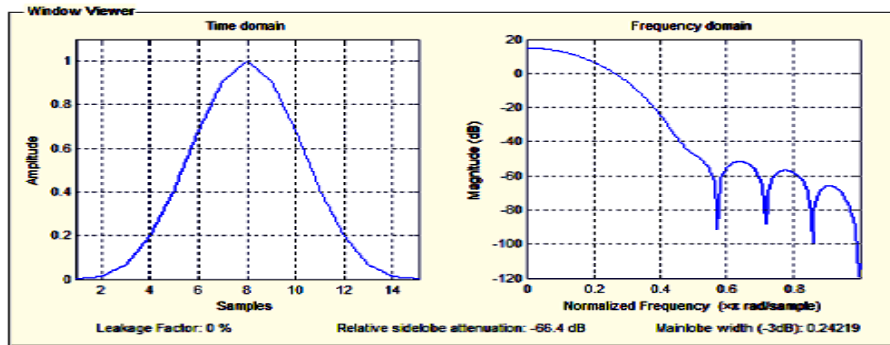


Figure 6. Response of FIR low pass filter using hybrid window with averaging operator

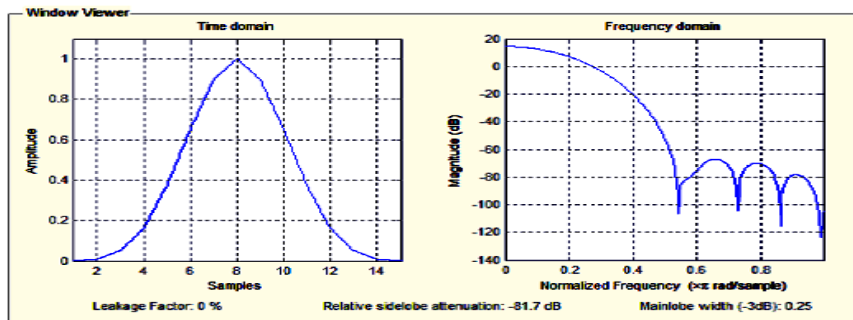


Figure 7. Response of FIR low pass filter using hybrid window with Ex-or operator

Table 3. Relative SLA values of FIR LPF for hybrid windows

Mathematical operation	Relative side lobe attenuation
Addition	-81 dB
Averaging	-81 dB
Multiplication	-104 dB
Ex-Or	-96.5 dB

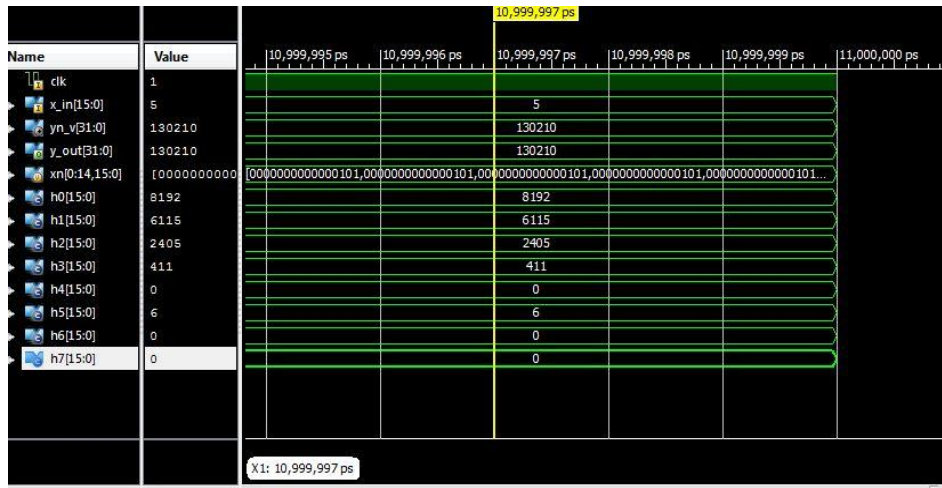


Figure 8. 15 tap FIR LPF output with binary representation

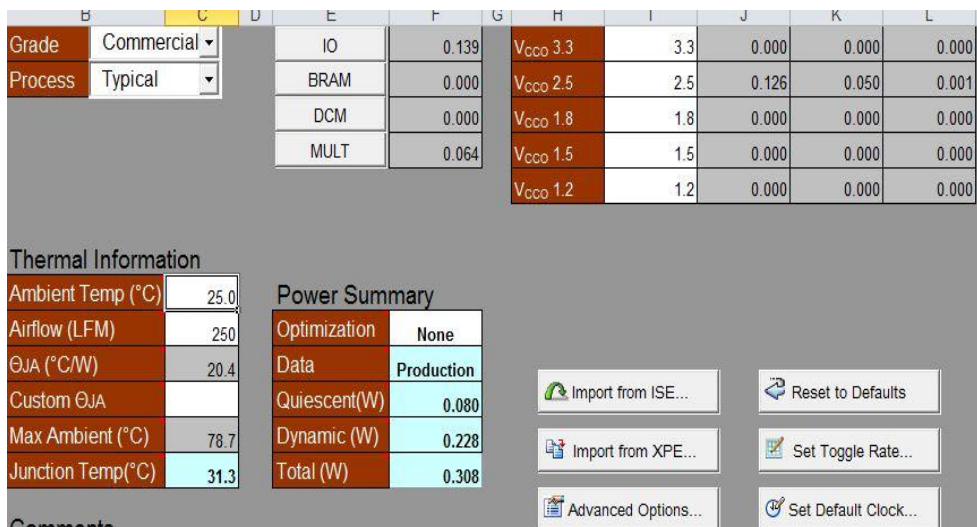


Figure 9. Power report of 15 tap FIR LPF with binary representation

Total 21.487ns (18.076ns logic, 3.411ns route)
(84.1% logic, 15.9% route)

Figure 10. Delay report of 15 tap FIR LPF with binary representation

Device utilization summary:

Selected Device : 3s500efg320-5

Number of Slices:	174	out of	4656	3%
Number of Slice Flip Flops:	48	out of	9312	0%
Number of 4 input LUTs:	327	out of	9312	3%
Number of IOs:	49			
Number of bonded IOBs:	49	out of	232	21%
Number of MULT18X18SIOs:	8	out of	20	40%
Number of GCLKs:	1	out of	24	4%

Figure 11. Synthesis report of 15 tap FIR LPF with binary representation



Figure 12. 15 tap FIR LPF output with CSD representation

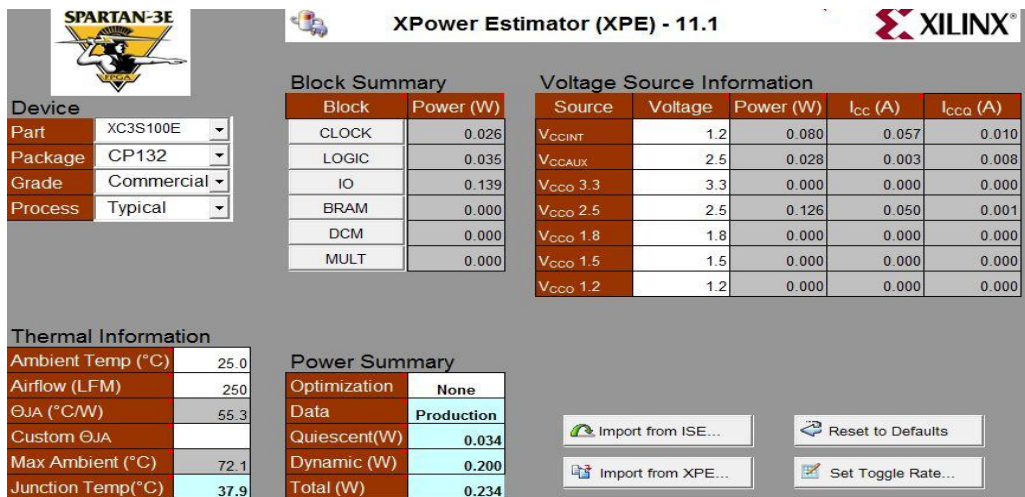


Figure 13. Power report of 15 tap FIR LPF with CSD representation

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Total                               52.765ns (34.841ns logic, 17.924ns route)
                                     (66.0% logic, 34.0% route)
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Figure 14. Delay report of 15 tap FIR LPF with CSD representation

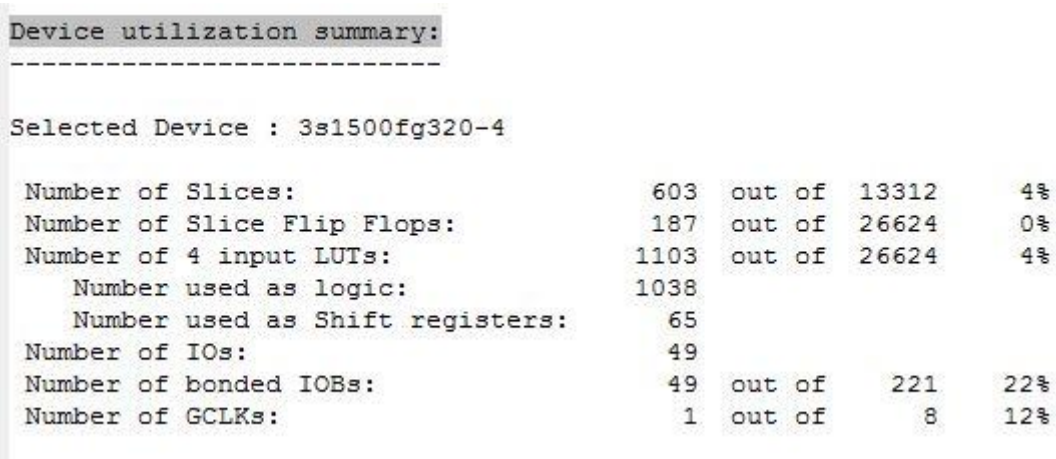


Figure 15. Synthesis report of 15 tap FIR LPF with CSD representation

Table 4.Performancecomparison in terms of power consumption

Method of Filter coefficient representation	Binary	CSD
Power consumption	308 mw	234 mw

V. Conclusion and Future Scope

With the advancement in VLSI technology as the Digital signal processing has become increasingly popular over the years, the high speed realization of FIR filter with less power consumption has become much more demanding. In this work, the FIR LPF is designed using hybrid windows for various mathematical operations like addition, averaging, ex-or and multiplication. The highest side lobe attenuation was obtained and achieved better magnitude response for the multiplication based design of FIR low pass filter. Moreover multiplication based design of filter is realized in direct form structure and the CSD algorithm is applied for the multiplication process and obtained the less power consumption compared to binary representation of filter coefficients. The simulation, synthesis and power reports are analyzed using Xilinx 13.1 ISE and XPower Estimator. In multiplication process more number of partial products will be generated. The future scope of this work is to use any partial products reduction technique to reduce further power consumption.

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